JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS, 2024, Vol. 165, No. 5, pp. 733–750

ELECTRODYNAMICS OF PLASMA SOLENOID AND ELECTROMAGNETIC PROPERTIES OF INDUCTIVE DISCHARGE

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Received November 20, 2023
Revised January 17, 2024
Accepted January 18, 2024

Abstract. The electrodynamic properties of a plasma solenoid with cold collisional magnetoactive plasma and the dynamics of wave excitation by azimuthal current on its surface have been studied at arbitrary ratiosarbitrary ratios between the external current source frequency, electron cyclotron frequency, and plasma frequency. Cases of unbounded and longitudinally bounded plasma solenoids have been considered. Their complex impedances and effective resistances as quantities characterizing the power absorbed in the plasma source have been calculated. It is shown that despite the limitation of the complex impedance concept to the quasi-stationary case, its real part coincides with the effective resistance even beyond the quasistationarity condition. The resonant dependencies of the calculated complex impedances and effective plasma resistances indicate that in the presence of an external magnetic field, resonant excitation of electromagnetic waves by azimuthal current with a significant longitudinal component of the electric field strength is possible in the plasma solenoid at frequencies lower than cyclotron and plasma frequencies.

DOI: 10.31857/S004445102405e122

1. INTRODUCTION

For several decades, inductive radio-frequency discharges and plasma sources based on them have been actively discussed in scientific literature (see reviews [1-4]). Unlike traditional capacitive and inductive radio-frequency discharges without an external magnetic field, its presence leads to a significant change in the electromagnetic properties of plasma in the discharge. In particular, it becomes possible to excite waves penetrating into the plasma in the low-frequency part of the spectrum with frequencies lower than the electron cyclotron frequency. Plasma sources based on inductive radio-frequency discharge in an external magnetic field have significantly fewer limitations on plasma density associated with reaching its critical value for a given generator frequency, that is, they are sources of dense plasma. Real electron concentrations reach values up to 10^{13} cm^{-3} [1].

Unique characteristics predetermine the wide practical applications of radio-frequency inductive plasma sources in an external magnetic field. One of their most important applications is their use as ion engines for spacecraft. In such sources, plasma is created by inputting radio-frequency power followed by electrostatic acceleration of ions. Plasma sources of various sizes (diameters 0.5–74 cm) [5] are considered as ion engines, including small ones (vacuum chamber diameter of several centimeters) [6, 7] and ultra-small ones (diameters 0.2–2 cm) [8]. The external magnetic field is created both using a traditional solenoid and using a system of permanent magnets. In particular, in [6, 7] to simplify and miniaturize the design, the magnetic field is created by a permanent ring magnet. Another application of radio-frequency inductive plasma sources is their use in microelectronics for plasma coating deposition and material etching [9]. Additionally, the absorption of electromagnetic waves with frequencies up to the electron cyclotron frequency is of significant importance as one of the plasma heating methods in controlled thermonuclear fusion facilities, particularly in ITER and DEMO projects [10,11]. Wave absorption in hot thermonuclear plasma occurs mainly due to collisionless Landau damping. The plasma parameters and external confining magnetic field differ significantly from those of traditional radio-frequency discharge. At electron concentrations of the order of 10^{14} cm^{-3} and magnetic field induction in the tokamak of 5 T, the electron Langmuir and cyclotron frequencies are of the same order and constitute $(5\ldots10)\times10^{11}$ rad/s, corresponding to the microwave region. This circumstance makes it relevant to advance existing theories of inductive radio-frequency discharges into the shorter wavelength region — up to the microwave range.

In the case of a generator frequency significantly lower than the electron cyclotron frequency, when power is deposited into dense plasma, these are referred to as helicon discharges. The study of helicon discharges and plasma sources based on them began with Boswell's works [12, 13]. The first of these works showed that in a tube placed in an external magnetic field of up to 1.5 kG, a discharge was ignited at a frequency of 8 MHz, and the power deposition source into the plasma was an excited standing wave of helicon type. In the theoretical description of the helicon discharge, the possibility of two radial propagating modes was indicated, one of which is associated with the helicon, while the other is currently commonly called the Trivelpiece-Gould mode or oblique Langmuir wave [14-17]. The work [18] examined the mechanisms of radio-frequency power deposition into unbounded magnetoactive plasma during the realization, depending on discharge parameters, of limiting cases of excited wave with dispersion laws corresponding to helicon wave and oblique Langmuir wave (Trivelpiece-Gould mode). In this work, we investigate the electromagnetic properties of a finite-length inductive discharge in an external magnetic field at various ratios between the exciting generator frequency, electron cyclotron frequency, and plasma frequency, which allows describing the electrodynamics of the plasma solenoid not only in the traditional radio-frequency range, but also for systems using microwave fields [19-23].

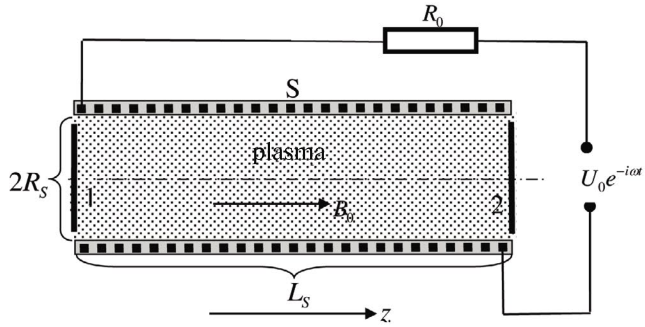

Fig. 1. Plasma solenoid

Typical plasma sources represent a vacuum chamber with gas pressure of units and tens of millitorr, having a diameter from several millimeters to tens of centimeters and length up to values exceeding a meter. The discharge is maintained by a system of currents flowing through an antenna, which can have various configurations. In this work, we will focus on the discharge parameters from works [24-26].

2. PROBLEM STATEMENT. BASIC EQUATIONS AND RESEARCH METHODS

Let's consider a solenoid S, completely filled with a homogeneous plasma (Fig. 1). The solenoid (inductor) represents a section of a cylinder with radius R_S, length L_S with a thin wire winding containing N_S turns. The solenoid winding is connected through resistance R_0 to a frequency source ω. There is also a uniform external magnetic field B_0, directed along the solenoid axis (axis z). The solenoid ends can be closed with conducting plugs "1" and "2". The described system represents a simplified scheme of inductive gas discharge [27].

Let's introduce cylindrical coordinates r,φ,z and consider only azimuthally symmetric distributions of the electromagnetic field in the solenoid ($\partial/\partial\varphi = 0$). Let's assume that the plasma dielectric permittivity tensor in the cylindrical coordinate system has the form ($i,j = r,\varphi,z$)

$$\varepsilon_{ij}(\omega) = \begin{pmatrix} \varepsilon_{rr}(\omega) & \varepsilon_{r\varphi}(\omega) & 0 \\ \varepsilon_{\varphi r}(\omega) & \varepsilon_{\varphi\varphi}(\omega) & 0 \\ 0 & 0 & \varepsilon_{zz}(\omega) \end{pmatrix}. \quad (1)$$

In the simplest model of cold electron magnetoactive plasma with collisions, the components of tensor (1) are determined by formulas [28]

$$\varepsilon_{rr}(\omega) = \varepsilon_{\varphi\varphi}(\omega) = \varepsilon_{\perp}(\omega) = 1 - \frac{\omega_{Le}^2(\omega + i\nu_e)}{\omega\left[(\omega + i\nu_e)^2 - \Omega_e^2\right]},$$
$$\varepsilon_{r\varphi}(\omega) = -\varepsilon_{\varphi r}(\omega) = ig(\omega) = -i\frac{\omega_{Le}^2\Omega_e}{\omega\left[(\omega + i\nu_e)^2 - \Omega_e^2\right]},$$
$$\varepsilon_{zz}(\omega) = \varepsilon_{\parallel}(\omega) = 1 - \frac{\omega_{Le}^2}{\omega(\omega + i\nu_e)}. \tag{2}$$

Here ω_{Le} — is the electron Langmuir frequency, ν_e — is the effective electron collision frequency, Ω_e — is the electron cyclotron frequency.

For plasma with a dielectric permittivity tensor (1) in the case of azimuthally symmetric monochromatic perturbations of frequency ω after transition to complex amplitudes, Maxwell's equations system is written as

$$\begin{gathered}
\frac{\partial E_\varphi}{\partial z} = -i\frac{\omega}{c}B_r, \\
\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = i\frac{\omega}{c}B_\varphi, \\
\frac{1}{r}\frac{\partial}{\partial r}\left(rE_\varphi\right) = i\frac{\omega}{c}B_z, \\
\frac{\partial B_\varphi}{\partial z} = i\frac{\omega}{c}\varepsilon_{rr}(\omega)E_r + i\frac{\omega}{c}\varepsilon_{r\varphi}(\omega)E_\varphi - \\
-\frac{4\pi}{c}j_{0r}(z,r), \\
\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = -i\frac{\omega}{c}\varepsilon_{\varphi r}(\omega)E_r - i\frac{\omega}{c}\varepsilon_{\varphi\varphi}(\omega)E_\varphi + \\
+\frac{4\pi}{c}j_{0\varphi}(z,r), \\
\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\varphi\right) = -i\frac{\omega}{c}\varepsilon_{zz}(\omega)E_z + \frac{4\pi}{c}j_{0z}(z,r), \\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\varepsilon_{rr}E_r\right) + \frac{\partial}{\partial z}(\varepsilon_{zz}E_z) + \frac{1}{r}\frac{\partial}{\partial r}(r\varepsilon_{r\varphi}E_\varphi) = \\
= 4\pi\rho_0(z,r).
\end{gathered} \tag{3}$$

Here $\mathbf{j}_0(z,r) = \{j_{0r}, j_{0\varphi}, j_{0z}\}$ and $\rho_0(z,r)$ — are complex amplitudes of current density and charge density of external field sources in plasma. In our case, the external source is a solenoid winding through which current flows from the source, and the charge density $\rho_0(z,r)$ equals zero. However, if plugs "1" and "2" were connected to some source, the charge density $\rho_0(z,r)$ would be caused by surface charges on these plugs, as on capacitor plates. This would involve a combined gas discharge – inductive-capacitive. We will limit ourselves to the case $\rho_0(z,r) = 0$. The resonant properties of capacitive discharge in a transverse magnetic field are considered in [29]. Since the current in the solenoid is predominantly azimuthal, we will assume that $j_{0r} = j_{0z} \equiv 0$.

Excluding quantities B_z, B_r, B_φ and considering formulas (2), we obtain from (3) the following equations for complex amplitudes of electric field intensity vector components E_z, E_r, E_φ:

$$\begin{gathered}
\frac{\partial}{\partial z}\left(\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}\right) + \frac{\omega^2}{c^2}\varepsilon_\perp E_r + i\frac{\omega^2}{c^2}gE_\varphi = 0, \\
\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}rE_\varphi\right) + \frac{\partial^2 E_\varphi}{\partial z^2} + \frac{\omega^2}{c^2}\varepsilon_\perp E_\varphi - i\frac{\omega^2}{c^2}gE_r = \\
= -i\frac{\omega}{c}\frac{4\pi}{c}j_{0\varphi}(z,r), \\
\frac{1}{r}\frac{\partial}{\partial r}r\left(\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z}\right) + \frac{\omega^2}{c^2}\varepsilon_\parallel E_z = 0.
\end{gathered} \tag{4}$$

Let us now discuss the boundary conditions for equations (4). Suppose that the solenoid winding is localized on the cylindrical surface $r = R_S$, i.e., the solenoid current is a surface current. In this case, the external source current density can be defined using the delta function

$$j_{0\varphi}(z,r) = J(z)\delta(r - R_S), \qquad 0 < z < L_S, \tag{5}$$

where $J(z)$ — is some function of only the longitudinal coordinate z. Let's take the second equation of system (4), substitute function (5) into it and integrate the equation over within the limits from $R_S - h$ to $R_S + h$ ($h \to +0$). As a result, we get the relation

$$\frac{\partial E_\varphi}{\partial r}(R_S + 0) - \frac{\partial E_\varphi}{\partial r}(R_S - 0) = -i\frac{\omega}{c}\frac{4\pi}{c}J(z). \tag{6}$$

As seen from the third and fifth equations of system (3), relation (6) is caused by the jump of the tangential component of magnetic field induction $B_z(r)$ on the current surface (5). In addition to (6), we will also need conditions for continuity of tangential components of electric field intensity

$$E_z(R_S + 0) - E_z(R_S - 0) = 0,$$
$$E_\varphi(R_S + 0) - E_\varphi(R_S - 0) = 0. \quad (7)$$

Relations (6) and (7) constitute the main boundary conditions for homogeneous equations (4)[1]. Boundary conditions for variable z will be discussed further in the course of presentation.

One of the complex aspects is determining the function $J(z)$. There is no complexity only in the quasi-stationary approximation, within which it is assumed that the instantaneous values of current in each point of the electrical circuit, including in each turn of the solenoid winding, are identical. Assuming this is true, we can write

$$\int_0^{L_S} dz \int_{R_S-h}^{R_S+h} dr j_{0\varphi}(z,r) = N_S I. \quad (8)$$

Here I — is the current in the circuit, and $2h$ — is the thickness of the solenoid winding, i.e., the integration in (8) is performed over the longitudinal cross-section of the solenoid winding. In the solenoid model with surface current (5), it is assumed that $h \to +0$. Substituting (5) into (8), we have

$$I = \frac{1}{N_S}\int_0^{L_S} J(z)dz. \quad (9)$$

If the density of the number of solenoid winding turns is constant and the number of turns is sufficiently large, then the function $J(z)$ can be represented as

$$J(z) = \frac{N_S I}{L_S} = \text{const}. \quad (10)$$

In practice, a situation where the density of solenoid turns is not constant[2] is common. In this case, the function $J(z)$ is considered known, for example, from experimental conditions. In any case, such a function must satisfy the normalization condition (9).

The applicability conditions of formula (8)

$$\frac{\omega}{c} R_S, \frac{\omega}{c} L_S \ll 1 \quad (11)$$

mean that the electromagnetic wavelength of frequency ω is large compared to the dimensions of the solenoid[3]. Under these conditions, the plasma solenoid, regardless of how complex the electromagnetic processes occurring in the plasma might be, acts as a lumped element for the electrical circuit, like ordinary inductance coils or capacitors. Therefore, in the quasi-stationary approximation for the electrical circuit, the plasma solenoid is characterized by a single parameter – the complex impedance Z_S.

To calculate the impedance Z_S let's consider that in a circuit with a solenoid, an induced EMF $\mathcal{E}_S$, acts, being "connected" in series with the external source. Therefore, Ohm's law for the complete circuit, shown in Fig. 1, for complex amplitudes takes the form $U_0 + \mathcal{E}_S = IR_0$. The induced EMF itself, by definition, equals the integral along the solenoid winding of the electric field strength component $E_\varphi(z,r)$. Considering that the length of the solenoid winding equals $2\pi R_S N_S$, and introducing the average value of the azimuthal component of the electric field strength along the solenoid length

$$\langle E_\varphi(z, R_S)\rangle = \frac{1}{L_S}\int_0^{L_S} E_\varphi(z, R_S)dz, \quad (12)$$

we can represent the induced EMF as

$$\mathcal{E}_S = 2\pi R_S N_S \langle E_\varphi(z, R_S)\rangle. \quad (13)$$

Substituting expression (13) into Ohm's law, we write the following relation:

$$U_0 = IR_0 - 2\pi R_S N_S \langle E_\varphi(z, R_S)\rangle, \quad (14)$$

which can be used when necessary as an additional condition for equations (4). Relations of type (14) are known in literature as external circuit equations [30].

To obtain the formula for the plasma solenoid impedance, we consider that the external source voltage is distributed between the voltage drop across the resistance R_0 and the voltage drop across the plasma solenoid, i.e. $U_0 = IR_0 + IZ_S$. Comparing the latter relation with expression (14), for impedance we have

$$Z_S = -2\pi R_S N_S \langle E_\varphi(z, R_S)\rangle / I. \quad (15)$$

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[1] Inside the solenoid at $r < R_S$ and outside the solenoid at $r > R_S$ the right side of the second equation of system (4) according to (5) equals zero.

[2] In real experiments on inductive gas discharge, solenoids of very complex shapes are often used. Therefore, they are not even called solenoids, but antennas[1, 3].

[3] The quasi-stationarity condition of the electrical circuit has the form (11) with the replacement of solenoid dimensions by the size of the entire circuit as a whole.

Due to boundary condition (6) and formula (9), the electric field strength in the solenoid is proportional to the current in the circuit. Consequently, value (15) does not depend on current but is determined only by the solenoid – its geometry and plasma parameters. Using the general relation $Z_S = -i\omega\Lambda_S$ we can also calculate the complex inductance of the plasma solenoid Λ_S.

Using impedance, the resonant properties of a plasma solenoid can be investigated, and the question of power dissipation from an external source in the plasma solenoid can be examined, which is of paramount importance for gas discharge physics. For the simplest series circuit (voltage source U_0 connected through active resistance R_0 to the plasma solenoid winding, Fig. 1), the power of the external source dissipated in the plasma is determined by the formula [31]

$$W = \frac{1}{2}|U_0|^2 \frac{Z'_S}{(R_0 + Z'_S)^2 + (Z''_S)^2}, \quad (16)$$

where $Z_S = Z'_S + iZ''_S$[4]. As a function of complex variable ω, complex impedance $Z_S(\omega)$ poles in the vicinity of which

$$Z_S = iA(\omega - \omega^{(\infty)})^{-1}, \quad (17)$$

and zeros near which

$$Z_S = iB(\omega - \omega^{(0)}), \quad (18)$$

where A and B — are constants, and $\omega^{(\infty)}$ and $\omega^{(0)}$ — are certain complex frequencies [31]. In the case of high circuit resistance, $R_0 \gg |Z_S|$, the power dissipated in the plasma is given by the formula

$$W = \frac{|U_0|^2}{2R_0^2} Z'_S. \quad (19)$$

As a function of frequency ω value (19) is maximum at the maxima of the real part of impedance, i.e., at points $\omega = \mathrm{Re}\,\omega^{(\infty)}$. It is known that at $\omega = \omega^{(\infty)}$ current resonance occurs in the circuit: impedance is maximum, and current in the circuit is minimum. With low resistance R_0 for power (16) we have

$$W = \frac{1}{2}|U_0|^2 \frac{Z'_S}{(Z'_S)^2 + (Z''_S)^2}. \quad (20)$$

Power (20) is maximum at zeros of the imaginary part of impedance $\omega = \mathrm{Re}\,\omega^{(0)}$, which occurs during voltage resonance – impedance is minimum, and current in the circuit is maximum[5]. Note that experimental observation of current and voltage resonances in gas discharges is based, among other things, on formulas (19) and (20).

The applicability range of formulas (16)-(20), like the applicability range of the impedance concept itself, is limited by the quasi-stationary condition of the electrical circuit [32]. When increasing the external source frequency ω, when the quasi-stationary condition is violated, electrodynamic methods should be used to calculate the power dissipated in the solenoid plasma. We start from the power density formula $W = \langle \mathbf{j}\cdot\mathbf{E}\rangle$, where $\mathbf{E}$ — is the electric field strength vector, $\mathbf{j}$ — is the current density vector induced in the plasma, and angle brackets denote averaging over period $2\pi/\omega$. Expressing current density through field strength and dielectric permittivity [33]

$$j_i = \frac{\omega}{4\pi i}(\varepsilon_{ij} - \delta_{ij})E_j, \quad (21)$$

after simple calculations for power density we have

$$W(z,r) = \frac{\omega}{8\pi}[\varepsilon''_{\perp}(|E_r(z,r)|^2 + |E_\varphi(z,r)|^2) + \\ +\varepsilon''_{||}|E_z(z,r)|^2 - 2g''\mathrm{Im}(E_r^*(z,r)E_\varphi(z,r))], \quad (22)$$

where the index "double prime" denotes the imaginary part. Performing integration in (22) over z from zero to L_S and over r with weight $2\pi r$ from zero to R_S, one can calculate the total power of the external source dissipated in the solenoid plasma $W(\omega)$. In general case, this can only be done numerically.

The electric field strength components included in (22) are calculated from the homogeneous system of equations (4) with boundary conditions (6) and (7). In this regard, the question of determining the function $J(z)$, arises again, but now beyond the applicability of the quasistationary

[4] The elementary information from electrical engineering presented here applies not only to the plasma solenoid but also to any other element of the electrical circuit.

[5] Historically, the terms current and voltage resonances emerged in connection with the study of parallel and series oscillatory circuits consisting of active resistance, capacitance, and inductance.

approximation, when the analysis of the system shown in Fig. 1 by electrical engineering methods is inadmissible. In this case, one should solve the complete electrodynamic problem for the solenoid, supply wires, voltage source, and even part of the surrounding space, which is hardly feasible. Our goal is to obtain compact, physically clear results useful for experimental research. Regardless of the frequency of the signal steadily supplied to the solenoid antenna, some current distribution $J(z)$ is established in it. By setting it from some reasonable physical considerations and determining the components of the electric field strength vector from system (4), one can calculate the energy dissipated in the solenoid plasma using formula (22) and, most importantly, investigate the resonant properties of the plasma-filled solenoid, which will be done further. Within the applicability limits of formula (9), the power dissipated in the plasma $W(\omega)$, can be represented as $W(\omega) = R_{eff} I^2 / 2$, where R_{eff} — is the effective plasma resistance. In the quasi-stationary frequency range, the effective resistance should, of course, coincide with the real part of the impedance.

3. LONG PLASMA SOLENOID WITH CONSTANT DENSITY OF WINDING TURNS

Let us assume that the inequality is satisfied

$$R_S \ll L_S, \tag{23}$$

i.e., the length of the solenoid significantly exceeds its radius. In this case, it is reasonable to assume that the processes near the longitudinal boundaries of the solenoid $z = 0, L_S$ (edge effects) have little effect on what occurs in its volume. Therefore, we can consider that the longitudinal distribution of the electromagnetic field in the solenoid is determined not by conditions at the longitudinal boundaries, but by other factors. Due to the linearity of equations (4) and boundary conditions (6) and (7), the only such factor is the density of the solenoid winding turns, i.e., the function $J(z)$. Let's start with the simplest case of constant current density (10).

Considering the electromagnetic field in the solenoid independent of coordinate , in equations (4) we assume $\partial / \partial z = 0$. In this case, the third equation of system (4) becomes independent of the first two equations. As a result, the field componentE_z, which is also not included in the boundary condition (6), becomes independent of componentsE_r and E_φ, and therefore we can set $E_z = 0$. Then from the first two equations of system (4), after eliminating E_r, the following equation is obtained:

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}rE_\varphi\right) + \frac{\omega^2}{c^2}\tilde{\varepsilon}_\perp E_\varphi = 0, \quad \tilde{\varepsilon}_\perp = \varepsilon_\perp - \frac{g^2}{\varepsilon_\perp}, \tag{24}$$

valid both inside the solenoid ($r < R_S$), and outside it ($r > R_S$); outside the solenoid $\tilde{\varepsilon}_\perp = 1$. The boundary conditions for equation (24), taking into account (6), (7) and (10), have the form

$$\begin{gathered} E_\varphi(R_S + 0) - E_\varphi(R_S - 0) = 0, \\ \frac{dE_\varphi}{dr}(R_S + 0) - \frac{dE_\varphi}{dr}(R_S - 0) = -i\frac{\omega}{c}\frac{4\pi}{c}\frac{N_S}{L_S}I. \end{gathered} \tag{25}$$

Additionally, the function $E_\varphi(r)$ must be bounded at zero and at infinity.

For $r < R_S$ the limited solution of equation (24) has the form $E_\varphi(r) = A_1 J_1\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp} r\right]$, where $J_1(x)$ — is the first-order Bessel function. In the region $r > R_S$ the solution should be taken as follows: $E_\varphi(r) = A_2 H_1^{(1)}\left[(\omega / c) r\right]$, where $H_1^{(1)}(x)$ — is the Hankel function of the first kind. By writing the solution through the Hankel function of the first kind, we used the causality principle (radiation condition [34]), according to which at $\omega = \omega + i\delta$ when $\delta \to +0$, the field should exponentially decay at $r \to \infty$ ($\sim \exp(-\delta r / c)$). Substituting the found solutions into the boundary conditions (25) and determining the constants $A_{1,2}$, we find the electric field in the solenoid volume ($r \le R_S$)

$$E_\varphi(r) = i\frac{4\pi}{c}\frac{N_S}{L_S} I \frac{J_1\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp} r\right] H_1^{(1)}\left[(\omega / c) R_S\right]}{\sqrt{\tilde{\varepsilon}_\perp} J_0\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp} R_S\right] H_1^{(1)}\left[(\omega / c) R_S\right] - J_1\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp} R_S\right] H_0^{(1)}\left[(\omega / c) R_S\right]}. \tag{26}$$

In the quasi-stationary approximation, considering the first inequality (11) and the asymptotics of cylindrical functions at small argument values, solution (26) transforms to

$$E_\varphi(r) = i\frac{4\pi}{c}\frac{N_S}{L_S}I\frac{J_1\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp}r\right]}{\sqrt{\tilde{\varepsilon}_\perp}J_0\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp}R_S\right]}. \quad (27)$$

Solution (27) can also be obtained directly from equation (24). Indeed, in the quasi-stationary approximation in equation (24) in the region $r > R_S$ the second term in the left part should be discarded. The solution bounded at infinity of the resulting equation has the form $E_\varphi(r) = A_2 / r$. Its substitution into the boundary conditions (25) again leads to formula (27). Note that even in the quasi-stationary approximation in the left part of equation (24) we do not discard the second term proportional to $\tilde{\varepsilon}_\perp(\omega^2 / c^2)R_S^2$. For example, at $\omega \approx \Omega_e$ this term is significant even in the quasi-stationary approximation. And in general, if this term is discarded, nothing would remain of the plasma.

There is an important difference between solutions (26) and (27). Solution (26) takes into account the electromagnetic radiation output from the solenoid through its lateral surface $r = R_S$. The quasi-stationary solution (27) naturally does not account for such radiation. But in the radio-frequency region $\omega > c / R_S$ radiation from the solenoid may play a significant role in the overall energy balance, as an additional channel for external source energy consumption. To exclude radiation through the lateral surface of the solenoid, we can assume that the solenoid is enclosed in a conducting cylindrical shell of radius $r = \mathcal{R} > R_S$. Then, supplementing the boundary value problem (24), (25) with the condition $E_\varphi(\mathcal{R}) = 0$ and finding its solution, it is not difficult to obtain for $E_\varphi(r)$ an expression that differs from (26) only by replacing the Hankel functions $H_{0,1}^{(1)}[(\omega / c)R_S]$ with functions $X_{0,1}[(\omega / c)R_S] = J_{0,1}[(\omega / c)R_S]N_1[(\omega / c)\mathcal{R}] - N_{0,1}[(\omega / c)R_S]J_1[(\omega / c)\mathcal{R}]\cdot$

Now let's calculate the impedance of the solenoid under consideration. For this, we will use the result of the quasi-stationary approximation (27), which when substituted into formula (15), gives

$$Z_S(\omega) = Z_S'(\omega) + iZ_S''(\omega) = \\ = -i\omega\Lambda_0\frac{2J_1\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp}R_S\right]}{(\omega / c)\sqrt{\tilde{\varepsilon}_\perp}R_S J_0\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp}R_S\right]}, \quad (28)$$

where $\Lambda_0 = 4\pi^2 R_S^2 N_S^2 c^{-2} L_S^{-1}$ — is the inductance of the solenoid without plasma filling. Let's also provide the formula for the impedance of a plasma solenoid enclosed in a conductive shield

$$Z_S(\omega) = -i\omega\Lambda_0\frac{2\left[(\omega / c)R_S\right]^{-1}J_1\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp}R_S\right]X_1\left[(\omega / c)R_S\right]}{\sqrt{\tilde{\varepsilon}_\perp}J_0\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp}R_S\right]X_1\left[(\omega / c)R_S\right] - J_1\left[(\omega / c)\sqrt{\tilde{\varepsilon}_\perp}R_S\right]X_0\left[(\omega / c)R_S\right]}. \quad (28a)$$

The inductance of the plasma solenoid can be determined by formula $\Lambda_S(\omega) = -Z_S''(\omega) / \omega$, and its active resistance equals the real part of the impedance $Z_S'(\omega)$.

Let's analyze expression (28) for plasma with dielectric permittivity (1), (2). If there are no collisions, then the active resistance of the plasma solenoid equals zero, and the expression for its inductance becomes:

$$\Lambda_S(\omega) = \Lambda_0\frac{2J_1(a)}{aJ_0(a)}, \quad a^2 = \frac{(\omega^2 - \omega_{Le}^2)^2 - \omega^2\Omega_e^2}{\omega^2 - \Omega_g^2}\frac{R_S^2}{c^2}, \quad (29)$$

where $\Omega_g = \sqrt{\omega_{Le}^2 + \Omega_e^2}$ — is the upper hybrid frequency. Let's consider the limiting cases. In the absence of plasma, i.e., when $\omega_{Le} = 0$, considering the quasi-stationary condition (11) for inductance (29), we have, as expected, $\Lambda_S(\omega) = \Lambda_0$. In the absence of external magnetic field ($\Omega_e = 0$) formula (29) transforms to

$$\Lambda_S(\omega) = \Lambda_0\frac{2J_1\left(\sqrt{\omega^2 - \omega_{Le}^2}R_S / c\right)}{\left(\sqrt{\omega^2 - \omega_{Le}^2}R_S / c\right)J_0\left(\sqrt{\omega^2 - \omega_{Le}^2}R_S / c\right)}. \quad (30)$$

In the quasi-stationary frequency range, the inductance (30) has neither zeros nor poles. At $\omega \to \omega_{Le}$ inductance (30) approaches the inductance of the solenoid without plasma filling Λ_0. In the case of $\omega < \omega_{Le}$ it is convenient to write formula (30) as follows:

$$\Lambda_S(\omega) = \Lambda_0 \frac{2I_1\left(\sqrt{\omega_{Le}^2 - \omega^2}R_S / c\right)}{\left(\sqrt{\omega_{Le}^2 - \omega^2}R_S / c\right)I_0\left(\sqrt{\omega_{Le}^2 - \omega^2}R_S / c\right)}. \quad (31)$$

Inductance (31) is less than Λ_0, which is related to the screening of the low-frequency transverse field in plasma. When $\omega_{Le}R_S / c \gg 1$ and $\omega \ll \omega_{Le}$ (dense plasma case) from (31) we have $\Lambda_S(\omega) = 2c\Lambda_0 / (\omega_{Le}R_S) \ll \Lambda_0$. In the limit of strong external magnetic field ($\Omega_e \to \infty$) from (29) we have $\Lambda_S(\omega) = \Lambda_0$. The latter is understandable: in a strong magnetic field, transverse electron motions are prohibited, which for the azimuthal electric field is equivalent to the absence of plasma. Thus, in the limiting cases of zero and very strong external magnetic field in the quasi-stationary frequency range, the inductance of the plasma solenoid does not have any resonant features.

At intermediate values of cyclotron and Langmuir frequencies, zeros and poles of inductance (29) and impedance (28) can fall into the low-frequency (quasi-stationary) region. From formula (28), it follows that in the absence of collisions, zeros and poles of the plasma solenoid impedance coincide with the roots of the equation

$$\frac{(\omega^2 - \omega_{Le}^2)^2 - \omega^2\Omega_e^2}{\omega^2 - \Omega_g^2} = \mu_n^2 \frac{c^2}{R_S^2}, \quad \mu_n = \begin{cases} \mu_{1n}, \\ \mu_{0n}. \end{cases} \quad (32)$$

At $\mu_n = \mu_{1n}$ $(n = 1,2,\ldots)$ from (32), zeros are determined $\omega = \omega_n^{(0)}$, and at $\mu_n = \mu_{0n}$ equation (32) gives poles $\omega = \omega_n^{(\infty)}$ of impedance (28). Here μ_{1n} — are roots of equation $J_1(x) = 0$, and μ_{0n} — are roots of equation $J_0(x) = 0$. It is easy to see that the roots of equation (32) fall into our region of interest, the quasi-stationary frequency region, only at $\Omega_e R_S / c < 1$ and $\omega_{Le}R_S / c < 1$. Otherwise, there are no resonant points for impedance (28) in the quasistationary frequency region.

Using the first inequality (11), it is easy to show that in our region of interest for frequencies and plasma parameters, the roots of equation (32) are determined by the following approximate formula::

$$\omega_n^{(0,\infty)} \approx \Omega_g \sqrt{\left(1 + \frac{\Omega_e^2}{\Omega_g^2}\frac{\Omega_e^2 R_S^2}{c^2\mu_n^2}\right)\left(1 + \frac{\Omega_e^2 R_S^2}{c^2\mu_n^2}\right)^{-1}} \le \Omega_g. \quad (33)$$

In obtaining (33), the fact was used that μ_n^2 are sufficiently large values (not less than $\mu_{01}^2 \approx 5.8$). At $\Omega_e \to 0$ all roots (33) become equal to ω_{Le}.

Points determined by formulas (33) are located quite close to each other. Therefore, even with small dissipation, adjacent zeros $\omega_n^{(0)}$ and poles $\omega_n^{(\infty)}$ become indistinguishable. Resonant absorption of external source energy in plasma still exists (despite the merging of resonances), occurring at frequency ω, close to the upper hybrid frequency Ω_g. The fact that resonant absorption should occur precisely near Ω_g is already evident from formula (33), since for all large n we have $\omega_n^{(0,\infty)} \approx \Omega_g$.

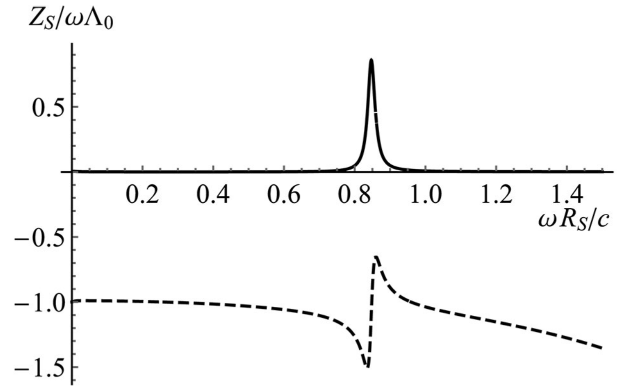

Fig. 2. Complex impedance of a homogeneous plasma solenoid without a shell: solid line – real part, dashed line – imaginary part

Fig. 2 shows the real (solid line) and imaginary (dashed line) parts of the impedance of a homogeneous plasma solenoid without a shell as a function of external source frequency at $\omega_{Le}R_S / c = 0.5$, $\Omega_e R_S / c = 0.7$ and $\nu_e = 0.03\omega_{Le}$. Significant absorption and perturbation of the imaginary part of the impedance are observed near frequencies (33), i.e., around Ω_g, which is associated with resonant excitation of electromagnetic oscillations of B-type in plasma. The dashed curve effectively represents the inductance of the plasma solenoid and, except for a narrow region near frequencies (33), the inductance of the plasma solenoid is close to the vacuum value Λ_0. Note that

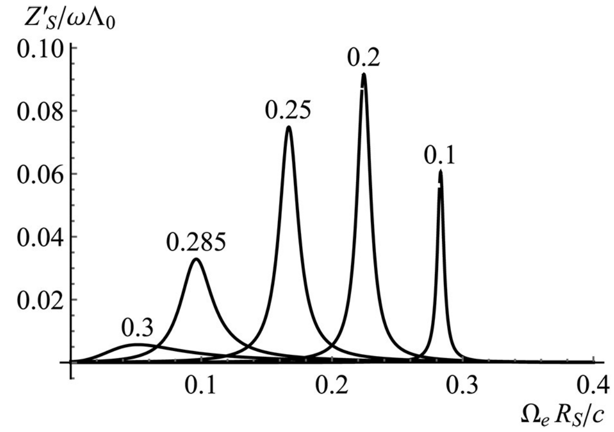

Fig. 3. Real part of impedance of a homogeneous plasma solenoid. Numbers near curves correspond to the value $\omega_{Le} R_S \ / \ c$

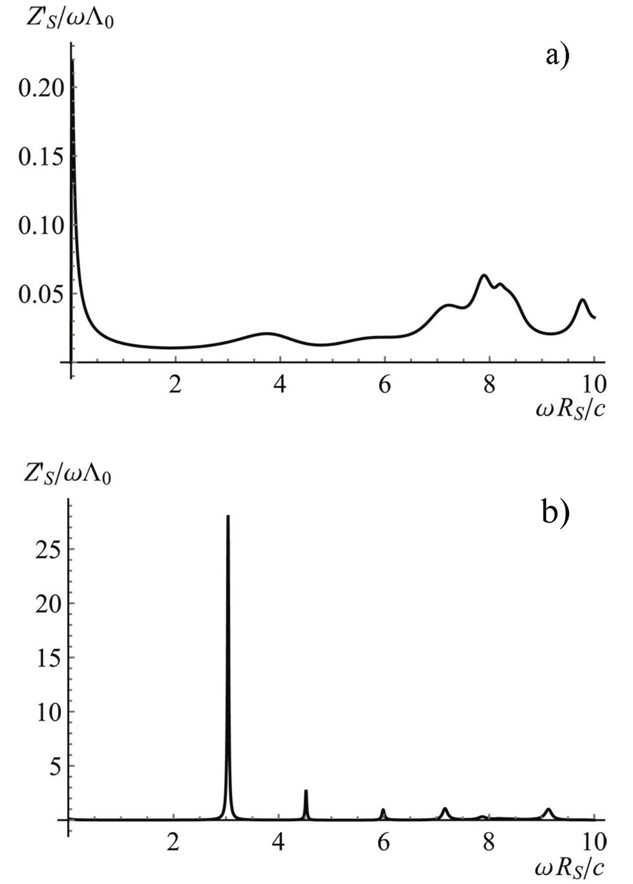

Fig. 4. Effective resistance of a uniform plasma solenoid considering radiation transfer (a) and in the presence of a limiting shell (b)

the parameter $\omega R_S \ / \ c$ takes rather large values in Fig. 2, and therefore Fig. 2 is at the limit of applicability of the quasi-stationary approximation. We have presented such a "not entirely reliable" figure here only to make the characteristic features of the impedance and inductance of the plasma solenoid more noticeable. When the parameter $\Omega_g R_S \ / \ c$ decreases, the structure of the dependencies shown in Figure 2 remains, but the features of the curves become less pronounced. Furthermore, as calculations show, the real part of the impedance coincides with good accuracy with the effective resistance calculated based on the expression for the volumetric energy density (22), even beyond the quasi-stationary approximation. Therefore, in what follows, we will use the notation Z'_S, for the effective resistance, the same as for the real part of the impedance.

According to the "canonical" formulas (17) and (18) at point $\omega = \omega^{(\infty)}$ (at current resonance), the imaginary part of the impedance becomes zero, and the real part reaches its maximum, while at point $\omega = \omega^{(0)}$ (at voltage resonance), the imaginary part of the impedance becomes zero. As seen from Fig. 2, the imaginary part of the impedance never actually becomes zero. This somewhat unusual behavior of impedance at the resonance point is due to the merging of closely located poles $\omega_n^{(\infty)}$ and zeros $\omega_n^{(0)}$. Since the real part of the impedance in Fig. 2 has a rather sharp maximum, the corresponding resonance should be unambiguously classified as current resonance. Note that the difference between current and voltage resonances in the plasma solenoid is determined by the magnitude of the field component $E_\varphi(r)$ on the solenoid winding (see formula (27)): when $E_\varphi(R_S)$ reaches its maximum value, current resonance occurs, and when $E_\varphi(R_S) \approx 0$ the voltage drop across the solenoid is small and voltage resonance occurs. Calculations show (see Fig. 2 and further) that voltage resonance is an atypical phenomenon for a plasma solenoid.

Fig. 3 shows the real parts of impedance (28) as a function of the dimensionless cyclotron frequency $\Omega_e R_S \ / \ c$ calculated at different plasma frequencies $\omega_{Le} R_S \ / \ c = 0.1$; 0.2; 0.25; 0.285; 0.3, constant source frequency $\omega R_S \ / \ c = 0.3$ and $\nu_e = 0.03\omega_{Le}$. Resonant cyclotron frequencies are determined from equations $\omega = \omega_n^{(\infty)}$, solving which with good accuracy we have $\Omega_e \approx \sqrt{\omega^2 - \omega_{Le}^2}$, which fully agrees with Fig. 3. As ω_{Le} increases, the resonances shift to the region of lower cyclotron frequencies, and at $\omega_{Le} > \omega$ they disappear completely. According to formula (19), the curves shown in Fig. 3 determine, in relative units, the source powers dissipated in the plasma solenoid when the circuit resistance is high.

When frequencies Ω_e and ω_{Le} increase, the resonant frequency extends beyond the quasistationary frequency range. In this case, to study resonance in the plasma solenoid, formula (22) should be used, which in the present case is written as

$$W(r) = \frac{\omega}{8\pi} q \,|\, E_\varphi(r) \,|^2,$$
$$q = \varepsilon_\perp'' \left(1 + |\, g \,|^2 \,/\, |\, \varepsilon_\perp \,|^2\right) - 2g'' \mathrm{Re}(g \,/\, \varepsilon_\perp). \quad (34)$$

Here, the first equation of system (4) was taken into account at $\partial / \partial z = 0$. Substituting field (26) into (34) and performing integration over the solenoid volume, one can obtain an expression for the total power of the external source dissipated in the plasma, which we do not present here due to its cumbersomeness, and the calculation result for a plasma solenoid with parameters $\omega_{Le} R_S / c = 5$, $\Omega_e R_S / c = 7$, $\nu_e = 0.03\omega_{Le}$ is shown in Fig. 4a. As can be seen, the peaks of effective resistance appear broad and diffused. All this indicates that the source power is not spent on exciting natural waves in the plasma but goes into radiation from the solenoid. This circumstance should be taken into account when interpreting experiments on inductive charges in the frequency range comparable to values c / R_S and c / L_S.

It was mentioned earlier that radiation from the solenoid can be avoided using a shielding shell surrounding the solenoid. Fig. 4*b* shows the calculation result of the source power for a solenoid with a shell at $\mathcal{R} / R_S = 1.6$. The calculation was performed using formula (34) with field (26), where Hankel functions were replaced by functions $X_{0,1}[(\omega / c) R_S]$. As we can see, the result is fundamentally different from that shown in Fig. 4*a*.

4. BOUNDED PLASMA SOLENOID WITHOUT EXTERNAL MAGNETIC FIELD

The theory of plasma solenoid becomes significantly more complex when the electromagnetic field in the solenoid depends on the longitudinal coordinate z. Such dependence naturally arises in a solenoid with variable density of the number of winding turns if the number of turns is not large or in a bounded solenoid where conducting planes are located at boundaries $z = 0$ and $z = L_S$ Let's consider the case of a bounded solenoid. On conducting planes, the tangential components of the electric field intensity are equal to zero. Therefore, the following boundary conditions apply:

$$E_r\big|_{z=0} = E_r\big|_{z=L_S} = 0, \quad E_\varphi\big|_{z=0} = E_\varphi\big|_{z=L_S} = 0. \quad (35)$$

Taking into account boundary conditions (35), we will seek the solution of equations (4) in the form

$$E_r(z,r) = \sum_{n=1} E_{rn}(r) \sin(k_{zn} z),$$
$$E_\varphi(z,r) = \sum_{n=1} E_{\varphi n}(r) \sin(k_{zn} z), \quad (36)$$
$$E_z(z,r) = \sum_{n=1} E_{zn}(r) \cos(k_{zn} z),$$

where $k_{zn} = \pi n / L_S$ — are longitudinal wave numbers of electromagnetic oscillations excited in the solenoid. Substituting expansions (36) into equations (4) leads to the following equations for functions $E_{rn}(r)$, $E_{\varphi n}(r)$ and $E_{zn}(r)$:

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr} r E_{\varphi n}\right) - k_{zn}^2 E_{\varphi n} + \frac{\omega^2}{c^2} \varepsilon_\perp E_{\varphi n} - $$
$$-i \frac{\omega^2}{c^2} g E_{rn} = 0,$$
$$k_{zn}\left(k_{zn} E_{rn} - \frac{dE_{zn}}{dr}\right) - \frac{\omega^2}{c^2} \varepsilon_\perp E_{rn} - \quad (37)$$
$$-i \frac{\omega^2}{c^2} g E_{\varphi n} = 0,$$
$$\frac{1}{r}\frac{d}{dr} r \left(\frac{dE_{zn}}{dr} - k_{zn} E_{rn}\right) + \frac{\omega^2}{c^2} \varepsilon_{||} E_{zn} = 0.$$

Here the components of the dielectric permittivity tensor are functions of coordinate r, therefore equations (37) are valid both in plasma and outside plasma. Substituting the second expression (36) into (14) and considering (12), we transform the external circuit equation to the form

$$U_0 = IR_0 - \pi R_S N_S \sum_{n=1} P_n E_{\varphi n}(R_S), \quad (38)$$

where

$$P_n = \int_0^\pi \sin nx dx \,/\, \int_0^\pi \sin^2 nx dx = 2\frac{1 - \cos \pi n}{\pi n}. \quad (39)$$

Finally, substituting the second expression (36) into the boundary condition (6), we obtain

$$\frac{dE_{\varphi n}}{dr}(R_S+0)-\frac{dE_{\varphi n}}{dr}(R_S-0)= \\ =-i\frac{\omega}{c}\frac{4\pi}{c}\frac{2}{L_S}\int_0^{L_S}J(z)\sin(k_{zn}z)dz. \quad (40)$$

If the solenoid winding density is constant along the solenoid length and the number of turns is sufficiently large, then boundary condition (40) simplifies. Then, taking into account the continuity of tangential components of electric field (7), we have the following boundary conditions for equations (37):

$$\begin{gathered} E_{\varphi n}(R_S+0)-E_{\varphi n}(R_S-0)=0, \\ E_{zn}(R_S+0)-E_{zn}(R_S-0)=0, \\ \frac{dE_{\varphi n}}{dr}(R_S+0)-\frac{dE_{\varphi n}}{dr}(R_S-0)= \\ =-i\frac{\omega}{c}\frac{4\pi}{c}\frac{N_S}{L_S}IP_n. \end{gathered} \quad (41)$$

Note that coefficients P_n depend on the design of the coil enclosing the plasma and on conditions at the longitudinal boundaries of plasma, i.e., in principle P_n can be determined by some other formulas. We will use formulas (39).

Excluding functions E_{rn}, we write the field equations (37) in the form

$$\begin{gathered} \frac{d}{dr}\frac{1}{r}\frac{d}{dr}\left(rE_{\varphi n}\right)-\left(\chi_n^2-\frac{\omega^4}{c^4\chi_n^2}g^2\right)E_{\varphi n}= \\ =i\frac{\omega^2}{c^2}\frac{k_{zn}}{\chi_n^2}g\frac{dE_{zn}}{dr}, \\ \frac{1}{r}\frac{d}{dr}\left(r\frac{\varepsilon_\perp}{\chi_n^2}\frac{dE_{zn}}{dr}\right)-\varepsilon_{||}E_{zn}= \\ =-ik_{zn}\frac{1}{r}\frac{d}{dr}\left(r\frac{g}{\chi_n^2}E_{\varphi n}\right), \end{gathered} \quad (42)$$

where

$$\chi_n^2=k_{zn}^2-\frac{\omega^2}{c^2}\varepsilon_\perp. \quad (43)$$

Equations (42) are valid both in the plasma volume (at $r < R_S$), and in the external space (at$r > R_S$).

Analysis of system (42) will begin with the case of absence of external magnetic field, when $g = 0$, $\varepsilon_\perp = \varepsilon_{||}$. In this case, the second equation of system (42) is decoupled from the first equation, and component E_z, since it is not excited by the azimuthal current, can be set to zero. The first equation then simplifies to:

$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}rE_{\varphi n}-\chi_n^2E_{\varphi n}=0. \quad (44)$$

If we assume that the inequality

$$\omega < \pi c / L_S, \quad (45)$$

which is consistent with the general quasi-stationarity condition, is satisfied, then the solution of equation (44) bounded at zero and infinity has the form

$$E_{\varphi n}(r)=\begin{cases} A_nI_1(\chi_nr), & r<R_S, \\ B_nK_1(\chi_{0n}r), & r>R_S, \end{cases} \quad (46)$$

where A_n and B_n — are constants, and χ_{0n}^2 — are quantities (43) taken at $\varepsilon_\perp = 1$.

For a long solenoid, or in the radio-frequency region, inequality (45) is too strict. If inequality (45) is not used, then there is difficulty in writing the bounded at infinity solution of equation (44). In this case, the solution can be expressed through the Hankel function $H_1^{(1)}(\sqrt{\omega^2/c^2-k_{zn}^2}r)$ (see derivation of formula (26)), or we can assume that the solenoid is enclosed in a conductive cylindrical shell of radius $r = \mathcal{R} > R_S$. In the presence of the shell, instead of (46), we have the following solution:

$$E_{\varphi n}(r)=\begin{cases} A_nI_1(\chi_nr), & r<R_S, \\ B_n[K_1(\chi_{0n}r)I_1(\chi_{0n}\mathcal{R})-I_1(\chi_{0n}r)K_1(\chi_{0n}\mathcal{R})]\equiv B_nX_1(\chi_{0n}r), & R_S<r<\mathcal{R}. \end{cases} \quad (47)$$

To determine the constants A_n and B_n solution (46) is matched at point $r = R_S$ using the first and third conditions (41). As a result, for the azimuthal component of the electric field intensity of the solenoid without a conductive shell, we obtain the following expression (at$r \le R_S$):

$$E_{\varphi n}(r) = i\frac{\omega}{c}\frac{4\pi}{c}\frac{N_S}{L_S}P_n\frac{I_1(\chi_n r)K_1(\chi_{0n}R_S)}{\chi_n I_0(\chi_n R_S)K_1(\chi_{0n}R_S) + \chi_{0n}I_1(\chi_n R_S)K_0(\chi_{0n}R_S)}I. \quad (48)$$

Substituting (48) into the circuit equation (38) and taking into account formula (15), we obtain the following expression for the impedance of a bounded plasma solenoid without a shell in the absence of an external magnetic field

$$Z_S(\omega) = -i\omega\Lambda_0\sum_{n=1}P_n^2\frac{I_1(\chi_n R_S)K_1(\chi_{0n}R_S)}{\chi_n R_S I_0(\chi_n R_S)K_1(\chi_{0n}R_S) + \chi_{0n}R_S I_1(\chi_n R_S)K_0(\chi_{0n}R_S)}. \quad (49)$$

In the case of a plasma solenoid with a conductive shell (in this case, instead of (46), solution (47) is taken), the complex impedance has the form

$$Z_S(\omega) = -i\omega\Lambda_0\sum_{n=1}P_n^2\frac{I_1(\chi_n R_S)X_1(\chi_{0n}R_S)}{\chi_n R_S I_0(\chi_n R_S)X_1(\chi_{0n}R_S) + \chi_{0n}R_S I_1(\chi_n R_S)X_0(\chi_{0n}R_S)}, \quad (49a)$$

where $X_0(\chi_{0n}r) = K_0(\chi_{0n}r)I_1(\chi_{0n}\mathcal{R}) + I_0(\chi_{0n}r)K_1(\chi_{0n}\mathcal{R})$. Formula (49) can only be used in the frequency range (45). In the case of formula (49a), there is no such limitation.

The denominators in expression (49)

$$D_{Bn}(\chi_n R_S) \equiv \chi_n I_0(\chi_n R_S)K_1(\chi_{0n}R_S) + \\ +\chi_{0n}I_1(\chi_n R_S)K_0(\chi_{0n}R_S) \quad (50)$$

are dispersion functions for waves B of plasma cylinder type with a free surface. Such waves do not exist in a plasma cylinder without an external magnetic field, since the dispersion equations $D_{Bn} = 0$ at $\nu_e = 0$ real solutions regarding frequency ω do not have[6]. Thus, the impedance (49) has no resonant features – zeros and poles. Consequently, there is no resonant power absorption from the external source in a bounded plasma solenoid without a casing and external magnetic field. Previously, the same result was obtained for a homogeneous solenoid, in the calculation of which it was assumed $\partial / \partial z = 0$ (see formulas (30) and (31)). The impedance determined by formula (49a) has resonant features, but only in the radio-frequency region, where the quasi-stationary approximation, and hence the very concept of impedance, are inapplicable.

[6] Complex solutions correspond to damping field perturbations. The damping is caused by free radiation into the space surrounding the plasma cylinder. The plasma cylinder with a free surface for B-type fields is not a waveguide.

Fig. 5 shows the complex impedance (49) of a bounded plasma solenoid with uniform winding density in the absence of an external magnetic field at $\omega_{Le}R_S / c = 0.5$, $\nu_e = 0.03\omega_{Le}$, $L_S / R_S = 2$. The monotonic nature of the presented dependencies indicates the absence of resonances associated with the excitation of natural waves B-type in a plasma cylinder with a free surface. The imaginary part of the impedance shown in Fig. 5 indicates the absence of frequency dependence of inductance. The impedance of the solenoid calculated using formula (49a) has approximately the same form.

The impedance and inductance of the bounded solenoid depend on its length L_S. The inductance of a vacuum solenoid also depends on length Λ_0, but in the case of a bounded plasma solenoid, this dependence is significantly stronger. Fig. 6 shows the relative inductance of the plasma solenoidL_S calculated Λ_S / Λ_0, as a function of length using impedance (49a). At large values of L_S it approaches the relative inductance of a long plasma solenoid, calculated using impedance (28a). It is intuitively clear that the longer the solenoid, the less influence its boundaries have $z = 0, L_S$. Fig. 6 confirms that this is indeed the case.

When the quasi-stationary condition is violated, formulas (49) and (49a) are unsuitable, and formula (48) for the harmonics of the azimuthal component of the electric field strength is also unsuitable, as it does not account for radiation through the lateral surface of the solenoid. Instead of accounting for radiation, let's assume the presence of a conductive shell with radius $\mathcal{R}$, enclosing the solenoid. The field in the plasma in this case is given by formula (48), in which functions $K_{0,1}(\chi_{0n}R_S)$ are replaced with $X_{0,1}(\chi_{0n}R_S)$. Substituting $E_{\varphi n}(r)$ into the second formula (36), and then into formula (22), after integration over the solenoid volume, we obtain the following expression for the source power dissipated in the plasma solenoid with shell in the absence of an external magnetic field:

$$W(\omega) = \omega\frac{\Lambda_0 I^2}{2}\varepsilon_\perp''\sum_n\left|\frac{(\omega / c)X_1(\chi_{0n}R_S)}{\chi_n I_0(\chi_n R_S)X_1(\chi_{0n}R_S) + \chi_{0n}I_1(\chi_n R_S)X_0(\chi_{0n}R_S)}\right|^2 \frac{P_n^2}{R_S^2}\int_0^{R_S} I_1(\chi_n r)I_1(\chi_n^* r)rdr \quad (51)$$

For a plasma solenoid with parameters $\omega_{Le}R_S / c = 5$, $\nu_e = 0.03\omega_{Le}$, $L_S / R_S = 2$ and $\mathcal{R} / R_S = 1.6$ the power (51) is shown in Fig. 7. It can be seen that beyond the quasi-stationary frequency range, power maxima have appeared. They are caused by resonant excitation by the source of natural waves of B-type in a waveguide of radius $\mathcal{R}$ with a plasma cylinder of radius R_S. It can be shown that the dispersion function of these waves is in the denominator of the expression under the modulus in (51) (the same function is in the denominator of expression (49a)).

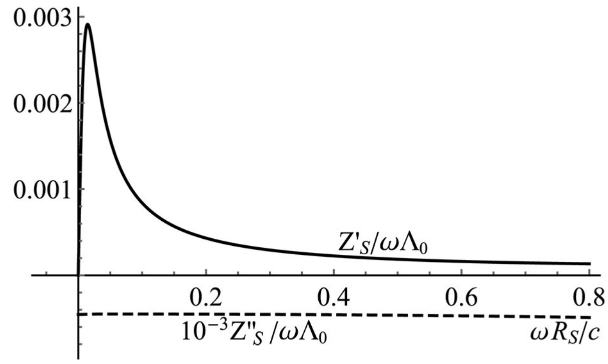

Fig. 5. Complex impedance of a limited plasma solenoid without shell in the absence of external magnetic field

5. BOUNDED PLASMA SOLENOID WITH MAGNETOACTIVE PLASMA

Let's now turn to the general case of magnetoactive plasma. In the region $r < R_S$ we seek the solution of equations in the form

$$E_{\varphi n} = A_n J_1(\kappa_n r),\quad E_{zn} = B_n J_0(\kappa_n r), \quad (52)$$

where A_n, B_n — are constants, and κ_n — are unknown eigenvalues. Substitution of (52) into equations (42) leads to a homogeneous system of two equations for A_n, B_n

$$\left[\kappa_n^2 + \left(\chi_n^2 - \frac{\omega^4}{c^4\chi_n^2}g^2\right)\right]A_n = i\frac{\omega^2}{c^2}\frac{k_{zn}\kappa_n}{\chi_n^2}gB_n,$$
$$\left(\kappa_n^2 + \frac{\varepsilon_{||}}{\varepsilon_\perp}\chi_n^2\right)B_n = ik_{zn}\kappa_n\frac{g}{\varepsilon_\perp}A_n. \quad (53)$$

From which we obtain the equation for determining eigenvalues κ_n

$$\varepsilon_\perp\kappa_n^4 + \left[\chi_n^2(\varepsilon_\perp + \varepsilon_{||}) + \frac{\omega^2}{c^2}g^2\right]\kappa_n^2 + \left(\chi_n^4 - \frac{\omega^4}{c^4}g^2\right)\varepsilon_{||} = 0 \quad (54)$$

and the relationship between constants in solution (52)

$$B_n = i\frac{k_{zn}\kappa_n g}{\varepsilon_{||}\chi_n^2 + \varepsilon_\perp\kappa_n^2}A_n \equiv i\beta(\kappa_n)A_n. \quad (55)$$

The roots of the biquadratic equation (54) can be written as κ_{n1}, $-\kappa_{n1}$, κ_{n2}, $-\kappa_{n2}$, where

$$\kappa_{n1,2}^2 = \frac{1}{2\varepsilon_\perp}\left\{-\left(\varepsilon_\perp + \varepsilon_{||}\right)\chi_n^2 - g^2\frac{\omega^2}{c^2} \pm \pm\sqrt{\left[\left(\varepsilon_\perp - \varepsilon_{||}\right)\chi_n^2 + g^2\frac{\omega^2}{c^2}\right]^2 + 4\varepsilon_{||}g^2k_{zn}^2\frac{\omega^2}{c^2}}\right\}. \quad (56)$$

Since $J_1(-z) = -J_1(z)$, $J_0(-z) = J_0(z)$, $\beta(-z) = -\beta(z)$, then considering (52) the general solution of the first two equations of system (42) is as follows:

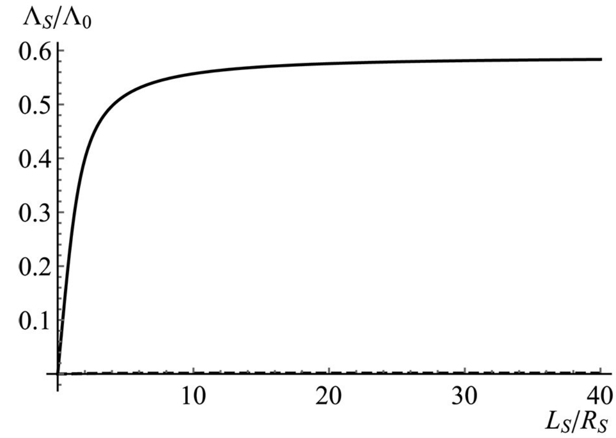

Fig. 6. Inductance of a bounded plasma solenoid depending on its length

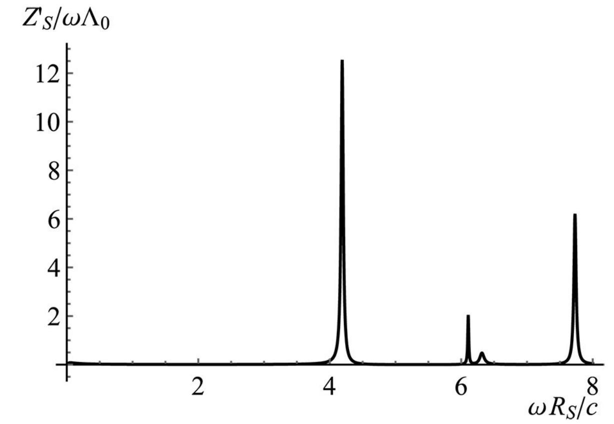

Fig. 7. Effective resistance of a bounded plasma solenoid with shell in the absence of external magnetic field

$$E_{\varphi n} = A_{n1}J_1(\kappa_{n1}r) + A_{n2}J_1(\kappa_{n2}r),$$
$$E_{zn} = i\beta(\kappa_{n1})A_{n1}J_0(\kappa_{n1}r) + i\beta(\kappa_{n2})A_{n2}J_0(\kappa_{n2}r), \quad (57)$$

where $A_{n1,2}$ — are constants.

In the vacuum region $r > R_S$ the bounded solutions of the first two equations (42) have the form

$$E_{\varphi n} = C_n K_1(\chi_{0n}r), \quad E_{zn} = D_n K_0(\chi_{0n}r), \quad (58)$$

where C_n and D_n — are constants. If the plasma solenoid has a conducting cylindrical shell, then at $\mathcal{R} > r > R_S$ the following solution is written:

$$E_{\varphi n} = C_n X_1(\chi_{0n}r),$$
$$E_{zn} = D_n[K_0(\chi_{0n}r)I_0(\chi_{0n}\mathcal{R}) - -I_0(\chi_{0n}r)K_0(\chi_{0n}\mathcal{R})] \equiv D_n Z_0(\chi_{0n}r), \quad (59)$$

where function $X_1(\chi_{0n}r)$ is given in (47).

To determine the constants $A_{n1,2}$, C_n and D_n of solutions (57) and (58), they should be matched at point $r = R_S$. However, three matching conditions – boundary conditions (41) – are insufficient to determine four constants in solutions (57) and (58). The missing fourth condition is obtained by integrating the second equation of system (42) in the vicinity of point$r = R_S$, which gives

$$\left\{\frac{\varepsilon_\perp}{\chi_n^2}\frac{dE_{zn}}{dr} + ik_{zn}\frac{g}{\chi_n^2}E_{\varphi n}\right\}_{r=R_S} = 0. \quad (60)$$

Using system (3), it can be shown that (60) is equivalent to the continuity condition at the plasma boundary of the tangential component of magnetic field induction B_φ, or the normal component of electric field induction $D_r = \varepsilon_\perp E_r + igE_\varphi$.

Matching solutions (57) and (58) using the obtained conditions, we express the constants $A_{n1,2}$, C_n, D_n and find the following expression:

$$E_{\varphi n}(r) = i\frac{\omega}{c}\frac{4\pi}{c}I\frac{N_S}{L_S}P_n K_1(\chi_{0n}R_S)\times \times\left(\frac{D_{En}(\kappa_{n2}R_S)}{D_n}J_1(\kappa_{n1}r) - \frac{D_{En}(\kappa_{n1}R_S)}{D_n}J_1(\kappa_{n2}r)\right), \quad (61)$$

where

$$D_n = D_{En}(\kappa_{n2}R_S)D_{Bn}(\kappa_{n1}R_S) - -D_{En}(\kappa_{n1}R_S)D_{Bn}(\kappa_{n2}R_S), \quad (62)$$

$$D_{Bn}(\kappa_{n1,2}R_S) = \chi_{0n}J_1(\kappa_{n1,2}R_S)K_0(\chi_{0n}R_S) + +\kappa_{n1,2}J_0(\kappa_{n1,2}R_S)K_1(\chi_{0n}R_S),$$
$$D_{En}(\kappa_{n1,2}R_S) = (J_0(\kappa_{n1,2}R_S)K_1(\chi_{0n}R_S)\chi_{0n}^{-1} - \varepsilon_\perp\kappa_{n1,2}J_1(\kappa_{n1,2}R_S)K_0(\chi_{0n}R_S)\chi_n^{-2})\beta(\kappa_{n1,2}) + +k_{zn}g\chi_n^{-2}J_1(\kappa_{n1,2}R_S)K_0(\chi_{0n}R_S). \quad (63)$$

Expressions D_n are dispersion functions of mixed, i.e., B- and E-type, electromagnetic waves of a magnetoactive plasma cylinder with a free surface. Keeping in mind the limit transition to zero external magnetic field, we can conditionally call functions $D_{Bn}(\kappa_{n1,2}R_S)$ dispersion functions of B-type waves, and $D_{En}(\kappa_{n1,2}R_S)$ — dispersion functions of E-type waves (see formula (50)).

Substituting (61) into the external circuit equation (38), we obtain in a standard way the following expression for the complex impedance of a solenoid filled with homogeneous magnetoactive plasma:

$$Z_S(\omega) = -i\omega\Lambda_0 \sum_{n=1} P_n^2 \frac{\left(D_{En}(\kappa_{n2}R_S)J_1(\kappa_{n1}R_S) - D_{En}(\kappa_{n1}R_S)J_1(\kappa_{n2}R_S)\right)K_1(\chi_{0n}R_S)}{R_S\left(D_{En}(\kappa_{n2}R_S)D_{Bn}(\kappa_{n1}R_S) - D_{En}(\kappa_{n1}R_S)D_{Bn}(\kappa_{n2}R_S)\right)}. \tag{64}$$

In zero magnetic field, formula (64) reduces to formula (49).

When deriving formula (64), we assumed that the solenoid had no conducting shell. If there is a conducting shell, then solution (59) should be used. Then, almost completely repeating the derivation of formula (64), we obtain the following expression for the impedance of a bounded plasma solenoid with a conducting shell in an external magnetic field:

$$Z_S(\omega) = -i\omega\Lambda_0 \sum_{n=1} P_n^2 \frac{\left(D_{En}(\kappa_{n2}R_S)J_1(\kappa_{n1}R_S) - D_{En}(\kappa_{n1}R_S)J_1(\kappa_{n2}R_S)\right)X_1(\chi_{0n}R_S)}{R_S\left(D_{En}(\kappa_{n2}R_S)D_{Bn}(\kappa_{n1}R_S) - D_{En}(\kappa_{n1}R_S)D_{Bn}(\kappa_{n2}R_S)\right)},$$

and the dispersion functions are determined by formulas

$$\begin{gathered} D_{Bn}(\kappa_{n1,2}R_S) = \chi_{0n}J_1(\kappa_{n1,2}R_S)X_0(\chi_{0n}R_S) + \kappa_{n1,2}J_0(\kappa_{n1,2}R_S)X_1(\chi_{0n}R_S), \\ D_{En}(\kappa_{n1,2}R_S) = \beta(\kappa_{n1,2})\Big(J_0(\kappa_{n1,2}R_S)Z_1(\chi_{0n}R_S)\chi_{0n}^{-1} - \varepsilon_\perp \kappa_{n1,2}J_1(\kappa_{n1,2}R_S)Z_0(\chi_{0n}R_S)\chi_n^{-2}\Big) + \\ + k_{zn}g\chi_n^{-2}J_1(\kappa_{n1,2}R_S)Z_0(\chi_{0n}R_S), \end{gathered} \tag{65}$$

where $Z_1(\chi_{0n}r) = K_1(\chi_{0n}r)I_0(\chi_{0n}\mathcal{R}) + I_1(\chi_{0n}r)K_0(\chi_{0n}\mathcal{R})$.

Solution (59) makes sense for any sign of values χ_{0n}^2 (unlike solution (58), obtained at $\chi_{0n}^2 > 0$), and the applicability of formula (64a) is not limited by condition (45). Therefore, in (64a), it becomes possible to take the limit transition to a solenoid of infinite length $L_S \to \infty$, or $k_{zn} \to 0$. In this limit, from formulas (56) and (55), we have $\kappa_{n1}^2 \to (\omega^2 / c^2)\varepsilon_{||}$, , $\kappa_{n2}^2 \to (\omega^2 / c^2)\tilde{\varepsilon}_\perp \equiv \kappa_2^2$ ($\tilde{\varepsilon}_\perp$ see in (24)), $\beta(\kappa_{n1}) \to \text{const} \neq 0$, $\beta(\kappa_{n2}) \to 0$. Then, considering (65), formula (64a) transforms to

$$Z_S(\omega) = -i\omega\Lambda_0 \frac{J_1(\kappa_2 R_S)X_1(i\omega R_S / c)}{R_S\left(\chi_{0n}J_1(\kappa_2 R_S)X_0(i\omega R_S / c) + \kappa_2 J_0(\kappa_2 R_S)X_1(i\omega R_S / c)\right)} \sum_{n=1} P_n^2. \tag{66}$$

Taking into account the relation

$$\sum_{n=1} P_n^2 = \frac{4}{\pi^2}\sum_{n=1}\left(\frac{1-(-1)^n}{n}\right)^2 = 2, \tag{67}$$

we see that formula (66) exactly transitions into formula (28a) of the inductance of a uniform solenoid. Thus, a solenoid of very large length can be considered uniform. But this is understandable, since with a large solenoid length, the conditions at its ends do not play a significant role.

Equation

$$\begin{aligned} D_n \equiv D(\omega, k_{zn}) = D_{En}(\kappa_{n2}R_S)D_{Bn}(\kappa_{n1}R_S) - \\ - D_{En}(\kappa_{n1}R_S)D_{Bn}(\kappa_{n2}R_S) = 0 \end{aligned} \tag{68}$$

determines the frequencies $\omega_s(k_{zn})$ of the natural waves of a plasma cylinder with radius R_S (with a free surface, or in a waveguide of radius $\mathcal{R}$) in an external magnetic field. Here, the index s denotes the number of the plasma wave branch, and n coincides with the summation index in (64) and (64a). These same frequencies determine the poles $\omega_{sn}^{(\infty)} = \omega_s(k_{zn})$ of impedances (64) and (64a). The rather complex equation (68) has been studied in various limiting cases in plasma waveguide theory [35]. The exact equation for the impedance zeros, due to the presence of an infinite sum in (64) and (64a), cannot be written in explicit form at all. Therefore, it is impossible to determine in general form whether impedances (64) and (64a) have zeros. In calculations performed for specific cases, impedance zeros were not found. Let us recall that current resonance occurs at impedance poles, while voltage resonance occurs at zeros. Thus, in a bounded plasma solenoid in an external magnetic field, current resonances are possible, while voltage resonances most likely do not exist. The same can be said about a uniform plasma solenoid, based on previous results.

Let us consider the mechanisms of energy transfer from an external source to magnetoactive plasma located in a solenoid. All these mechanisms are taken into account in formulas (64) and (64a). The radio-frequency electromagnetic field of the solenoid has a B-type structure. Forced oscillations of this type will be excited in the plasma. However, in the presence of a finite external magnetic field in the plasma, oscillations of B-type and E-type become coupled with each other. Thus, in a solenoid with magnetoactive plasma, mixed forced electromagnetic oscillations of both types are excited at the frequency of the external source ω. The most intense excitation of oscillations in the plasma occurs when the source frequency coincides with the frequencies of the natural waves of the plasma system, i.e., at $\omega \approx \omega_{sn}^{(\infty)} = \omega_s(k_{zn})$, where $\omega_s(k_{zn})$ are determined from equations (68). Naturally, the strongest absorption of external source energy in the plasma occurs at the same frequencies. It is precisely the coupling of electromagnetic waves of B-type and E-type that distinguishes the case of magnetoactive plasma in a solenoid from the case of a plasma solenoid without a magnetic field (only B-type oscillations are excited) and from the case of plasma in a capacitor (only E-type oscillations are excited) [18]. Consequently, the use of magnetoactive plasma allows engaging plasma waves of all types as inductive channels for resonant energy transfer from the source to the plasma. Of particular interest are waves of E-type due to their low frequency. Moreover, in a plasma cylinder, there are no natural electromagnetic waves of B-type in the low-frequency region at all (except for waves (33) with frequencies close to the upper hybrid frequency).

It is known [35] that in a plasma waveguide with a weak external magnetic field, when $\Omega_e < \omega_{Le}$, there are three groups of E-type waves. In the low-frequency region $\omega < \Omega_e$ the frequencies of bulk oblique cyclotron waves are located. In the intermediate frequency range $\Omega_e < \omega < \omega_{Le}$ the frequency of the surface Langmuir wave lies. In the radio-frequency region $\omega_{Le} < \omega < \Omega_g$ the frequencies of oblique Langmuir waves are located. In the case of a strong external magnetic field, when $\Omega_e > \omega_{Le}$, there is no surface wave, the frequencies of oblique Langmuir waves are located in the region

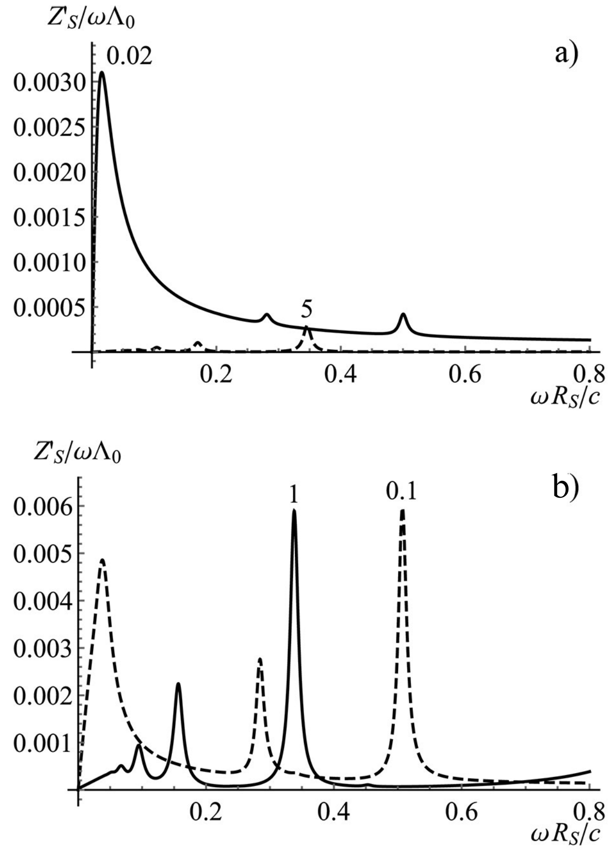

Fig. 8. The real part of the impedance of a bounded plasma solenoid without a shield in an external magnetic field for $\omega_{Le} R_S / c = 0.5$, $\nu_e = 0.03\omega_{Le}$, $L_S / R_S = 2$. The numbers near the curves correspond to the value of $\Omega_e R_S / c$

$\omega < \omega_{Le}$, and the frequencies of oblique cyclotron waves belong to the range $\Omega_e < \omega < \Omega_g$.

Figure 8 shows the calculation results of the real part of the impedance of a bounded plasma solenoid, performed using formula (64) for $\omega_{Le} R_S / c = 0.5$, $\nu_e = 0.03\omega_{Le}$, $L_S / R_S = 2$ and various values of $\Omega_e R_S / c = 0.02, 0.1, 1, 5$. At a small magnetic field value, the curve is similar to the curve in Fig. 5, constructed for the case of zero magnetic field. In the absence of an external magnetic field, there are only E-type waves, which are not excited by azimuthal current and there is no resonant energy absorption in the plasma. With a non-zero magnetic field value, waves of E-type and B-type become coupled, and excitation of such a wave by azimuthal current becomes possible. This manifests in the appearance of resonant absorption peaks in Fig. 8 for the curve corresponding to $\Omega_e R_S / c = 0.02$. The right peak corresponds to the excitation of a bulk Langmuir wave with a frequency close to the Langmuir frequency, and the left one corresponds to a surface wave with a frequency of order $\omega_{Le} / \sqrt{2}$. When the magnetic field increases to values $\Omega_e R_S / c = 0.1$ the coupling between waves strengthens and absorption peaks increase, including the formation of a low-frequency cyclotron absorption peak. In a strong magnetic field ($\Omega_e > \omega_{Le}$), at $\Omega_e R_S / c = 1$, Fig. 8 shows only peaks of low-frequency Langmuir waves in the quasi-stationary frequency region. These peaks persist in the case of $\Omega_e R_S / c = 5$, but their magnitude decreases noticeably, corresponding to the fact that in a strong magnetic field, the waves transform into E-type waves, which are not excited by azimuthal current. In all considered cases, the imaginary part of the impedance practically does not depend on the external magnetic field magnitude and coincides with the dependence shown in Fig. 5.

6. CONCLUSION

Based on Maxwell's equations in cold collisional magnetoactive plasma, the electrodynamic properties of a plasma column and the dynamics of electromagnetic field excitation by azimuthal current on its surface are examined for arbitrary ratios between the exciting generator frequency, electron cyclotron frequency, and plasma frequency. Cases of unbounded and longitudinally bounded plasma solenoid are considered. The complex impedance of the system and effective resistance as a quantity characterizing the power absorbed in the plasma were calculated. Despite the limitation of the complex impedance concept to the quasistationary case, nevertheless, the real part of the impedance coincides with the effective resistance, whose concept has a broader range of applicability not limited by quasi-stationarity conditions. The resonant properties of the complex impedance and effective plasma resistance are associated with the possibility of exciting natural electromagnetic waves in the system with their subsequent collisional dissipation. The broadening of resonance lines is determined by both the electron collision frequency and the possibility of radial energy transfer in an open system. In a system without a casing in either sufficiently weak or, conversely, sufficiently strong external magnetic field, only E-type waves become dominant, which are not excited by azimuthal current, and the resonant nature of energy absorption in plasma does not manifest. At intermediate magnetic field values E-type and B-type waves become coupled, and the excitation of such a wave by azimuthal current becomes possible. This manifests in the appearance of resonant absorption peaks associated with the excitation of bulk or surface Langmuir waves, as well as cyclotron waves.

FUNDING

This work was supported by the Russian Science Foundation (project No. 22-29-00642).

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