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QUANTUM SUPERMEMBRANES AND ADS/CFT DUALITY

A. A. Tseytlin [a,b,c*]

[a] *Blackett Laboratory, Imperial College*
SW7 2AZ, London, UK

[b] *Institute for Theoretical and Mathematical Physics, Moscow State University*
119991, Moscow, Russia

[c] *Lebedev Physical Institute, Russian Academy of Sciences*
119991, Moscow, Russia

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Dedicated to the memory of Valery Rubakov

Testing AdS/CFT duality [1, 2] is an important direction of research leading to novel non-trivial results on both gauge theory and dual string theory sides. Most of the work so far was done for duality of 4d super Yang-Mills theory and type IIB 10d string theory in $AdS_5 \times S^5$ background. The aim of this paper is to review and extend some recent work [3, 4] on testing AdS/CFT correspondence between supersymmetric $U(N)_k \times U(N)_{-k}$ Chern-Simons-matter 3d gauge theory and M-theory in $\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$ background [5, 6] (ABJM theory).

Membrane theory with the bosonic action dating back to Dirac [7] remains an enigma. While the existence of a consistent quantum theory of bosonic membranes may be in doubt, it may happen to be well defined for the 11d supermembrane or M2 brane theory [8]. The large amount of supersymmetry and possibly some unknown hidden symmetries may lead to its UV finiteness despite formal power-counting nonrenormalizability. This may be true, in particular, for the supermembrane in the maximally supersymmetric $\mathrm{AdS}_4 \times S^7$ or $\mathrm{AdS}_7 \times S^4$ backgrounds [9, 10].

While the M2 brane action is highly non-linear, expanded near a classical solution with non-degenerate induced 3d metric it can be quantized in a static gauge. Then the leading 1-loop result for its partition function is UV finite and thus unambiguous [3, 4, 11, 12].

The M2 brane action in 11d background is related to the type IIA string in the corresponding 10d background by a double dimensional reduction [13]. Considering M2 brane world volume of topology $\Sigma^2 \times S^1$ and expanding 3d fields in Fourier modes in S^1 coordinate one gets an effective 2d string action on Σ^2 coupled to an infinite tower of massive 2d fields. Choosing a static gauge in the M2 brane action one gets a static gauge Nambu-Goto action for the massless transverse string modes coupled to a tower of the massive "Kaluza-Klein" 2d modes. This "effective string" 2d action is essentially equivalent to the original M2 brane action and thus may inherit some of its hidden symmetries.

The work described below provides a remarkable evidence that direct semiclassical quantization of the M2 brane in $\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$ background reproduces the results of large N localization computations [14–16] of the 1/2-BPS Wilson loop and the instanton contributions to free energy in the ABJM gauge theory.

Our first example is the 1/2-BPS Wilson loop. For fixed k, the large N expansion of the Wilson loop operator in the ABJM theory corresponds to the expansion in the large effective M2 brane tension $R^3 T_2 \sim \sqrt{Nk}$, where R is the curvature radius of $\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$ and $T_2 = 1/((2\pi)^2 \ell_P^3)$. The analytic expression for the expectation value of the 1/2-BPS circular Wilson loop in the ABJM theory was derived

[*] E-mail: atseytlin@gmail.com

using supersymmetric localization in [15]. In order to compare to the semiclassical expansion in the M2 brane world-volume theory, one is to expand this expression at large N with fixed k, which gives

$$\langle W_{\frac{1}{2}}\rangle = \frac{1}{2\sin(\frac{2\pi}{k})} e^{\pi\sqrt{\frac{2N}{k}}} \times \times \Big[1 - \frac{\pi\left(k^2+32\right)}{24\sqrt{2}\,k^{3/2}}\frac{1}{\sqrt{N}} + O(\frac{1}{N})\Big]. \quad (1)$$

The Wilson loop has a dual description in terms of an M2 brane wrapped on $\mathrm{AdS}_2\times S^1$ [17] in the M-theory background $\mathrm{AdS}_4\times S^7/\mathbf{Z}_k$. One finds [3] that the exponential factor in (1) is reproduced by the classical action of the M2 brane with $\mathrm{AdS}_2\times S^1$ world-volume, while the k-dependent prefactor $(2\sin(2\pi k))^{-1}$ is matched precisely by the 1-loop correction coming from the functional determinants of the quantum fluctuations around this M2 brane solution.

The non-perturbative part of the ABJM theory free energy found from localization is [16]

$$F^{\mathrm{np}}(N,k) = F_1^{\mathrm{inst}}(N,k)\times \times\Big[1+\frac{\pi}{\sqrt{2k}}\frac{k^2-40}{12k}\frac{1}{\sqrt{N}}+...\Big],$$
$$F_1^{\mathrm{inst}}(N,k) = -\frac{1}{\sin^2(\frac{2\pi}{k})}\,e^{-2\pi\sqrt{\frac{2N}{k}}}\,. \quad (2)$$

Here F_1^{inst} is the leading large N term in the 1-instanton contribution. In the type IIA string theory regime (i.e., for large N and k with $= N/k$=fixed) it may be interpreted as the contribution of the string world-sheet instanton (wrapping CP^1 in CP^3 [18]). In the M-theory regime (i.e. for large N with fixed k), the world-sheet instantons correspond to the M2 brane instantons wrapping the 11d circle and a CP^1 in CP^3, i.e., $S^3/\mathbb{Z}_k \subset S^7/\mathbf{Z}_k$. Computing the corresponding classical M2 brane action and 1-loop fluctuation determinants [4] one matches precisely the exponential and its prefactor in (2).

As another example of the quantum M2 brane computation in $\mathrm{AdS}_4\times S^7/\mathbf{Z}_k$ one may consider the 1-loop correction to the partition function expanded near the classical M2 brane solution generalizing infinitely long rotating folded string [19, 20] in AdS_4. It determines leading large λ corrections at each order in $1/N$ to the cusp anomaly function in ABJM theory [21]. The predicted generalization of the planar cusp anomaly in ABJM theory [22–24] to include leading at large terms at each order in $1/N$ reads

$$\hat{f}(k) = c_0 + f(k) = -\frac{5}{2\pi}\log 2 + \frac{2\pi}{3k^2} + \frac{2\pi^3}{45k^4} + \ldots = = -\frac{5}{2\pi}\log 2 + \frac{2\pi\lambda^2}{3N^2} + \frac{2\pi^3\lambda^4}{45N^4} + \ldots$$

From the point of view of the 't Hooft limit of large N and large $k = N/\lambda$ these corrections are non-planar corrections to the cusp anomaly. This result is a non-trivial strong-coupling prediction about non-planar correction to the anomalous dimension of a high-spin operator in the ABJM theory. Being non-planar it is unlikely to be captured by integrability methods. Also, as the cusp anomaly corresponds to a non-supersymmetric observable (anomalous dimension of a non-BPS operator) it is unlikely to be captured by a localization computation.

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