SPACE PLASMA

NONLINEAR PERIODIC DUSTY SOUND WAVES IN THE MAGNETOSPHERE OF SATURN

 Izvekova Yu. N.[a, *], Kopnin S. I.[a], Shokhrin D. V.[b], Popel S. I.[a]

[a] *Space Research Institute of the Russian Academy of Sciences, Moscow, Russia*

[b] *National Research University "Higher School of Economics", Moscow, Russia*

e-mail: besedina_yn@mail.ru

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Abstract. A characteristic feature of Saturn's magnetosphere is the presence of two types of electrons, obeying kappa distributions - hot and cold. Electrons, magnetospheric ions and dust particles, which were discovered within the Cassini mission, form a plasma-dust system in Saturn's magnetosphere. Nonlinear periodic dust sound waves of arbitrary amplitude, which can propagate in the dusty magnetosphere of Saturn, are considered. The obtained results are important for the interpretation of future space observations.

Keywords: *dusty plasma, nonlinear dust sound waves, kappa distribution, Saturn's magnetosphere*

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1. INTRODUCTION

Studies of dusty plasma in the Solar system are currently relevant due to the accumulated data from space missions, the development of theoretical models allowing to investigate such systems, as well as in light of planned space exploration. Dust and dusty plasma are important objects of study near the surface of the Moon, Mercury, Mars' satellites and other airless cosmic bodies [1-13], in atmospheres and magnetospheres of planets [14-22], planetary rings, comet tails [23-25], and interplanetary space. Saturn's magnetospheric plasma was first investigated during the Voyager 1 and Voyager 2 missions in the 1980s. Evidence of waves [26] in Saturn's magnetospheric plasma was obtained at that time. Theoretical studies of ion-acoustic waves in Saturn's magnetosphere were conducted in [27]. During the Cassini mission [28, 29], dusty plasma was discovered in the Saturn system near its satellite Enceladus. Research within this mission also pointed to a possible cause of its appearance — the Cassini spacecraft discovered fountains of dust particles and small water ice particles (which can also be interpreted as dust particles) hundreds of kilometers high, erupting from four cracks near Enceladus' south pole [30]. It was also discovered [26, 31, 32] that two types of electrons — hot and cold — are simultaneously present in Saturn's magnetosphere, with electron distributions described by kappa distributions [32].

Kappa distributions are typical for planetary magnetospheres. In collisionless magnetospheric plasma, the relaxation of particle distribution functions formed as a result of acceleration and transport initially leads to kappa distributions and much later to distributions close to Maxwellian (see, for example, [33]). The formation of kappa distributions is due to the existence of long-range correlations in collisionless magnetospheric plasma and the action of turbulent acceleration and turbulent particle transport processes.

In dust plasma with parameters corresponding to the conditions in Saturn's magnetosphere, nonlinear waves can exist, primarily dust acoustic waves. Previously, the possibility of the existence of solitary nonlinear dust acoustic waves—solitons—in this system has been demonstrated [14]. It was shown [17] that under the conditions of Saturn's magnetosphere, there are solutions to the Zakharov-Kuznetsov equation describing one-dimensional and three-dimensional solitons; a two-dimensional description of the dusty plasma in Saturn's magnetosphere was also conducted [15], resulting in solutions to the Kadomtsev-Petviashvili equation in the form of one-dimensional solitons and two-dimensional N -solitons. In this paper, we propose to consider nonlinear periodic dust acoustic waves, taking into account the presence of two types of electrons. The analysis is conducted for arbitrary amplitudes of nonlinear waves, which is important from the perspective of interpreting data that may be obtained in future missions.

2. BASIC EQUATIONS

Let us write the basic equations for nonlinear periodic dust acoustic waves in Saturn's magnetosphere. We will consider one-dimensional perturbations along the coordinate x . Dust acoustic nonlinear waves are described by a system of equations [14], consisting of the Poisson equation for the potential and equations defining the concentrations of plasma components. Let us write the Poisson equation as

$$\frac{\partial^2 \varphi}{\partial x^2} = 4\pi e(n_e - n_i - n_d Z_d), \tag{1}$$

where φ is the self-consistent potential in the plasma, $n_{d(i,e)}$ are the concentrations of dust particles (ions, electrons), $-e$ is the electron charge, Z_d is the dust particle charge, expressed in the number of electrons. Since dust in Saturn's magnetosphere can be considered unmagnetized, the continuity and Euler equations can be used to describe its dynamics, presented in the following form:

$$\frac{\partial n_d}{\partial t}+\frac{\partial\left(n_d v_d\right)}{\partial x}=0, \tag{2}$$

$$\frac{\partial v_d}{\partial t}+v_d\frac{\partial v_d}{\partial x}=-\frac{eZ_d}{m_d}\frac{\partial\varphi}{\partial x}, \tag{3}$$

where v_d is the directed velocity of dust particles, m_d is the mass of a dust particle.

On dust sound scales, electron and ion distributions have time to establish. Ions follow the Boltzmann distribution

$$n_i=n_{i0}e^{-e\varphi/T_i}, \tag{4}$$

where T_i is the ion temperature expressed in energy units, and the index "0" here and further will denote unperturbed states. It should be taken into account that in Saturn's magnetospheric plasma there are two types of electrons - cold ones with concentration $n_{e,c}$ and hot ones with concentration $n_{e,h}$, where

$$n_e=n_{e,c}+n_{e,h}. \tag{5}$$

Electrons of both types follow kappa distributions [34]

$$n_{e,c}=n_{e,c0}\left(1-\frac{1}{\kappa_c-3/2}\frac{e\varphi}{T_{ec}}\right)^{-\kappa_c+1/2}, \tag{6}$$

$$n_{e,h}=n_{e,h0}\left(1-\frac{1}{\kappa_h-3/2}\frac{e\varphi}{T_{eh}}\right)^{-\kappa_h+1/2}, \tag{7}$$

where $T_{ec(h)}$ is the temperature of cold (hot) electrons expressed in energy units, $\kappa_{c(h)}$ is the kappa distribution parameter of cold (hot) electrons, where $\kappa_{c(h)}>3/2$.

The quasi-neutrality condition for unperturbed concentration values takes the form

$$n_{i0}+Z_d n_{do}=n_{e,c0}+n_{e,h0}=n_{e0}. \tag{8}$$

For convenience, let's introduce a coefficient for the ratio between cold and hot electron concentrations α . Then we have

$$n_{e,c0} = \alpha\left(n_{i0} + Z_d n_{d0}\right), \quad n_{e,\mathrm{h}0} = \left(1-\alpha\right)\left(n_{i0} + Z_d n_{d0}\right). \tag{9}$$

The characteristic time scales of dust sound waves significantly exceed the characteristic time of dust particle charge variation [35], i.e. dust sound waves are slow enough and dust particle charges have time to adjust to plasma parameters. Further, for simplicity, we consider regions in Saturn's magnetosphere where the photoelectric current is negligibly small compared to any of the microscopic currents of electrons and ions to the dust particle, which is easily realized, for example, in magnetospheric regions shadowed from solar radiation by Saturn. Thus, dust particle charges can be determined from the balance of electron and ion currents to the particle surface i.e. dust sound waves are slow enough and the charges of dust particles have time to adjust to the plasma parameters. Further, for simplicity, we consider regions in Saturn magnetosphere in which the photocurrent is negligibly small compared to any of the microscopic currents of electrons and ions on a dust particle, which is easily realized, for example, in magnetosphere regions shaded from solar radiation by Saturn. Thus, the charges of dust particles can be determined from the balance of electron and ion currents on the particle surface

$$I_e\left(Z_d\right) + I_i\left(Z_d\right) = 0. \tag{10}$$

Microscopic currents of cold (hot) electrons to the dust particle surface are determined by the expression [36]

$$I_{e,c(h)}\left(Z_d\right) = 2\sqrt{\pi}a^2 e n_{e0,c(h)} \frac{\left(\kappa_{c(h)} - 3/2\right)^{1/2}}{\kappa_{c(h)}\left(\kappa_{c(h)} - 1\right)} \frac{\Gamma\left(\kappa_{c(h)} + 1\right)}{\Gamma\left(\kappa_{c(h)} - 1/2\right)} \times \sqrt{\frac{T_{e,c(h)}}{m_e}} \left(1 - \frac{1}{\kappa_{c(h)} - 3/2} \frac{e^2 Z_d}{a T_{e,c(h)}}\right)^{-\kappa_{c(h)}+1}, \tag{11}$$

and the ion current equals [32]

$$I_i\left(Z_d\right) = 4\pi a^2 e n_{i0} \sqrt{\frac{T_i}{2\pi m_i}} \left(1 - \frac{e^2 Z_d}{a T_i}\right), \tag{12}$$

where a is the dust particle size, $m_{e(i)}$ is the mass of electron (ion), $\Gamma\left(\kappa_{c(h)}\right)$ is the gamma function.

As a result of the self-consistent solution of system (10)-(12) for a given value n_{i0} we obtain the values of dust particle charge numbers Z_d and electron concentrations n_{e0} . Figure 1 shows the dependencies of dust particle charge numbers and electron concentrations on dust sizes for various dust particle concentrations.

3. NONLINEAR PERIODIC WAVES

The resulting system (1)-(7) will be solved using the Sagdeev potential method. To obtain a wave moving at a constant velocity, we will transition to a reference frame moving in the positive direction along the OX axis with velocity M : $\xi = x - Mt$. We will seek the solution in dimensionless form, using the following dimensionless quantities:

$$e\varphi/T_i \to \varphi, \quad M/C_{Sd} \to M, \quad \xi/\lambda_{Di} \to \xi, \quad C_{Sd} = \sqrt{T_i/m_d}, \quad \lambda_{D_i} = \sqrt{T_i/4\pi n_{i0} e^2}.$$

To search for nonlinear dust acoustic waves in dust plasma in Saturn's magnetosphere, the system of equations (1)-(7) in dimensionless variables can be represented as

$$\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2 + V(\varphi) = E, \tag{13}$$

$$V(\varphi) = 1 - e^{-\varphi} + \alpha(1 + Z_d d)\tau_c\left[1 - \left(1 - \frac{1}{\kappa_c - 1.5}\frac{\varphi}{\tau_c}\right)^{-\kappa_c + 3/2}\right] + (1-\alpha)(1 + Z_d d)\tau_h\left[1 - \left(1 - \frac{1}{\kappa_h - 1.5}\frac{\varphi}{\tau_h}\right)^{-\kappa_h + 3/2}\right] + dM\left(M - \sqrt{M^2 - 2Z_d\varphi}\right), \tag{14}$$

where $d = n_{d0}/n_{e0}$, $\tau_{c(h)} = T_{ec(h)}/T_i$, $V(\varphi)$ is the Sagdeev potential, and E is some constant. In particular, by taking E =0, we obtain a solution in the form of solitary waves. For the existence of nonlinear dust acoustic structures, it is necessary that the Sagdeev potential has a local maximum at φ =0. In our case, this condition is satisfied if

$$M \geq \left[dZ_d^2 \left(1 + \frac{(1-\alpha)(1+Z_d d)}{\tau_h}\frac{\kappa_h - 0.5}{\kappa_h - 1.5} + \frac{\alpha(1+Z_d d)}{\tau_c}\frac{\kappa_c - 0.5}{\kappa_c - 1.5} \right)^{-1} \right]^{1/2} \tag{15}$$

The shape of the Sagdeev potential is shown in Fig. 2a, c, $V_{\min}$ is the depth of the potential well formed by the Sagdeev potential. To find the solution in the form of a nonlinear periodic wave, $j(x)$ it is necessary to integrate the expression that follows from (13), (14):

$$\xi(\varphi) - \xi(\varphi_{\min}) = \int_{\varphi_{\min}}^{\varphi} \frac{d\Phi}{\sqrt{2(E - V(\Phi))}}, \qquad (16)$$

where φ ranges from values $\varphi_{\min}$ to $\varphi_{\max}$, where $\varphi_{\min}$ and $\varphi_{\max}$ are the values of the electrostatic potential corresponding to the intersection points of the function $V(\varphi)$ with the horizontal line $V = E$.

For calculations, we will use the following parameters: T_i=100 K, T_{ec}=10 eV, T_{eh}=700 eV, $\alpha = 1/2$, $\kappa_h = \kappa_c = 2$ [27, 31, 32]. It is assumed that the electron concentration in the absence of dust is 10 cm $^{-3}$, and the ion concentration satisfies the plasma quasi-neutrality condition.

In Fig. 2, calculations are performed for $n_{d0} = 10^{-2}$ см$^{-3}$ and M =40. Figs. 2a, c show the Sagdeev potentials $V(\varphi)$, and Figs. 2b, d show nonlinear periodic waves $\varphi(\xi)$. Figs. 2a, b correspond to particle sizes of 0.2 μm, Figs. 2c, d correspond to particle sizes of 2 μm. Nonlinear waves *1* (b, d) are obtained for values of $E = 10^{-3} V_{\min}$, and nonlinear waves *2* (b, d) correspond to values of $E = V_{\min}/2$.

Figs. 3a, c show the Sagdeev potentials $V(\varphi)$ and Figs. 3b, d show nonlinear periodic waves $\varphi(\xi)$ for dust particle concentration $n_{d0} = 10^{-3}$ см$^{-3}$ and M=100. In this case, Figs. 3a, b correspond to particle sizes of 0.2 μm, Figs. 3c, d correspond to particle sizes of 2 μm. Nonlinear waves *1* (*2*) in Figs. 3b, d are obtained for values of $E = 10^{-3} V_{\min}$, ($E = V_{\min}/2$).

Figs. 4a, c show the Sagdeev potentials $V(\varphi)$ and Figs. 4b, d show nonlinear periodic waves $\varphi(\xi)$ for dust particle concentration $n_{d0} = 10^{-4}$ см$^{-3}$. In this case, Figs. 3a, b correspond to particle sizes of 0.2 μm and M =60, Figs. 3c, d correspond to particle sizes of 2 μm and M =300. Nonlinear waves *1, 2* (b, d) are obtained for values of $E = 10^{-3} V_{\min}$ ($E = V_{\min}/2$).

As can be seen from Fig. 2-4, changing the free parameter E allows to change the period and amplitude of nonlinear periodic dust acoustic waves, with a decrease in E the period and amplitude of the nonlinear wave increase. The characteristic period of nonlinear periodic waves ranges from several values λ_{Di} (tens of centimeters) to significantly larger values corresponding to very small values of E . A similar situation in the Earth's atmosphere was considered earlier [18], with discussions of possible manifestations of nonlinear periodic waves accessible to an observer on the Earth's surface. The consideration of dust acoustic waves in Saturn's magnetosphere is carried out for the case when the photoeffect is insignificant and dust particles acquire negative charges due to the greater mobility of electrons compared to ions. Thus, in the entire domain of definition of the amplitude of the electrostatic potential, nonlinear periodic dust acoustic waves in Saturn's magnetosphere are negative. Their absolute values can reach values of the order of T_i / e (about 10^{-3} CGSE units), which indicates the possibility of observing these wave structures in future space missions.

4. CONCLUSION

The paper shows the possibility of propagation of nonlinear periodic dust acoustic waves in the dusty plasma of Saturn's magnetosphere, which includes hot and cold electrons, magnetospheric ions, and charged dust particles. The amplitudes of nonlinear dust acoustic waves reach fairly large values and are in the region of negative potentials. In order to ensure the possibility of observing nonlinear periodic dust acoustic waves in Saturn's magnetosphere in future space missions, it is necessary to equip the spacecraft with instruments that allow high-precision measurements of electric fields. An example is the equipment placed on the Freja spacecraft [37], which was used to observe lower hybrid solitons in the Earth's magnetosphere.

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FIGURE CAPTIONS

Fig. 1. Dependence of charge numbers Z_d (curves 1–3) and electron concentrations n_{e0} (curves 1'–3') on particle size a, obtained at different values of dust particle concentration n_{d0}: curves 1, 1' corresponds to $n_{d0} = 10^{-4}$ см$^{-3}$, curves 2, 2' corresponds $n_{d0} = 10^{-3}$ см$^{-3}$, curves 3, 3' corresponds $n_{d0} = 10^{-2}$ см$^{-3}$. The calculations were carried out at $T_i = 100$ TO, $T_{ec} = 10$ eV, $T_{eh} = 100$ eV, $\alpha = 1/2$, $\kappa_c = \kappa_h = 2$. The electron concentration in the absence of dust was 10 cm-3. The ion concentration satisfied the plasma quasi-neutrality condition.

Fig. 2. Sagdeev potentials $V(\varphi)$ (a, c) and nonlinear periodic waves $\varphi(\xi)$ (b, g) for concentration of dust particles $n_{d0} = 10^{-2}$ см$^{-3}$ and M =40. In this case (a, b) correspond to particle sizes of 0.2 μm, (c, d) correspond to particle sizes of 2 μm. Nonlinear waves 1 (b, d) were obtained for values of E= 10-3Vmin, and nonlinear waves 2 (b, d) correspond to values of E= Vmin/2.

Fig. 3. Sagdeev potentials $V(\varphi)$ (a, c) and nonlinear periodic waves $\varphi(\xi)$ (b, g) for concentration of dust particles $n_{d0} = 10^{-3}$ см$^{-3}$ and M = 100. In this case (a, b) correspond to particle sizes of 0.2 μm, (c, d) correspond to particle sizes of 2 μm. Nonlinear waves 1 (b, d) were obtained for values of E = 10-3Vmin, and nonlinear waves 2 (b, d) correspond to values of E = Vmin/2.

Fig. 4. Sagdeev potentials $V(\varphi)$ (a, c) and nonlinear periodic waves $\varphi(\xi)$ (b, g) for concentration of dust particles $n_{d0} = 10^{-4}$ см$^{-3}$. In this case (a, b) corresponds to particle sizes of 0.2 μm and M = 60, (c, d) corresponds to particle sizes of 2 μm and M = 300. Nonlinear waves 1 (b, d) were obtained for values of E = 10-3Vmin, and nonlinear waves 2 (b, d) correspond to values of E = Vmin/2.

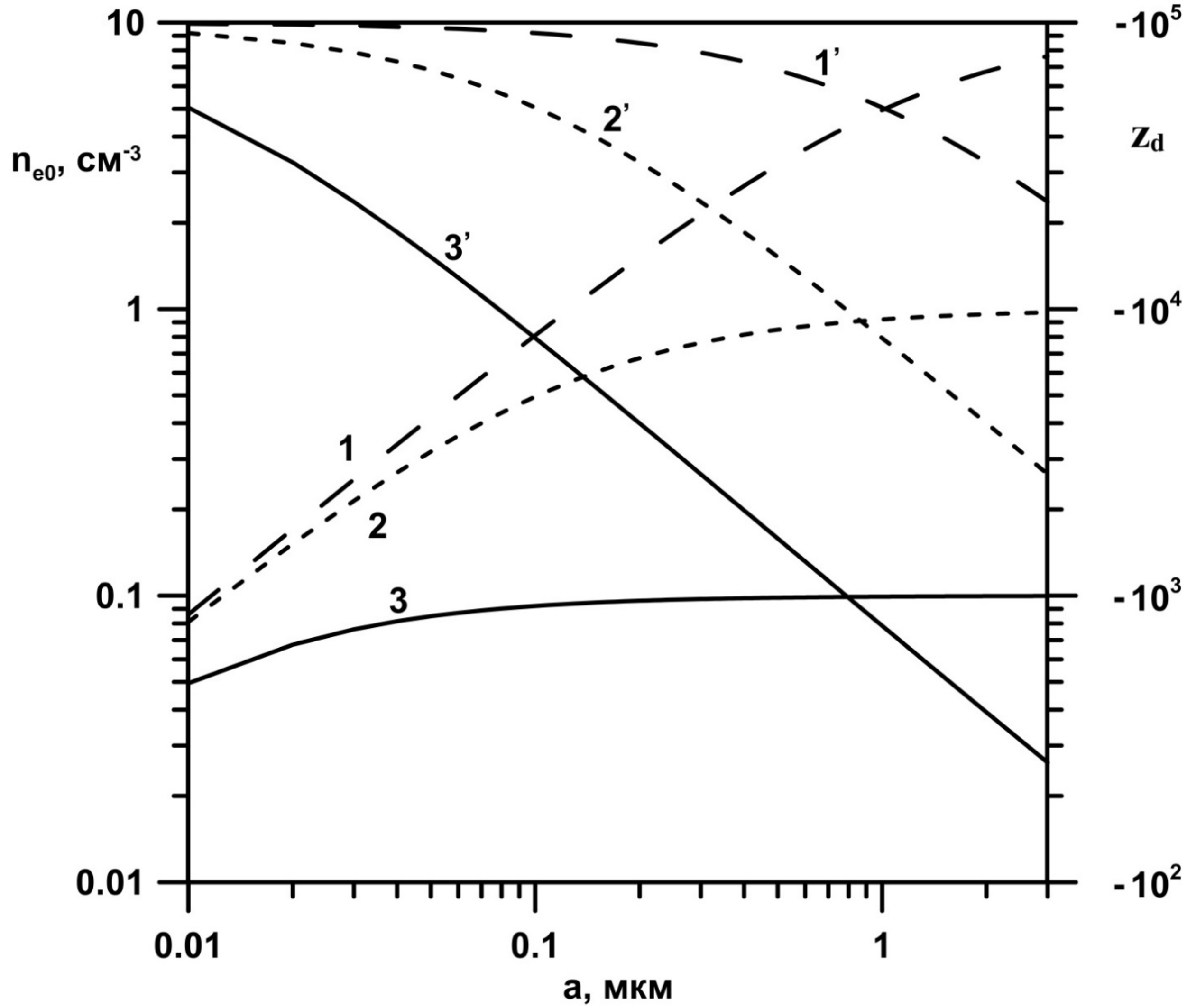

Fig. 1

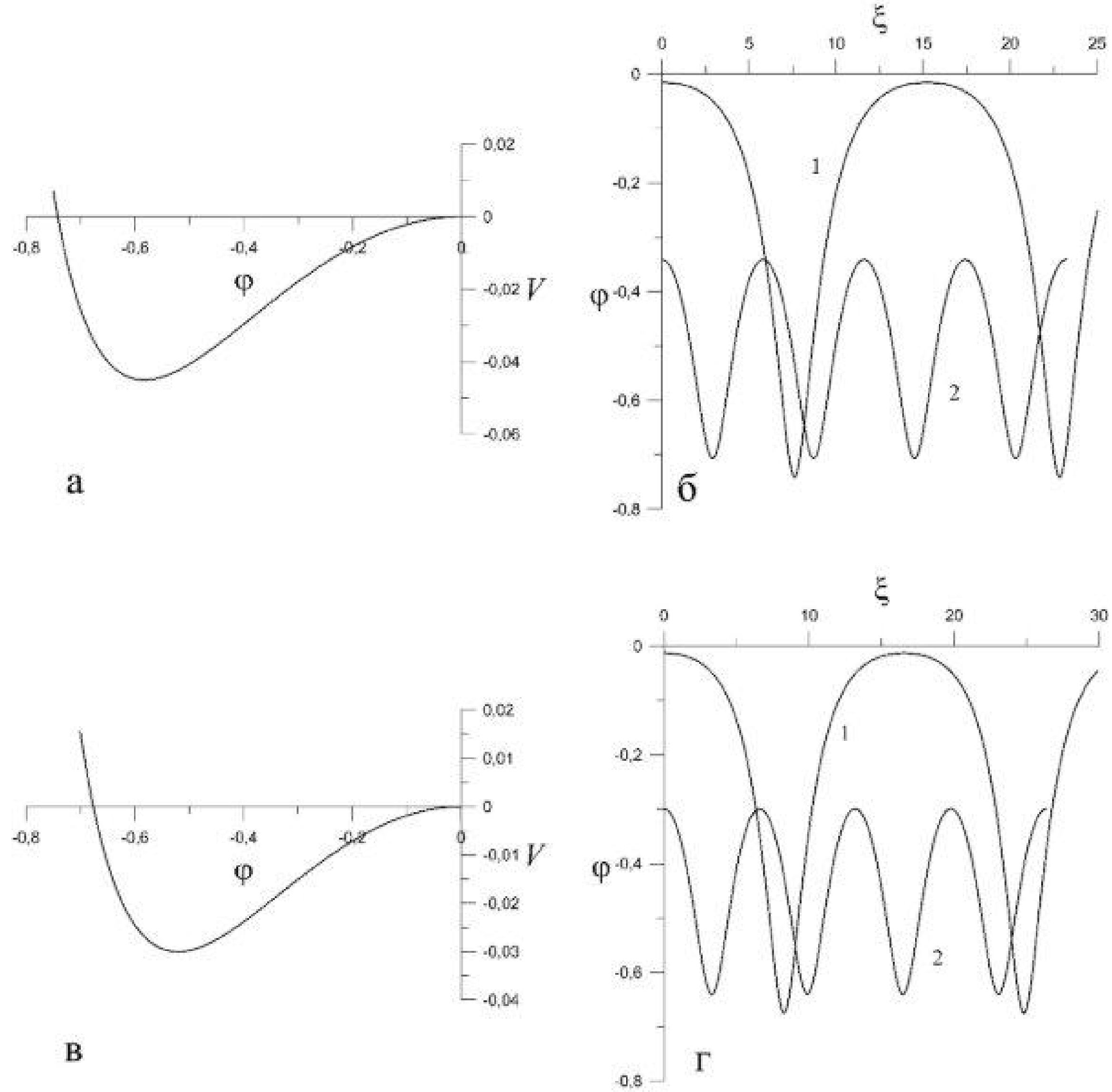

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Fig. 2

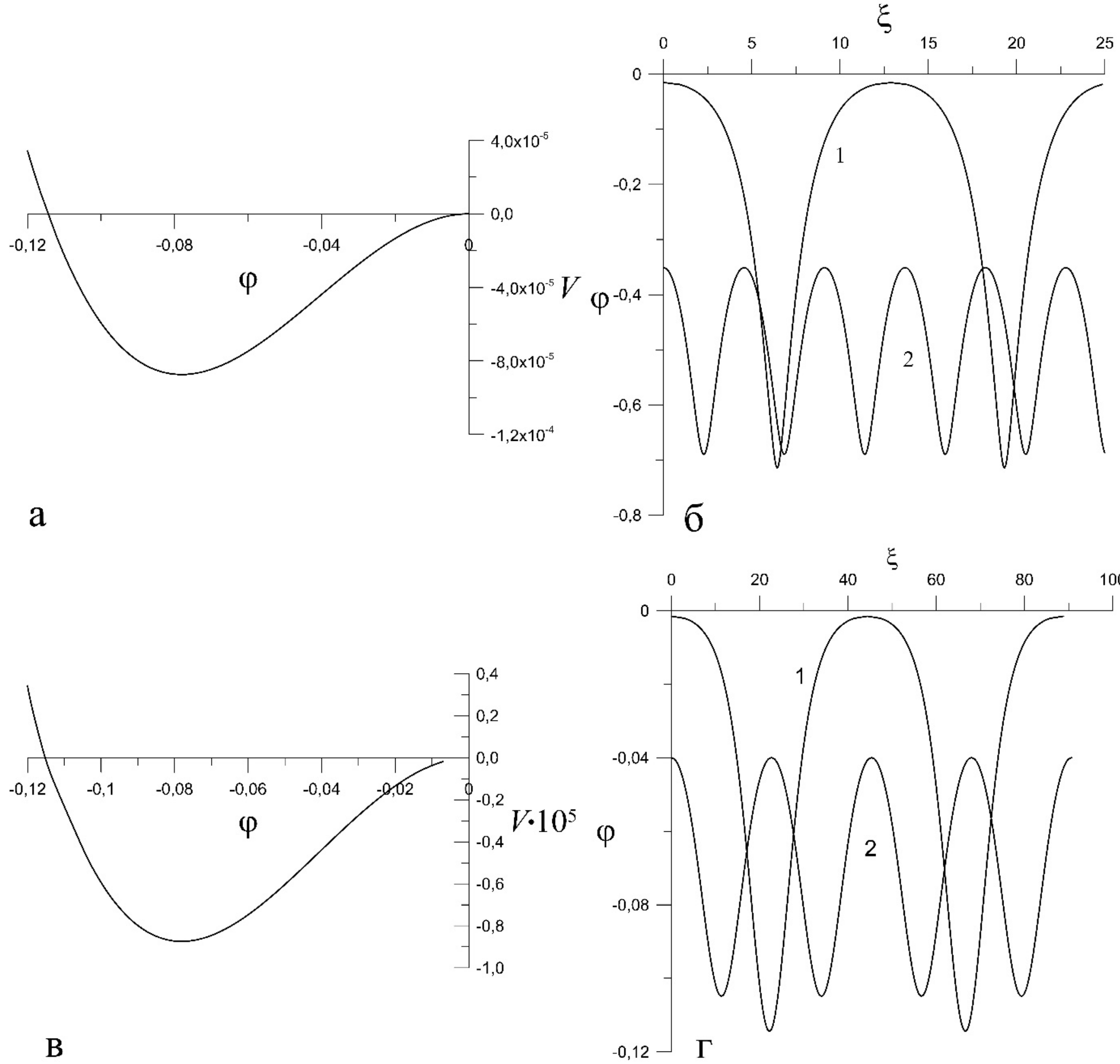

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Fig. 3

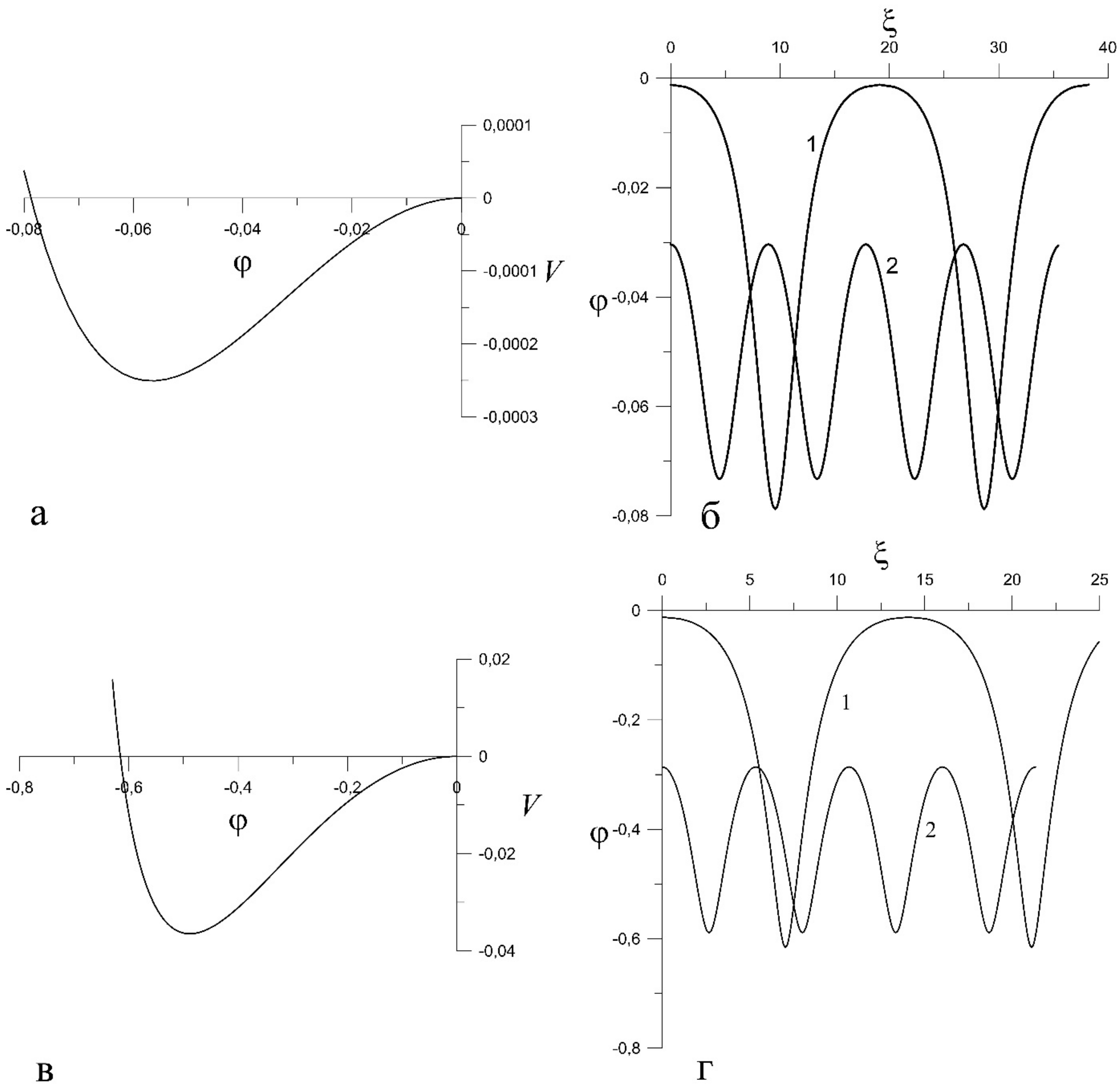

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Fig. 4