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INTERFERENCE CORRECTION TO OPTICAL CONDUCTANCE OF A MAGNETO-ACTIVE MEDIUM WITH SCATTERING INHOMOGENEITIES

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Abstract. The interference contribution to the optical conductance (total transmission) of a disordered sample is calculated. It is shown that wave interference in the medium is suppressed due to helicity-flip scattering events. As a result, when the cross-section of this process changes resonantly, as in the case of scattering by Mie particles near the first Kerker point, the spectral dependence of the interference contribution also becomes resonant. When waves propagate through a magneto-active medium, the applied magnetic field does not disrupt the interference of waves with given helicity but suppresses it if the helicity changes along different parts of the trajectory. This leads to a decrease in the interference contribution to conductance with increasing magnetic field. A similar phenomenon — negative magnetoresistance — is known as a consequence of weak localization of electrons in metals with impurities. It is found that with increasing magnetic field, the change in the interference correction to the optical conductance approaches a certain limit value, depending on the ratio of transport mean free path to helicity-flip scattering length. The possibility of controlling the transition to strong "Anderson" localization in the quasi-one-dimensional case (magneto-active waveguide) using the field is discussed.

Keywords: *disordered sample, weak localization, Faraday effect, optical conductance, magneto-active medium*

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1. INTRODUCTION

Optical analogues of mesoscopic effects observed in electron transport in solid-state structures [1, 2], underlie many modern achievements in the manipulation of coherent light fields [3]. As examples illustrating the analogy between optical and quantum electronic phenomena, one can point to Anderson localization of light in random layered structures [4, 5], optical Tamm states in photonic crystals [6], universal fluctuations of optical conductance [7, 8], and topological effects in photonic systems [9].

As known [2], interference of time-reversed waves leads to the weak localization effect. For electron waves in solids, this effect leads to decreased conductivity. When an external magnetic field is applied, which breaks time-reversal symmetry and suppresses interference, the weak localization effect manifests as negative magnetoresistance [2, 10, 11]. For electromagnetic radiation, the interference of waves traveling along time-reversed trajectories causes such a well-known phenomenon as coherent backscattering enhancement [2, 12]. This effect manifests as a sharp peak in the intensity of light scattered exactly in the backward direction. Along with coherent backscattering, the interference of time-reversed waves should lead to an optical analogue of the effect of decreased electron conductivity — the emergence of a negative correction to optical conductance. It is defined as the product of the transmission coefficient from a diffuse source by the number of propagating modes. Although optical conductance has been discussed in many theoretical and experimental works (see [8, 12]) in the context of studying its fluctuations in Q1D systems (waveguides), the contribution of weak localization of electromagnetic waves to conductance has not been studied yet. Unlike electrons, for which various methods of manipulating the interference contribution to conductance are known [10, 11, 13], this issue has not been considered in relation to optical conductance.

According to [14–17], the parameters that control wave interference in a medium are the attenuation lengths of different polarization modes. These lengths depend on the optical characteristics of individual scatterers and the applied external magnetic field when dealing with scatterers in a magnetically active matrix. Depolarization processes with changes in helicity have a significant impact on the interference of time-reversed waves [14, 15]. Therefore, a strong change in the interference contribution to optical conductance should occur near the first Kerker point in the Mie resonant scattering region, where the helicity-flip scattering cross-section with changes sharply depending on wavelength [18]. The mechanism of wave interference destruction in a magnetic field is directly related to helicity-flip scattering processes [19-22]. The magnetic field's influence on interference is due to the Faraday effect. Unlike electronic waves, whose interference is always disrupted by a magnetic field, the applied field does not affect the interference of optical waves at all if their helicity remains unchanged [21, 22]. The mechanism of interference destruction by the field is only activated due to the depolarization process, helicity-flip scattering. For waves with opposite helicity, the magnetic field-induced phase shifts cancel out, while for waves with the same helicity, they add up. Random phase shifts between interfering waves occur due to helicity changes at different points along the trajectory.

In the present work, the interference contribution to the optical conductance of a disordered sample is calculated. The calculations are based on a system of diffusion equations for two cooperon modes describing the interference of time-reversed waves with given helicity. The sensitivity to the value of helicity-flip scattering length is illustrated with the example of resonant spectral dependence of the interference contribution near the first Kerker point. It is shown that an external magnetic field causes suppression of the interference contribution to the optical conductance of a magneto-active sample, similar to the negative magnetoresistance in metals with impurities. In the strong magnetic field limit, a "saturation" effect occurs: the interference contribution to optical conductance reaches a limiting value that is independent of field strength and is determined by the ratio of transport elastic scattering length to circular polarization decay length. In the case of a waveguide (Q1D geometry), the interference contribution decreases inversely proportional to the field strength and sample length L. This allows concluding about the possibility to control using the field the critical length L at which the transition to Anderson localization regime occurs. Some of the results presented below were briefly outlined in [23].)

2. THEORETICAL MODEL OF WAVE INTERFERENCE IN MAGNETO-ACTIVE MEDIUM WITH SCATTERING PARTICLES

2.1. General Relations

Let us consider electromagnetic wave propagation through a sample of non-absorbing magneto-active medium containing scattering particles. It is assumed that its linear dimensions L_x, L_y and $L_z = L$ significantly exceed the l mean free path, L_x, L_y, $L \gg l$, and the weak localization condition is satisfied, $k_0 l \gg 1$ (k_0 — wave number). The number of transverse modes N , in which electromagnetic waves propagate through the sample is large,

$$N = \frac{k_0^2 A}{4\pi} \gg 1,$$

здесь $A = L_x L_y$ — cross-sectional area of the sample [8].

According to [2, 3, 12], the conductance (or total transmission) of the sample is determined by the sum of transmission coefficients T_{ab} linking input and output modes a and b respectively,

$$T = \sum_{a,b} T_{ba}. \tag{1}$$

Under the weak localization conditions, $k_0 \gg 1$, the interference contribution $\langle \delta T \rangle$ to conductance

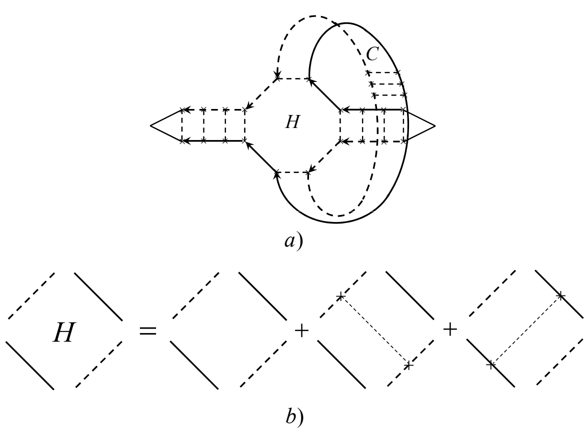

Fig. 1. *a* — Interference contribution to conductance. *b* — Hikami vertex [10]. Solid lines correspond to average Green's functions in the medium. Dashed lines denote scattering by medium inhomogeneities. The incoming *i* and outgoing *f* ladder propagators contain summation over modes

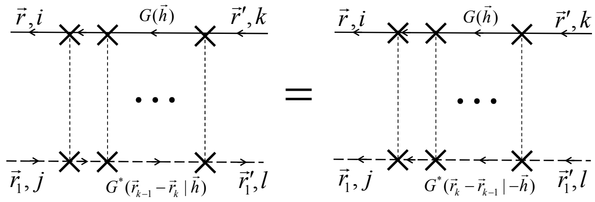

Fig. 2. Transformation of the Cooperon to a sum of ladder diagrams

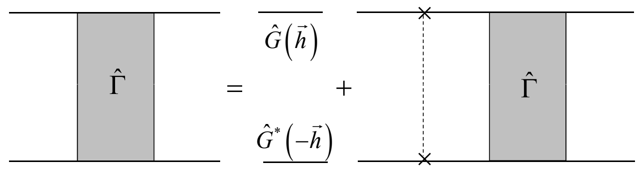

Fig. 3. Integral transport equation for the propagator $\hat{G}$

averaged over scatterer positions is determined by a diagram containing one Hikami vertex (see Fig.1) [2, 10, 13]. The average Green's function $\langle G_{ik}(\mathbf{r},\mathbf{r}'|\mathbf{h})\rangle$ included in the diagram describes electromagnetic wave propagation in a magnetoactive medium between scattering events. It is defined by the expression [22, 24–28]

$$\langle G_{ik}(\mathbf{r},\mathbf{r}'|\mathbf{h})\rangle = \langle G_{scal}(|\mathbf{r}-\mathbf{r}'|)\rangle \times \left[P_{ik}^{(+)}(\mathbf{n})e^{-i\mathbf{h}(\mathbf{r}-\mathbf{r}')/2} + P_{ik}^{(-)}(\mathbf{n})e^{i\mathbf{h}(\mathbf{r}-\mathbf{r}')/2}\right] \quad (2)$$

where $\langle G_{scal}(|\mathbf{r}|)\rangle$ — is the scalar Green's function [2], $\mathbf{n} = (\mathbf{r}-\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|$ and

$$P_{ik}^{(\pm)}(\mathbf{n}) = \frac{1}{2}(\delta_{ik} - n_i n_k \pm i e_{ikj} n_j) \quad (3)$$

— are projection operators for field states with given helicity (e_{ikj} is the antisymmetric tensor). Vector $\mathbf{h}$ in (2) is proportional to the applied magnetic field strength $\mathbf{H}$, $\mathbf{h} = 2V\mathbf{H}$ (V is the Verdet constant [29]).

We will assume that the magnetic field is sufficiently weak, $hl \ll 1$, and the phase accumulation due to the field occurs as a result of multiple scattering events. In this approximation, one can neglect the magnetic field's influence on single scattering amplitude and disregard the field when calculating the Hikami vertex. The diagram shown in Fig. 1 can be calculated by "linking" the input propagators in the formula for the correlation function of intensity fluctuations of polarized light obtained in [16, 17]. The resulting internal propagator in the diagram (see "loop" in Fig. 1*a*) — the Cooperon $\hat{C} = \langle G_{ik}(\mathbf{r},\mathbf{r}'|\mathbf{h})G^*_{jl}(\mathbf{r}_1,\mathbf{r}'_1|\mathbf{h})\rangle_C$, describes the interference of waves propagating towards each other (i.e., along time-reversed trajectories). Using the equality $G^*_{jl}(\mathbf{r}_1,\mathbf{r}'_1|\mathbf{h}) = G^*_{lj}(\mathbf{r}'_1,\mathbf{r}_1|-\mathbf{h})$ the Cooperon can be reduced to a sum of ladder diagrams where indices and coordinates are transposed in one of the Green's functions and the magnetic field direction is reversed (see Fig. 2 and [25, 26]),

$$\hat{C} = \langle G_{ik}(\mathbf{r},\mathbf{r}'|\mathbf{h})G^*_{jl}(\mathbf{r}_1,\mathbf{r}'_1|\mathbf{h})\rangle_C = \langle G_{ik}(\mathbf{r},\mathbf{r}'|\mathbf{h})G^*_{lj}(\mathbf{r}'_1,\mathbf{r}_1|-\mathbf{h})\rangle_L. \quad (4)$$

Assuming that successive scattering events occur in the wave zone, the correlator $\langle G_{ik}(\mathbf{h})G^*_{lj}(-\mathbf{h})\rangle_L$ included in (4) can be written in the Wigner representation,

$$\langle G_{ik}(\mathbf{r},\mathbf{r}'|\mathbf{h})G^*_{lj}(\mathbf{r}_1,\mathbf{r}'_1|-\mathbf{h})\rangle_L = \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{k}'}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}_1)-i\mathbf{k}'(\mathbf{r}'-\mathbf{r}'_1)} \times \frac{\delta(k-k_0)}{k^2}\frac{\delta(k'-k_0)}{k'^2}, \quad (5)$$

$$\Gamma_{il,kj}\left(\frac{\mathbf{r}+\mathbf{r}_1}{2},\mathbf{n} = \frac{\mathbf{k}}{k_0}\,\middle|\,\frac{\mathbf{r}'+\mathbf{r}'_1}{2},\mathbf{n}' = \frac{\mathbf{k}'}{k_0}\right)$$

where the propagator $G_{il,kj}(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}')$ obeys the transport equation (see Fig. 3 and [22, 24]).

When going from the laboratory reference frame to the concomitant frame, the matrix transforms as

$$G_{il,kj}(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') = \sum_{\alpha,\beta,\gamma,\delta} e_i^{(\alpha)}(\mathbf{n})\left(e_l^{(\beta)}(\mathbf{n})\right)^*,$$
$$G_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}')e_k^{(\gamma)}(\mathbf{n}')\left(e_j^{(\delta)}(\mathbf{n}')\right)^*, \qquad (6)$$

where vectors $\mathbf{e}_i^{(\alpha)}(\mathbf{n})$ are unit polarization vectors in the concomitant coordinate system in linear or circular basis. For example, in the circular basis formed by the triad of vectors

$$\mathbf{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta),$$

$$\mathbf{e}^{(\pm)}(\mathbf{n}) = (\partial\mathbf{n}/\partial\theta \mp i[\mathbf{n}\times\partial\mathbf{n}/\partial\theta])/\sqrt{2},$$

where indices $\alpha,\beta,\gamma,\delta$ take values ± 1 (see, for example, [30]).

As is known (see, for example, [10, 13]), the magnitude of the interference correction in the 3D case is determined by wave propagation trajectories with length less than or on the order of l_{tr}. However, what is observable is not the correction itself, but its change depending on factors limiting the length of interfering wave trajectories. This change is determined by long trajectories and, accordingly, by modes that decay slowly on the scale of the transport length l_{tr}. Only circularly polarized modes [14,22,31,32]. can decay slowly in a scattering medium. Linear polarization always decays on scales of order l_{tr} [14,31,32]. Taking this into account, let's keep in (6) only the terms corresponding to two circularly polarized modes with given helicity. For this, in sum (6) we need to preserve terms with $\alpha = \beta$ and $\gamma = \delta$ [22]. Then instead of (6) we obtain

$$G_{il,kj}(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') =$$
$$= \sum_{\alpha,\beta} P_{il}^{(\alpha)}(\mathbf{n})G_{\alpha\beta}(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}')P_{jk}^{(\beta)}(\mathbf{n}'), \qquad (7)$$

where

$$P_{il}^{(\alpha)}(\mathbf{n}) = e_i^{(\alpha)}(\mathbf{n})\left(e_l^{(\alpha)}(\mathbf{n})\right)^*$$

are projection operators (3).

In the case of electromagnetic wave propagation through a thick ($L \gg l_{tr}$) sample, as for electrons [2, 13, 10, 11], when calculating the incoming and outgoing ladder propagators in the diagram shown in Fig. 1a, one can use the standard diffusion approximation (see, for example, [16, 17]). As a result, in the two-mode approximation (7) we arrive at the expression for the interference contribution to conductance,

$$\langle\delta T(\mathbf{h})\rangle = -\frac{2}{3}\left(\frac{1}{4\pi}\right)^2\left(\frac{l_{tr}}{L}\right)^2 \times$$
$$\times\, n_0\int d\mathbf{r}\int d\mathbf{n}\,d\mathbf{n}'(1-(\mathbf{n}\mathbf{n}'))\times$$
$$\times\,\mathrm{Tr}\left[\hat{a}(\mathbf{n},\mathbf{n}')\hat{G}(\mathbf{r},\mathbf{n}'\,|\,\mathbf{r},-\mathbf{n})\right] \qquad (8)$$

where n_0 is the number of scatterers per unit volume.

Matrix $a_{\alpha\beta}$ is expressed through the single scattering matrix $d_{ik,jl}$ [30] using the relation [22]

$$a_{\alpha\beta}(\mathbf{n},\mathbf{n}') = P_{il}^{(\alpha)}(\mathbf{n})d_{il,kj}(\mathbf{n},\mathbf{n}')P_{jk}^{(\beta)}(\mathbf{n}'). \qquad (9)$$

For spherically symmetric scatterers, matrix $a_{\alpha\beta}$ has the form

$$a_{\alpha\beta}(\mathbf{n}\mathbf{n}') = \begin{pmatrix} a_+(\mathbf{n}\mathbf{n}') & a_-(\mathbf{n}\mathbf{n}') \\ a_-(\mathbf{n}\mathbf{n}') & a_+(\mathbf{n}\mathbf{n}') \end{pmatrix} \qquad (10)$$

where $a_\pm(\mathbf{n}\mathbf{n}')$ are expressed through the single scattering amplitudes of co- and cross- polarized waves, $A_\parallel$ and $A_\perp$, $a_\pm(\mathbf{n}\mathbf{n}') = |A_\parallel \pm A_\perp|^2/4$.

The propagator $\hat{G}^{(c)}$ appearing in (8) obeys the transport equation [22]

$$\left\{\left[\mathbf{n}\frac{\partial}{\partial\mathbf{r}} + \sigma\right]_{\alpha\gamma} + i(\mathbf{n}\mathbf{h})(\hat{\sigma}_z)_{\alpha\gamma}\right\}G_{\gamma\beta}(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') =$$
$$= n_0\int d\mathbf{n}''a_{\alpha\gamma}(\mathbf{n},\mathbf{n}'')G_{\gamma\beta}(\mathbf{r},\mathbf{n}''\,|\,\mathbf{r}',\mathbf{n}') +$$
$$+\,\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')\delta(\mathbf{n}-\mathbf{n}'), \qquad (11)$$

where σ is the scattering coefficient; $\hat{\sigma}_z$ is the Pauli matrix. The medium is assumed to be non-absorbing.

In the absence of a magnetic field, the system of equations for propagators $G_{\pm\pm}$ and $\Gamma_{\pm\mp}$, which describe the interference of waves with given helicity, can be reduced to equations for the first (intensity) I and fourth V Stokes parameters of circularly polarized light. In this case, the propagators $\Gamma_{\pm\pm}$ and $\Gamma_{\pm\mp}$ are related to the Stokes parameters I abd V by relations

$$\Gamma_{++} = \Gamma_{--} = (I + V)/2,$$

$$\Gamma_{+-} = \Gamma_{-+} = (I - V)/2.$$

If we define the intensities of waves with given helicity as

$$I_{\pm} = (I \pm V)/2,$$

then their connection with the Stokes parameters I and V can be written as a linear transformation

$$\begin{pmatrix} I_+ \\ I_- \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} I \\ V \end{pmatrix}. \tag{12}$$

Under conditions of magneto-optical effect, the system of equations (11) can no longer be reduced to equations for Stokes parameters, and therefore the corresponding quantities $\Gamma_{\pm\pm}$ and $\Gamma_{\mp\pm}$ in the magnetic field cannot be expressed through the first and fourth Stokes parameters. However, for convenience, we will retain the notations I and V for linear combinations of these quantities. Using transformation (12), equation (11) transforms to

$$\begin{pmatrix} \mathbf{n}\dfrac{\partial}{\partial \mathbf{r}} + \sigma & i\eta \\ i\eta & \mathbf{n}\dfrac{\partial}{\partial \mathbf{r}} + \sigma \end{pmatrix} \times \begin{pmatrix} I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') & I_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') \\ V_I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') & V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') \end{pmatrix} = n_0 \int d\mathbf{n}_1 \begin{pmatrix} a_1(\mathbf{n}\mathbf{n}_1) & 0 \\ 0 & a_2(\mathbf{n}\mathbf{n}_1) \end{pmatrix} \times \begin{pmatrix} I(\mathbf{r},\mathbf{n}_1\,|\,\mathbf{r}',\mathbf{n}') & I_V(\mathbf{r},\mathbf{n}_1\,|\,\mathbf{r}',\mathbf{n}') \\ V_I(\mathbf{r},\mathbf{n}_1\,|\,\mathbf{r}',\mathbf{n}') & V(\mathbf{r},\mathbf{n}_1\,|\,\mathbf{r}',\mathbf{n}') \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\delta(\mathbf{n}-\mathbf{n}')\delta(\mathbf{r}-\mathbf{r}'), \tag{13}$$

where

$$a_1(\mathbf{n}\mathbf{n}) = (|A_{\parallel}|^2 + |A_{\perp}|^2)/2,$$

$$a_2(\mathbf{n}\mathbf{n}) = \mathrm{Re}\, A_{\parallel}A_{\perp}^*$$

are differential scattering cross-sections entering into the transport equations for intensity and the fourth Stokes parameter [30, 33].

The off-diagonal elements in equation (13) appear due to the Faraday effect. In the absence of a magnetic field, system (13) splits into two independent transport equations for I and V. [14, 15, 18].

2.2. Diffusion approximation

Assuming that the modes in (13) decay on spatial scales exceeding l_{tr}, we transform the system of equations (13) to the diffusion form. In the diffusion approximation, in the expansion of propagators entering into (13) in spherical harmonics, we should keep the first two terms. For example, for I we have (see, for example, [2])

$$I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') = \frac{1}{(4\pi)^2}[I(\mathbf{r},\mathbf{r}') + 3(\mathbf{n}-\mathbf{n}')\mathbf{J}(\mathbf{r},\mathbf{r}')], \tag{14}$$

where

$$I(\mathbf{r},\mathbf{r}') = \int d\mathbf{n}\, d\mathbf{n}'\, I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}')$$

is density propagator, $\mathbf{J}(\mathbf{r},\mathbf{r}')$ is the corresponding current (flux density). Similar expansions are valid for other propagators, $V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}')$, $I_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}')$ and $V_I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}')$, entering the system of equations (13).

Substituting the diffusion formulas for quantities I and V into (8), we obtain the optical conductance in the two-mode approximation:

$$\langle \delta T(\mathbf{h})\rangle = -\frac{2}{3}\frac{l_{tr}^2}{L^2}\int d\mathbf{r}\left[\sigma_{tr} I(\mathbf{r},\mathbf{r}) + \sigma_{tr}^{(2)} V(\mathbf{r},\mathbf{r})\right], \tag{15}$$

where

$$\sigma_{tr} = n_0 \int d\mathbf{n}'(1 - (\mathbf{n}\mathbf{n}'))a_1(\mathbf{n}\mathbf{n}'),$$

$$\sigma_{tr}^{(2)} = n_0 \int d\mathbf{n}'(1 - (\mathbf{n}\mathbf{n}'))a_2(\mathbf{n}\mathbf{n}')$$

are the transport scattering coefficients for intensity and the fourth Stokes parameter. Formula (15) transforms to the corresponding result [23] under conditions of slow decay of circular polarization ($\sigma_{tr} - \sigma_{tr}^{(2)} \ll \sigma_{tr}$), as well as under strong depolarization ($I \gg V$).

Density propagators $I(\mathbf{r},\mathbf{r}')$ and $V(\mathbf{r},\mathbf{r}')$, included in (15), obey the system of diffusion equations

$$\begin{pmatrix} \sigma_{tr}^{(V)}\Delta - \sigma_{tr}h^2 & i\left(\sigma_{tr} + \sigma_{tr}^{(V)}\right)(\mathbf{h}\nabla) \\ i\left(\sigma_{tr} + \sigma_{tr}^{(V)}\right)(\mathbf{h}\nabla) & \sigma_{tr}\Delta - \sigma_{tr}^{(V)}h^2 - 3\sigma_{dep}\sigma_{tr}^{(V)}\sigma_{tr} \end{pmatrix}$$

$$\cdot \begin{pmatrix} I(\mathbf{r},\mathbf{r}') & I_V(\mathbf{r},\mathbf{r}') \\ V_I(\mathbf{r},\mathbf{r}') & V(\mathbf{r},\mathbf{r}') \end{pmatrix}$$

$$= -3\sigma_{tr}\sigma_{tr}^{(V)}\delta(\mathbf{r}-\mathbf{r}')\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{16}$$

where

$$\sigma_{tr}^{(V)} = \sigma_{tr}^{(2)} + \sigma_{dep},$$

$$\sigma_{dep} = n_0 \int d\mathbf{n}'(a_1(\mathbf{nn}') - a_2(\mathbf{nn}'))$$

is the depolarization coefficient of circularly polarized light [14, 32, 18]. The helicity-flip scattering coefficient with helicity reversal is two times smaller, $\sigma_{dep}/2$. The system of equations (16) is a generalization of independent diffusion equations for I and V in the absence of magnetic field [14, 15].

Under a relatively weak magnetic field, $h \ll 1/l_{circ}$, where

$$l_{circ} = \sqrt{3\sigma_{dep}\sigma_{tr}^{(V)}} \tag{17}$$

is the attenuation length of circular polarization, the field's influence on interference occurs at distances $|\mathrm{r} - \mathrm{r}'|$, exceeding l_{circ}. [14, 15, 32] In this situation, the main contribution to the magnetic field-dependent part of the interference correction (15) will be given by the value I(r; r') surviving at large $|\mathrm{r} - \mathrm{r}'|$. Mode $V(\mathbf{r},\mathbf{r}')$ decays on the scale of l_{circ} and is independent of the magnetic field. The equation for mode V in the considered case reduces to the diffusion equation in the absence of a magnetic field. In equation (16) for mode I, the term proportional to V_I can be neglected (see Appendix). As a result, this equation takes the form

$$\left[\nabla^2 - \frac{\sigma_{tr}}{\sigma_{tr}^{(V)}}h^2\right]I(\mathbf{r},\mathbf{r}') = -3\sigma_{tr}\delta(\mathbf{r}-\mathbf{r}'). \tag{18}$$

Under conditions of slow circular polarization decay 14, 18, 34, 35, the difference between the cross-sections $a_1(\mathbf{nn}')$ and $a_2(\mathbf{nn}')$ entering into (13) is small and, consequently,

$$\sigma_{dep} \ll \sigma_{tr},\ \sigma_{tr} - \sigma_{tr}^{(2)} \ll \sigma_{tr},\ l_{circ} \gg l_{tr}.$$

Under strong depolarization, $\sigma_{dep} \sim \sigma_{tr}$, mode V decays on scales of the order of transport length l_{tr}. For example, under Rayleigh scattering conditions ($\sigma_{dep} = \sigma, \sigma_{tr} = \sigma, \sigma_{tr}^{(2)} = -\sigma/2$[22]), the attenuation length of mode V is $1.176l$ [32]. For Mie scattering near the second Kerker point [18], when scattering to backward hemisphere predominates ($\sigma_{dep} = 2\sigma$, $\sigma_{tr} = 3\sigma/2$, $\sigma_{tr}^{(2)} = -\sigma_{tr}$), the corresponding length equals $1.038l$. In the considered case, the diffusion expansion cannot be applied for mode V, and therefore the system of equations (16) can only be used for qualitative analysis of electromagnetic wave interference in a magneto-optical medium [22, 24]. In this situation, the contribution of I to the interference correction is still determined by the first term in (15), but for calculating the contribution of V, a more general formula (8) should be used. It is essential that the contribution of V at $\sigma_{dep} \sim \sigma_{tr}$ is determined by short trajectories (length less than or of the order of l_{tr}), and under conditions of relatively weak magnetic field, $h \ll 1/l_{tr}$, will not depend on h. Under strong depolarization conditions, system (13) leads to a separate diffusion equation for mode I (see Appendix)

$$\left(\nabla^2 - \frac{\sigma_{tr}}{\sigma}h^2\right)I(\mathbf{r},\mathbf{r}') = -3\sigma_{tr}\delta(\mathbf{r}-\mathbf{r}'). \tag{19}$$

The change in the circular polarization decay regime is reflected in the coefficient value before h^2 in equation (19) compared to (18).

3. INTERFERENCE CORRECTION IN THE ABSENCE OF MAGNETIC FIELD

Let's calculate the interference correction $\langle\delta T\rangle$ to the conductance of a flat layer, $L_x, L_y \gg L \gg L_{tr}$, in the absence of field, $h = 0$. As known [2, 10, 13], the main contribution to $\langle\delta T\rangle$ in the 3D case comes from small distances, less than or of the order of transport length l_{tr}. However, what is observed experimentally is not the interference correction itself, but the contribution of long trajectories sensitive to the factor destroying interference. In the considered case, this factor is depolarization caused by scattering with helicity change.

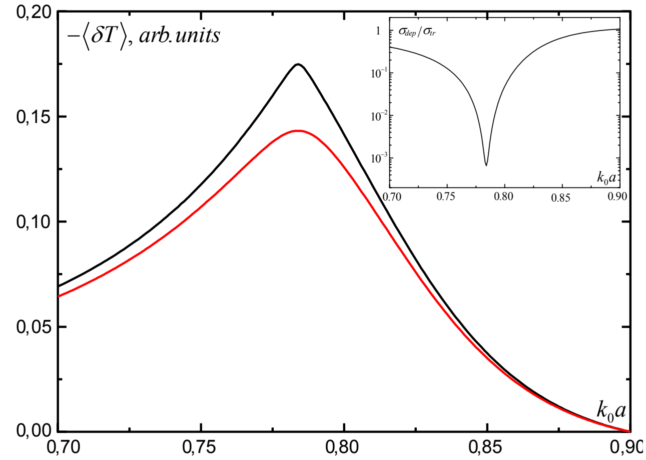

Fig. 4. Spectral dependence of interference correction to conductance near the first Kerker point (showing the change of $\langle\delta T\rangle$ relative to the value at $k_0a = 0.9$). Particles Si, refractive index and particle radius are n = 3.5 and 0.2 μm respectively. Calculations were performed using formulas (23) (upper curve) and (24) (lower curve, $L/l_{tr} = 10$). The inset shows spectral dependence of σ_{dep}

Let's analyze how the interference correction $\langle\delta T\rangle$ changes depending on σ_{dep} The influence of the depolarization process on the contribution of long trajectories to $\langle\delta T\rangle$ is determined by the difference

$$\langle\delta T(\sigma_{dep})\rangle = \langle\delta T\rangle_V - \langle\delta T\rangle_I = -\frac{2l_{tr}^2}{3L^2}\int d\mathbf{r}\left[\sigma_{tr}^{(2)}V(\mathbf{r},\mathbf{r}) - \sigma_{tr}I(\mathbf{r},\mathbf{r})\right] \quad (20)$$

In the absence of helicity-flip scattering $V = I$, $\sigma_{tr}^{(2)} = \sigma_{tr}$ and expression (20) turns to zero.

In experiment, the dependence $\langle\delta T\rangle$ on σ_{dep} can be observed in light scattering by Mie particles in the resonant spectral region near the first Kerker point [18, 36], where the difference between differential scattering cross-sections a_1 and a_2, entering equations (16), is minimal. The position of the first Kerker point is determined by the condition $\lambda \approx 2.29\,na$, where λ is the wavelength of light, n and a are the refractive index and particle radius [18, 36, 37]. In this case, in a narrow spectral range, the helicity-flip scattering cross-section can vary hundreds of times (while the transport cross-section remains practically unchanged). At the Kerker point itself, circular polarization decays on scales much exceeding the transport length [18, 36].

In the absence of magnetic field, the equations in system (16) become independent:

$$\Delta I(\mathbf{r},\mathbf{r}') = -3\sigma_{tr}\delta(\mathbf{r}-\mathbf{r}'),$$
$$\left[\Delta - 3\sigma_{dep}\sigma_{tr}^{(V)}\right]V(\mathbf{r},\mathbf{r}') = -3\sigma_{tr}^{(V)}\delta(\mathbf{r}-\mathbf{r}'). \quad (21)$$

The solution of the diffusion equation for the Fourier component over variables parallel to layer boundaries is determined by expression

$$V(z,z\,|\,\mathbf{q}) = 3\gamma l_{tr}\gamma z\gamma(L-z)\gamma L, \quad (22)$$

$$\gamma = \sqrt{q^2 + 1/l_{circ}^2}.$$

Similar expression for $I(z,z\,|\,\mathbf{q})$ follows from (22) at $\gamma = q$.

Substitution of solution (22) into formula (20) gives the following expression for the relative change in the interference correction

$$\langle\delta T(\sigma_{dep})\rangle = \frac{1}{2\pi}\frac{A}{Ll_{circ}}. \quad (23)$$

In (23) it is assumed that

$$\sigma_{tr} - \sigma_{tr}^{(2)} \ll \sigma_{tr}.$$

Note that, unlike the resonant spectral dependence of σ_{dep}, the values of transport coefficients in the vicinity of the first Kerker point remain practically unchanged. Result (23) is valid in the limit $L \gg l_{circ}$. It can also be obtained if, when calculating V we neglect the finite size of the sample and use the solution in the infinite medium approximation.

Taking into account the finite value of L somewhat complicates the expression for the interference contribution to conductance

$$\langle\delta T(\sigma_{dep})\rangle = \frac{1}{2\pi}\frac{A}{L^2}\ln\left(\frac{\mathrm{sh}(L/l_{circ})}{(L/l_{circ})}\right) \quad (24)$$

The spectral dependence $\langle\delta T\rangle$, calculated using formulas (23) and (24), is shown in Fig. 4. The value of the length l_{circ} entering into (23) and (24) was calculated using Mie theory [38]. As follows from the calculation results, the resonant dependence of σ_{dep} directly affects the behavior of the interference contribution to conductance. Taking into account the finite thickness of the sample only leads to a decrease in peak amplitude.

4. WAVE INTERFERENCE CORRECTION IN MAGNETO-ACTIVE MEDIUM

Let us proceed to analyze the dependence of the interference contribution to optical conductance on the magnetic field. As in the previous section, we consider a sample in the form of a flat layer and assume that $L_x, L_y \gg L \gg L_{tr}$.

In the absence of depolarization ($\sigma_{dep} = 0$, $\sigma_{tr}^{(2)} = = \sigma_{tr}$) the systems of equations for pairs of quantities I, V_I and V in (16) coincide with each other. The magnetic field in the equations for linear combinations $I \pm V_I$ and $V \pm I_V$ can be eliminated by transformation

$$I(\mathbf{r},\mathbf{r}';\mathbf{h}) \pm V_I(\mathbf{r},\mathbf{r}';\mathbf{h}) = \exp(\mp i\mathbf{h}(\mathbf{r}-\mathbf{r})')F(\mathbf{r},\mathbf{r}'),$$

$$V(\mathbf{r},\mathbf{r}';\mathbf{h}) \pm I_V(\mathbf{r},\mathbf{r}';\mathbf{h}) = \exp(\mp i\mathbf{h}(\mathbf{r}-\mathbf{r})')F(\mathbf{r},\mathbf{r}').$$

Therefore, the values $I(\mathbf{r},\mathbf{r}';\mathbf{h})$ and $V(\mathbf{r},\mathbf{r}';\mathbf{h})$ entering into equation (15) with matching arguments are independent of the magnetic field. Thus, in the absence of depolarization, the magnetic field does not affect the magnitude of the interference contribution to conductance.

The dependence of the interference correction to conductance on the magnetic field arises only due to depolarizing collisions, which change the helicity at different sections of wave propagation trajectories.

Consider the case of a relatively strong magnetic field, $1/L \ll h \ll 1/l_{tr}$, when calculating propagators I and V, the medium can be considered unbounded, but the diffusion approximation is applicable (see the system of equations (16)). In this situation, the interference contribution does not depend on the field direction. The orientational dependence manifests only at $h \leqslant 1/L$, when effects due to the finiteness of the sample become significant. However, at $h \leqslant 1/L$ the field's influence on change of the interference correction is insignificant.

At $h \gg 1/L$, expanding the solution of equation system (16) in a Fourier integral, from formula (15) we obtain

$$\langle \delta T(\mathbf{h})\rangle = -\frac{A}{4\pi^3 L}\times \int \frac{\Phi^2(Q)d\mathbf{Q}}{\Phi^2(Q) - 4(\mathbf{Qh})^2}\left[\frac{1}{Q^2 + (\sigma_{tr}/\sigma_{tr}^{(V)})h^2} + \frac{\sigma_{tr}^{(2)}\sigma_{tr}^{(V)}}{\sigma_{tr}^2}\frac{1}{Q^2 + (\sigma_{tr}^{(V)}/\sigma_{tr})h^2 + 1/l_{circ}^2}\right] \quad (25)$$

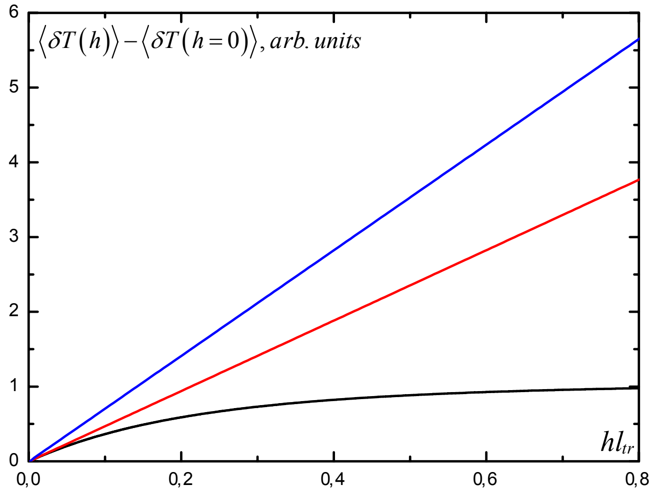

Fig. 5. Change in interference contribution (δT (h)) − (δT (h = 0)) to optical conductance with increasing magnetic field. Change in interference contribution to optical conductance with increasing magnetic field under conditions of slow circular polarization decay effect ($\sigma_{tr}/\sigma_{dep} = 10$), Rayleigh scattering ($\sigma_{tr}/\sigma_{dep} = 1$) and at the second Kerker point Spectral dependence of interference correction to conductance near the first Kerker point ($\sigma_{tr}/\sigma_{dep} = 0.75$) from lower to upper curves.

where

$$\Phi^2(Q) = \frac{4\sigma_{tr}\sigma_{tr}^{(V)}}{\left(\sigma_{tr} + \sigma_{tr}^{(V)}\right)^2}\times\left(Q^2 + \frac{\sigma_{tr}}{\sigma_{tr}^{(V)}}h^2\right)\left(Q^2 + \frac{\sigma_{tr}^{(V)}}{\sigma_{tr}}h^2 + \frac{1}{l_{circ}^2}\right) \quad (26)$$

As known [2, 10, 13], in the 3D case, the interference correction itself diverges in the diffusion approximation (each contribution in formula (25) is a divergent quantity at large Q). The observable is the relative change in the interference correction depending on the magnetic field,

$$\langle \delta T(\mathbf{h})\rangle - \langle \delta T(\mathbf{h}=0)\rangle$$

Integrating (25) over the direction of vector $\mathbf{Q}$, this quantity can be represented as

$$\langle \delta T(\mathbf{h})\rangle - \langle \delta T(\mathbf{h}=0)\rangle = -\frac{A}{\pi^2 L}\int_0^\infty Q^2 dQ \times \left\{\left[\frac{1}{Q^2 + (\sigma_{tr}/\sigma_{tr}^{(V)})h^2}\right.\right. \times$$

$$\times\left[\frac{\Phi(Q)}{4Qh}\ln\frac{\Phi(Q)+2Qh}{\Phi(Q)-2Qh}-\frac{1}{Q^2}\right]+$$

$$+\frac{\sigma_{tr}^{(2)}\sigma_{tr}^{(V)}}{\sigma_{tr}^{2}}\left[\frac{1}{Q^2+(\sigma_{tr}^{(V)}/\sigma_{tr})h^2+1/l_{circ}^2}\times\right.$$

$$\left.\times\left(\frac{\Phi(Q)}{4Qh}\ln\frac{\Phi(Q)+2Qh}{\Phi(Q)-2Qh}-\frac{1}{Q^2+1/l_{circ}^2}\right)\right]. \quad (27)$$

Under conditions of circular polarization "memory" effect,

$$\sigma_{dep} \ll \sigma_{tr}$$

(note that in this case $\sigma_{tr}-\sigma_{tr}^{(V)} \ll \sigma_{tr}$), two regions can be distinguished $\langle\delta T\rangle$ on $\mathbf{h}$. At $h \ll 1/l_{circ}$ $\langle\delta T\rangle$ changes linearly with magnetic field h:

$$\langle\delta T(\mathbf{h})\rangle-\langle\delta T(\mathbf{h}=0)\rangle=\frac{1}{2\pi}\frac{Ah}{L}. \quad (28)$$

Only the first term contributes to formula (28) in (15) and, accordingly, in (27). In this case, the interference correction

$$\langle\delta T(\mathbf{h})\rangle-\langle\delta T(\mathbf{h}=0)\rangle$$

is determined by long trajectories exceeding l_{circ}, and mode I is determined by equation (18).

When $h \gg 1/l_{circ}$ the interference contribution $\langle\delta T\rangle$ reaches a plateau (see Fig. 5) and tends to the value

$$\langle\delta T(\mathbf{h})\rangle-\langle\delta T(\mathbf{h}=0)\rangle=\frac{\sqrt{2}-1}{2\pi}\frac{A}{Ll_{circ}}. \quad (29)$$

The plateau in the dependence of $\langle\delta T(\mathbf{h})\rangle-\langle\delta T(\mathbf{h}=0)\rangle$ on the magnetic field can be explained by the decrease in the probability of depolarizing collisions over the length h^{-1} with increasing field. Under conditions of "memory" of circular polarization ($\sigma_{dep} \ll \sigma_{tr}$, $\sigma_{tr}^{(V)}=\sigma_{tr}$) the integral in formula (25) takes the form

$$\int d\mathbf{Q}\frac{Q^2+h^2+1/(2l_{circ}^2)}{(Q^2+h^2+1/(2l_{circ}^2))^2-4(\mathbf{Qh})^2-1/(4l_{circ}^4)}$$

(30)

In a strong field, $h \gg 1/l_{circ}$, neglecting the term $1/l_{circ}^4$ in the denominator of equation (30), this expression can be written as

$$\frac{1}{2}\int d\mathbf{Q}\left[\frac{1}{(\mathbf{Q}-\mathbf{h})^2+1/(2l_{circ}^2)}+\right.$$

$$\left.+\frac{1}{(\mathbf{Q}+\mathbf{h})^2+1/(2l_{circ}^2)}\right] \quad (31)$$

Field $\mathbf{h}$ is eliminated from (31) by shifting in the integration variable $\mathbf{Q}$, and the interference contribution to conductance ceases to depend on the magnetic field.

In the absence of the circular polarization "memory" effect, $\sigma_{dep} \sim \sigma_{tr}$, (for example, in the case of Rayleigh scattering or in the vicinity of the second Kerker point), the magnetic field- dependent part of the interference correction is determined only by the first term in expression (15), and, accordingly, by the solution of equation (19). The slope in the linear dependence $\langle\delta T(\mathbf{h})\rangle-\langle\delta T(\mathbf{h}=0)\rangle$ on h at $h \ll 1/l_{circ}$ changes compared to (29) due to the change in the coefficient before h^2 in equation (19),

$$\langle\delta T(\mathbf{h})\rangle-\langle\delta T(\mathbf{h}=0)\rangle=\frac{1}{2\pi}\frac{Ah}{L}\sqrt{\frac{\sigma_{tr}}{\sigma}}. \quad (32)$$

Mode V does not contribute to the linear dependence (32) on the magnetic field. In strong magnetic fields, $h \gg 1/l_{tr}$, the diffusion description loses its applicability.

5. OPTICAL CONDUCTANCE OF A WAVEGUIDE

Besides a flat layer, another frequently used type of samples in research is a waveguide with scattering inhomogeneities [39, 40]. When electromagnetic waves propagate in a waveguide, L_x, $L_y \ll L$ (Q1D-geometry), the situation changes. In this case, if we do not consider a waveguide with a specially selected transverse profile of refractive index, the effect of circular polarization preservation is suppressed. At each reflection from the lateral boundaries of the waveguide, the sign of circular polarization changes to the opposite [41]. The main contribution to the interference correction (15) is made by the quantity I(r; r') surviving at large at large L $I(\mathbf{r},\mathbf{r}')$ which in the absence of a magnetic field corresponds to the scalar intensity mode. Mode

$V(\mathbf{r},\mathbf{r}')$ decays on the scale $\min(L_x, L_y, l_{circ})$, and its contribution turns out to be a small value. In this situation, in the equation for mode I, the term proportional to V_I (see (16)) can be neglected (see Appendix), and we arrive at equation (18). For the waveguide, equation (18) should be supplemented with boundary conditions

$$\left.\frac{\partial}{\partial x} I(\mathbf{r},\mathbf{r}')\right|_{x=\pm L_x/2} = 0, \qquad \left.\frac{\partial}{\partial y} I(\mathbf{r},\mathbf{r}')\right|_{y=\pm L_y/2} = 0 \tag{33}$$

on the lateral surfaces of the waveguide and

$$\left. I(\mathbf{r},\mathbf{r}')\right|_{z=0,L} = 0 \tag{34}$$

at the input and output cross-sections of the waveguide.

Equation (18) is valid when modes I and V_I propagate in the diffusion regime. Under conditions of strong depolarization, when circular polarization decay occurs on scales of the order of mean free path l, equation (19) should be used instead of equation (18) (see Appendix). The reduction in depolarization length is reflected in the coefficient value before h^2 in the equation for $.I$.

The solution of equation (18) with boundary conditions (33) can be sought in the form of expansion in eigenmodes $\cos q_{xn}x \cos q_{ym}y$, where

$$q_{xn} = (2\pi n / L_x), \quad q_{ym} = (2\pi m / L_y),$$

$$n, m = 0, \pm 1, \pm 2, \ldots$$

Then formula (15) transforms as follows:

$$\langle \delta T(\mathbf{h}) \rangle = -23 l_{tr} L^2 \left[\sum_{\mathbf{q}} \int_0^L dz\, I_{\mathbf{q}}(z,z) \right] \tag{35}$$

where $I_{\mathbf{q}}(z,z')$ are the expansion coefficients of value $I(\mathbf{r},\mathbf{r}')$ over the waveguide's transverse eigenmodes. From the solution of equation (18) with boundary condition (34), one can obtain the expression for them

$$I_{\mathbf{q}}(z,z') = \frac{3}{\gamma l_{tr}} \frac{\sinh \gamma z (\gamma (L - z))}{\sinh \gamma L}, \tag{36}$$

where

$$\gamma^2 = \mathbf{q}^2 + \tilde{h}^2, \quad \tilde{h} = \sqrt{\frac{\sigma_{tr}}{\sigma_{tr}^{(V)}}} h. \tag{37}$$

Under conditions of strong depolarization, only the proportionality coefficient between $\tilde{h}$ and h changes (see (19)).

In Q1D-geometry ($L \gg L_x, L_y$) the mode with $\mathbf{q} = 0$ is characterized by the lowest attenuation. Keeping only the term with $\mathbf{q} = 0$ in (35), for the interference contribution to conductance we obtain

$$\langle \delta T(\mathbf{h}) \rangle = -\frac{1}{(\tilde{h}L)} \left[(\tilde{h}L) - \frac{1}{\tilde{h}L} \right]. \tag{38}$$

In the limit $h = 0$, equation (38) leads to the known result [42] for the interference contribution to the conductance of scalar waves $\langle \delta T \rangle = -1/3$. With increasing magnetic field, the magnitude of the interference correction monotonically decreases. At large h, the value of $\langle \delta T \rangle$ decreases as $-1/\tilde{h}L$. The change in $\langle \delta T \rangle$ with increasing field qualitatively resembles the behavior of the interference correction to electronic conductance [11, 10], however, the corresponding functional dependencies differ.

The results obtained above refer to the case of a waveguide with a sufficiently large cross- section, $l \ll \sqrt{A}$, when the conditions for going from the system of transport equations (13) to equation (18) (or (19), see Appendix) are met.

The situation changes in a waveguide with small cross-section, $l \gg \sqrt{A}$. It can be assumed that the circularly polarized mode propagates in ballistic regime and is destroyed upon reflection from boundaries at scales of order $\sqrt{A}$. In this approximation, we arrive at a diffusion equation for I, similar to (18) (or (19)), but with an additional factor proportional to $\sqrt{A} / l_{tr}$ before h^2. In the case of a waveguide with circular cross-section, the corresponding numerical coefficient changes from $8\sqrt{\pi}$ in longitudinal field to $4\sqrt{\pi}$ in transverse field.

As the waveguide cross-section decreases, the influence of magnetic field on the value of interference correction decreases as well. The length at which interference destruction occurs increases with decreasing A as $(l_{tr}^{1/2} / A^{1/4})(1/h)$. Since the applicability of the diffusion treatment used for I in this work is limited by fields $h < 1/l_{tr}$, this length always exceeds $(l_{tr}^{3/2} / A^{1/4})$. To observe the influence of magnetic field on interference, this value should not be greater than the localization length $l_{loc} = N l_{tr} \sim l_{tr} A / \lambda^2$. Therefore, the above results are limited by the condition on the waveguide cross-section $A > \lambda^2 (l_{tr} / \lambda)^{2/5}$. In the considered case, in the weak localization limit $\lambda \ll l_{tr}$, the waveguide can be considered multi-mode.

6. DISCUSSION OF RESULTS

Let us analyze how the interference contribution to optical conductance changes depending on the ratio $\sigma_{dep} / \sigma_{tr}$ and the magnetic field strength.

In the absence of a magnetic field ($h = 0$) the negative interference correction $\langle \delta T \rangle$ to conductance is maximal in magnitude in the limit when there is no depolarization at all (i.e., no mixing of polarizations due to scattering, and $I = V$ in (15)). In this case

$$\langle \delta T \rangle = 2 \langle \delta T^{(sc)} \rangle,$$

where $\langle \delta T^{(sc)} \rangle < 0$ — corresponds to the result in the scalar wave approximation. The scattering of light on an ensemble of Mie particles near the first Kerker point ($\lambda = 2.29na$, where λ is the wavelength of light, a and n are the radius and refractive index of the particle) corresponds most closely to the limiting situation described above [18,36,37]. In this case, for particles with a high refractive index, the helicity-flip scattering cross-section can be two orders of magnitude smaller than the transport scattering cross-section [18, 36]. Taking into account the small but finite value of σ_{dep} leads to deviation of $\langle \delta T \rangle$ from $2\langle \delta T^{(sc)} \rangle$. Under conditions of rare helicity-flip collisions ($\sigma_{dep} \ll \sigma_{tr}$) the interference contribution to optical conductance acquires an additional term (see (23))

$$\langle \delta T \rangle = 2 \langle \delta T^{(sc)} \rangle + \frac{1}{2\pi} \frac{A}{L l_{circ}}. \tag{39}$$

The deviation from $2\langle \delta T^{(sc)} \rangle$ in (39) is determined by the second term of expression (15), which increases sharply as it moves away from the first Kerker point. The resonant behavior of σ_{dep} directly affects the magnitude of the interference correction (see Fig.4).

In the case of strong wave depolarization, when helicity changes occur in each scattering event, $\langle \delta T \rangle = \langle \delta T^{(sc)} \rangle$ (this follows directly from formula (15) if we neglect the contribution of V).

If there were no wave scattering with helicity change, then the interference contribution to optical conductance would remain unchanged, $\langle \delta T \rangle = 2\langle \delta T^{(sc)} \rangle$ when applying a magnetic field. This is because the magnetic field can be eliminated from equation (16) and, accordingly, formula (15) (see section 4). Depolarization turns on the influence of the magnetic field on the value of $\langle \delta T \rangle$. Under conditions of slow depolarization ($\sigma_{dep} \ll \sigma_{tr}$) the change in the interference contribution with increasing magnetic field is described by expressions (28), (29), and at $h \gg 1 / l_{circ}$ the value of $\langle \delta T \rangle$ tends to

$$\langle \delta T \rangle = 2 \langle \delta T^{(sc)} \rangle + \frac{1}{\sqrt{2}\pi} \frac{A}{L l_{circ}}, \tag{40}$$

i.e., the difference between $\langle \delta T \rangle$ and $2\langle \delta T^{(sc)} \rangle$ changes by approximately one and a half times when the field varies from zero to large values.

The saturation of the interference correction dependence on h with increasing magnetic field can be explained by the decrease in the probability of depolarizing collisions over the length h^{-1}. Before the first helicity-flip scattering event modes I and V manage to mix strongly, and "hybridization" of modes I and V occurs. Only two modes with given helicity survive, $I + V_I$ and $V + I_V$, which are characterized by an attenuation length $\sqrt{2}$ times greater than the attenuation of mode V in the absence of a magnetic field (see expression (31)).

In the case of strong depolarization, the interference contribution to conductance tends to zero as the magnetic field strength increases. Under conditions of wave diffusion through a Q1D sample (waveguide), this is precisely the case that is realized. With rapid attenuation of mode V the main contribution to (15) comes from intensity I. According to (35), the transition to Q1D geometry occurs when the term with $q = 0$. becomes predominant in (35). In the absence of a magnetic field, the contribution to (35) from non-zero harmonics can be estimated as

$$\langle \delta T^{(sc)} \rangle = -\frac{2}{3} \frac{A l_{tr}}{L^2} \int \frac{d\mathbf{q}}{(2\pi)^2} \int_0^L dz I_{\mathbf{q}}(z, z) \sim -\frac{A}{l_{tr} L}, \tag{41}$$

where it is taken into account that the integral over $\mathbf{q}$ is cut off at values $q \sim 1 / l_{tr}$. The requirement that (41) be small compared to the contribution from the zero harmonic $\langle \delta T^{(sc)} \rangle_{q=0} = -1/3$ [42] leads to the inequality $A \ll l_{tr} L$, i.e., the cross-sectional area of the sample should be smaller than the average square of the transverse displacement of the light beam during diffusion in the medium layer of thickness L.

It should be noted that the disorder-averaged conductance in the diffusion approximation in the first $1/k_0 l_{tr}$ order equals

$$\langle T \rangle = \frac{8}{3}\frac{\mathcal{N} l_{tr}}{L}$$

(see, for example, [8, 17]. The interference correction to conductance $\langle \delta T \rangle$ has the order of

$$\langle \delta T \rangle \sim \frac{1}{(k_0 l_{tr})^2}\frac{\mathcal{N} l_{tr}}{L},$$

until the transition to Q1D geometry occurs. In this geometry, in the absence of a magnetic field $\langle \delta T \rangle = -1/3$. Therefore, in a long $L \sim l_{loc} = \mathcal{N} l_{tr}$ [42], waveguide, when $\langle T \rangle$ becomes of the order of $\langle \delta T \rangle$, a transition to the Anderson localization regime occurs. Since the interference correction $\langle \delta T \rangle$ in the presence of the magnetic field decreases in the Q1D case in the presence of a magnetic field with increasing L as $1/L$ (see (38)), the transition to the Anderson localization regime at $\tilde{h} > h_c \sim 1/\mathcal{N} l_{tr}$ should be disrupted. Due to the destruction of time-reversed wave interference, the localization length should increase (see, for example, [42]). In multimode optical fiber ($\mathcal{N} \gg 1$) the critical value of the magnetic field h_c turns out to be much smaller than that which leads to a noticeable effect in coherent backscattering from a Faraday medium [19,22]. In electron transport through Q1D systems (wires) it is difficult to observe such an effect, since with increasing L there is destruction of coherence of electron waves due to inelastic interactions, temperature, and other factors [11,13].

In conclusion, we consider the conditions under which experimental observation of the interference contribution to conductance is possible. The simplest scheme is the one for measuring the total transmission coefficient through a waveguide with scattering inhomogeneities. Let's assume that the measurement sensitivity allows registering relative changes in the transmission coefficient of about one percent. Since in Q1D geometry the interference correction varies from $-1/3$ to zero (when a magnetic field is applied), this imposes a requirement on the value of $\langle T \rangle$, which should not be greater than 10^2. Such value of $\langle T \rangle$ can be provided in a waveguide with length $L = 10$ cm, cross-sectional area $\mathcal{A} = 10^{-2}$ mm^2, number of modes $\mathcal{N} = 10^4$ at wavelength $\lambda = 1$ μm with a typical transport length of $l_{tr} = 100$ μm for disordered media. In this case, the ratio $\mathcal{A}/(L l_{tr})$ (see (41)) equals 10^{-3}, so the Q1D geometry approximation is certainly justified. Since in a magnetic field the interference correction changes significantly at $h \sim 10/L$ (see (38)), then for characteristic values of the Verdet constant ($V = 0.06$ mm^{-1} T for $\lambda = 1.064$ μm, see , for example [43, 44]) the correction to conductance can be observed in a field of 1 T. With increasing waveguide length, the amplitude of conductance changes increases with magnetic field, which makes it possible to study the transition to strong localization regime under conditions of broken T-invariance.

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APPENDIX

Assuming that mode V decays faster than mode I (i.e., in the approximation of relatively rapid depolarization), the interference correction to conductance is determined only by mode I. According to the system of transport equations (13), the coupled equations for modes I and V_I have the form

$$\left\{\mathbf{n}\frac{\partial}{\partial \mathbf{r}} + \sigma_{tot}\right\} I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') + i(\mathbf{n}\mathbf{h}) V_I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') = \int d\mathbf{n}_1\, a_1(\mathbf{n}\mathbf{n}_1) I(\mathbf{r},\mathbf{n}_1\,|\,\mathbf{r}',\mathbf{n}') + \delta(\mathbf{n}-\mathbf{n}')\delta(\mathbf{r}-\mathbf{r}'), \quad (42)$$

$$\left\{\mathbf{n}\frac{\partial}{\partial \mathbf{r}} + \sigma_{tot}\right\} V_I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') + i(\mathbf{n}\mathbf{h}) I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') = \int d\mathbf{n}_1\, a_2(\mathbf{n}\mathbf{n}_1) V_I(\mathbf{r},\mathbf{n}_1\,|\,\mathbf{r}',\mathbf{n}'). \quad (43)$$

Writing the expression for V_I as a convolution of the Green's function of equation (43) with the term $i(\mathbf{n}\mathbf{h})I$ in (43) and substituting V_I into (42), we obtain a closed equation for mode I

$$\left\{\mathbf{n}\frac{\partial}{\partial \mathbf{r}} + \sigma_{tot}\right\} I(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') + (\mathbf{n}\mathbf{h})\int d\mathbf{r}'' d\mathbf{n}'' G_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}'',\mathbf{n}'')(\mathbf{n}''\mathbf{h}) \times I(\mathbf{r}'',\mathbf{n}''\,|\,\mathbf{r}',\mathbf{n}') = \int d\mathbf{n}_1 a_1(\mathbf{n}\mathbf{n}_1) I(\mathbf{r},\mathbf{n}_1\,|\,\mathbf{r}',\mathbf{n}') + \delta(\mathbf{n}-\mathbf{n}')\delta(\mathbf{r}-\mathbf{r}'), \quad (44)$$

where $G_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}')$ is the Green's function of equation (43) with a source in the form of $\delta(\mathbf{n}-\mathbf{n}')\delta(\mathbf{r}-\mathbf{r}')$. Performing the standard procedure of transition to the diffusion approximation in (44) (see, for example, [45]), we arrive at the equation

$$\Delta I(\mathbf{r},\mathbf{r}') + 3\frac{\partial}{\partial x_i}\int d\mathbf{r}''\int d\mathbf{n}\,d\mathbf{n}'\,n_i(\mathbf{n}\mathbf{h})\int d\mathbf{n}'' \times G_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}'',\mathbf{n}'')(\mathbf{n}''\mathbf{h}) I(\mathbf{r}'',\mathbf{n}''\,|\,\mathbf{r}',\mathbf{n}') - 3\sigma_{tr}\int d\mathbf{r}''\int d\mathbf{n}\,d\mathbf{n}'(\mathbf{n}\mathbf{h})\int d\mathbf{n}'' \times G_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}'',\mathbf{n}'')(\mathbf{n}''\mathbf{h}) I(\mathbf{r}'',\mathbf{n}''\,|\,\mathbf{r}',\mathbf{n}') = -3\sigma_{tr}\delta(\mathbf{r}-\mathbf{r}'). \quad (45)$$

Assuming further that function G_V decreases at distances much smaller than the characteristic scale of mode I-variation, we can expand I in the vicinity of $\mathbf{r}''=\mathbf{r}$.

Keeping only the first term in the expansion, we obtain

$$\Delta I(\mathbf{r},\mathbf{r}') - \sigma_{tr}\sigma_{tr}^{(V)} h^2 I(\mathbf{r},\mathbf{r}') + 3h_j h_k \sigma_{tr}^{(V)} - \sigma_{tr}\sigma_{tr}^{(V)} \times \left[\int d\mathbf{n}\,d\mathbf{n}'\,n_j n_k' \int d\mathbf{r}''(\mathbf{n}\nabla) G_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}'',\mathbf{n}')\right] \times I(\mathbf{r},\mathbf{r}') = -3\sigma_{tr}\delta(\mathbf{r}-\mathbf{r}'). \quad (46)$$

Then we transform the third term in the left part of equation (46). Substituting the equality that follows from the reciprocity theorem (see, for example, [46])

$$(\mathbf{n}\nabla_{\mathbf{r}}) G_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}'',\mathbf{n}') = -(\mathbf{n}'\nabla_{\mathbf{r}''}) G_V(\mathbf{r}'',-\mathbf{n}'\,|\,\mathbf{r},-\mathbf{n}), \quad (47)$$

we can represent it as

$$3h_j h_k \sigma_{tr}^{(V)} - \sigma_{tr}\sigma_{tr}^{(V)} \times \left[\int d\mathbf{n}\,d\mathbf{n}'\,n_j' n_k \oint_S (\mathbf{n}'d\mathbf{s}) G_V(\mathbf{r}_S,-\mathbf{n}'\,|\,\mathbf{r},-\mathbf{n})\right] \times I(\mathbf{r},\mathbf{r}'), \quad (48)$$

where integration is performed over the sample surface, and $\mathbf{r}_s$ is taken at its boundary. When the distance from point $\mathbf{r}$ to the sample boundary exceeds the decay length of the circularly polarized mode l_{circ}, the contribution of term (48) to equation (46) can be neglected. As a result, equation (46) reduces to equation (18). This statement remains valid as long as the linear dimensions of the sample are much larger than the transport length, $(L,\sqrt{A}) \gg l_{tr}$, and to calculate the Green's function G_V one can use the diffusion approximation, setting

$$G_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}'',\mathbf{n}') = G_V(\mathbf{r},\mathbf{r}'')/(4\pi)^2,$$

where

$$G_V(\mathbf{r},\mathbf{r}'') = \int d\mathbf{n}\,d\mathbf{n}' G_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}'',\mathbf{n}')$$

is the density propagator of the circularly polarized mode V. In this case, contribution (48) vanishes due to the density propagator becoming zero at the boundaries of the integration region [45].

If in the expansion of I in the vicinity of $\mathbf{r}''=\mathbf{r}$ we keep the next term in (45), this will lead to renormalization of the coefficient before ΔI in equation (46) by a small addition of order $(hl_{tr})^2 \ll 1$, which can be neglected in the considered approximation.

The results obtained above (see (18)) rely on the diffusion approximation when calculating the Green's function G_V. Under conditions of strong depolarization, $\sigma_{dep} \sim \sigma_{tr}$, the attenuation length of the circularly polarized mode turns out to be close to the mean free path l [18, 32], and in the first approximation for calculating the function, G_V one can use the ballistic approximation, i.e., assume that the main contribution to G_V comes from non-scattered waves,

$$G_V(\mathbf{r},\mathbf{n}\,|\,\mathbf{r}',\mathbf{n}') =$$
$$= \delta(\mathbf{n}-\mathbf{n}')\delta\left(\mathbf{n}-\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\right)\frac{e^{-\sigma|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|^2}. \qquad (49)$$

Substituting (49) into (44), and assuming that the mode I is a smoother function of coordinates and directions than G_V, we arrive at a diffusion equation for I of the form (18), in which $\sigma_{tr}^{(V)}$ should be replaced with σ (see (19)).

Equations (18) and (19) remain valid for the waveguide, $L \gg \sqrt{A}$. The applicability of equation (18) under conditions of slow circular polarization decay is related to the use of the diffusion approximation for G_V. Due to the fact that upon reflection from the waveguide's lateral boundaries the sign of circular polarization changes to the opposite, the density propagator G_V vanishes at the waveguide boundaries (i.e., when the scattering medium is placed in a waveguide with reflecting boundaries, the boundary conditions for G_V, unlike the mode I, do not change). Equation (19) under conditions of strong depolarization is valid as long as the cross-section is sufficiently large, $\sqrt{A} > l$. In a waveguide with a small cross- section but remains multimode, $\lambda \ll \sqrt{A} \ll l$, substitution of the ballistic Green's function (49) into (44) after intermediate integration over spatial variables leads to a diffusion equation for $I_{q=0}(z,z')$, similar to (19), but with an additional factor proportional to $\sqrt{A}/l_{tr}$ before h^2.

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