ON THE POSSIBILITY OF PROPAGATING A SOLITARY ELECTROMAGNETIC WAVE IN ARBITRARY DIRECTIONS IN THE PLANE OF A TWO-DIMENSIONAL GRAPHENE-BASED SUPERLATTICE

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Abstract. The possibility of propagation of a plane solitary electromagnetic wave in the plane of a square two-dimensional graphene-based superlattice at various angles to its axes is investigated. A nonlinear equation describing the vector potential of a solitary electromagnetic wave is obtained for the case of weak nonadditivity of the energy spectrum of charge carriers in the collisionless approximation. It is shown that the propagation of plane solitary waves is possible either along the axes of the superlattice or at an angle of 45° to them.

Keywords: *solitary electromagnetic waves, solitons, graphene, two-dimensional superlattices, sine-Gordon equation, double sine-Gordon equation*

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INTRODUCTION

In recent years, numerous studies of the physical properties of graphene-based structures have been developed, particularly in the optics of graphene superlattices (GSRs). Despite the fact that today there are certain technological difficulties and limitations in the fabrication of graphene superstructures, theoretical and experimental studies of both one-dimensional [1-8] and two-dimensional (2D) [9-16] GSRs are actively conducted and a large number of interesting results have been obtained. Undoubtedly, in the near future, graphene superstructures will have important practical significance for the generation and amplification of electromagnetic waves, in particular ultra-short solitary electromagnetic waves (SEWs) [17,18]. Recently, interest has arisen in 2D SMWs formed by periodically staggered alternating staggered rectangular regions of SiO_2dioxide and SiC silicon carbide. In contrast to silicon carbide, silicon dioxide does not affect the energy spectrum of graphene, while silicon carbide causes the appearance in its spectrum of the forbidden zone ("slit") with a width of approximately 0.26 eV and due to the alternation of sites of slit and slitless modifications of graphene there is a miniband spectrum. The model energy spectrum of such 2D GSR is investigated in [5], and in [14-16] the peculiarities of UEB propagation are considered. In [16], the interaction of plane UEBs propagating along the axes of a square GSR in mutually perpendicular directions is studied. In the present work, the possibility of propagation of a plane UEB along arbitrarily chosen directions for a square 2D GSR is investigated.

BASIC EQUATIONS

The spectrum of 2D GSR, consisting of alternating staggered rectangular regions of slot and slotless graphene, in the one-minison approximation is [5]

$$\varepsilon(\vec{p}) = \pm\sqrt{\Delta_0^2 + \Delta_1^2(1-cos(p_x d_1)) + \Delta_2^2\big(1-cos(p_y d_2)\big)}, \qquad (1)$$

where p_x, p_y– projections of the electron quasi-momentum on the SR axes, d_1, d_2- periods of 2D GSR (hereinafter$\hbar = 1$). The nonparabolicity of the spectrum of the 2D graphene superlattice entails the appearance of nonlinear properties, in particular, the possibility of propagation of UEB in it.

The evolution of the UEB is described by the d'Alambert equation for the vector potential

$$\frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial y^2} - \frac{1}{V^2}\frac{\partial^2 \vec{A}}{\partial t^2} + \frac{4\pi}{c}\vec{j}(A_x,A_y) = 0, \quad (2)$$

where $V = c\chi^{-1/2}$– the velocity of the electromagnetic wave in the absence of electrons, χ– the effective dielectric constant of the medium. The vector potential is related to the electric field strength$\vec{E} = -(1/c)\partial\vec{A}/\partial t$. In solving the problem, we choose the Coulomb calibration of the vector potential, and neglect collisions.

The electric current density is defined in the form

$$\vec{j} = -e\sum n(\vec{p})\vec{v}\left(\vec{p} + \frac{e}{c}\vec{A}(\vec{r},t)\right), \quad (3)$$

where $n(\vec{p})$– electron distribution function,$v(\vec{p})$ =$(\partial\varepsilon/\partial_{p(x)},\partial\varepsilon/\partial_{p(y)})$– electron velocity.

Decomposing the electron velocity into a two-dimensional Fourier series and assuming the electron gas to be nondegenerate, we have an expression for the current density

$$\vec{j} = -\frac{en_0}{a}\begin{pmatrix} \frac{\Delta_1^2 d_1}{\Delta_0}\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} B_{nm}\, sin(n\phi_x)\, cos(m\phi_y), \\ \frac{\Delta_2^2 d_2}{\Delta_0}\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} C_{nm}\, sin(n\phi_y)\, cos(m\phi_x) \end{pmatrix}, \quad (4)$$

where n_0– is the concentration of 2D electrons, *and*– is the thickness of graphene, $\vec{\phi} = \frac{e}{c}(A_x d_1, A_y d_2)$– dimensionless vector potential, B_{nm}= $a_{nm}I_{(nm)}/I_{00}$, ,$I_{nm} =$

$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} cos(nx)\,cos(my)\,exp[-\sqrt{\Delta_0^2+\Delta_1^2(1-cos(x))+\Delta_2^2(1-cos(y))}/$

$kT]dxdy\; a_{nm} = \frac{1}{2\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\frac{sin(x)\,sin(nx)\,cos(my)dxdy}{\sqrt{1+\frac{\Delta_1^2}{\Delta_0^2}(1-cos(x))+\frac{\Delta_2^2}{\Delta_0^2}(1-cos(y))}}$. $C_{(nm)}$is calculated similarly

to $B_{(nm)}$) by Fourier series expansion of the projection of electron velocity on the *y-axis*. In what follows, we restrict ourselves to the special case of a square lattice ,$d_1 = d_2 = d$ $\Delta_1=\Delta_2=\Delta$. In this case, $B_{nm}= C_{nm}$.

In general, the solution of equation (2) with the current density in the form (4) is only possible numerically. The components of the current density along the x and y axes are generally different from zero and the resulting current density vector depends on coordinates and time in a complex way. However, it can be shown that in our considered case of a square symmetric superlattice there are such values of the angle θ between the *x-axis* and the direction of wave propagation at which the direction of the current density vector will not change relative to the direction of the vector potential and, moreover, will coincide with it. In this case, it is possible to introduce a new coordinate system in which one of the coordinate axes coincides with the direction of the current density vector, which automatically means that the perpendicular component of the current density in this new coordinate system is equal to zero. Thus, it is possible to reduce the (2+1)-dimensional problem to a one-dimensional (1+1) one.

Let us choose the direction of propagation of the plane UEB at an angle θ to the *x-axis* and denote it by x'. Due to the symmetry of the system, we will consider the angles θ, which are in the first quarter of the quadrant. The geometry of the problem is shown in Fig. 1. Let us pass to the one-dimensional case by projecting (2) onto the *y-*

axis′ and taking into account , ,$\phi = \phi_{y'} = edA/c\phi_{x'} = 0\omega_0^2 = 2\pi e^2 n_0 \Delta^2 d^2/c^2\Delta_0$, we obtain

$$\frac{\partial^2\phi}{\partial t^2} - V^2\frac{\partial^2\phi}{\partial x'^2} + \omega_0^2(-\sin\theta\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} B_{nm}\sin(n\phi_x)\cos(m\phi_y) +$$

$$+\cos\theta\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} B_{nm}\sin(n\phi_y)\cos(m\phi_x)) = 0. \quad (5)$$

Here the projection of the current density on the *y-axis′* is obtained using the projections of the current density on the *x*- and y-axes

$$j_{y'} = -j_x\sin\theta + j_y\cos\theta. \quad (6)$$

The relationship between the components of the vector potential in the unshaded and shaded coordinate systems is given as follows: $\phi_x\cos\theta = -\phi_y\sin\theta$, , .$\phi_x = -\phi\sin\theta\ \phi_y = \phi\cos\theta$

The projection of equation (2) to the direction of propagation of the plane UEW, to the *x-axis′*, leads to the condition $j_{x'}=0$. In our case, the vector potential and current density are co-directional and perpendicular to the direction of propagation of the plane wave. It is also convenient to express the projection of the current density on the *x-axis′* through the projections of the current density on the *x* and *y* axes

$$j_{x'} = j_x\cos\theta + j_y\sin\theta. \quad (7)$$

From the condition $j_{x'}=0$, we have

$$-\cos\theta\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} B_{nm}\sin(n\phi\sin(\theta))\cos(m\phi\cos(\theta)) +$$

$$+\sin\theta\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} B_{nm}\sin(n\phi\cos(\theta))\cos(m\phi\sin(\theta)) = 0. \quad (8)$$

Equality (8) is satisfied at arbitrary φ at certain values of the angle θ , corresponding to the direction of wave propagation at angles 0° and 45° to the GSR axes. Thus, equation (5) describes the vector potential of a plane UEB propagating in a square GSR at angles 0° and 45° to its axes.

Let us consider the case of weak non-additivity of the energy spectrum, in which we can restrict ourselves to the first summands in (5)

$$\frac{\partial^2\phi}{\partial t^2} - V^2\frac{\partial^2\phi}{\partial x'^2} + \omega_0^2\left(B_{10}\,sin(\phi\,sin\,\theta)\left(1 + \frac{2B_{11}}{B_{10}}cos(\phi\,cos\,\theta)\right)sin\,\theta + \right.$$

$$\left. + B_{10}\,sin(\phi\,cos\,\theta)\left(1 + \frac{2B_{11}}{B_{10}}cos(\phi\,sin\,\theta)\right)cos\,\theta\right) = 0. \qquad (9)$$

Let's switch to dimensionless variables$\eta = x'\omega_0/V, \tau = t\omega_0$ and introduce the notation$\beta = 2B_{11}/B_{10}$. We finally obtain:

$$\frac{\partial^2\phi}{\partial \tau^2} - \frac{\partial^2\phi}{\partial \eta^2} + sin\,\theta\,sin(\phi\,sin\,\theta)\,(1 + \beta\,cos(\phi\,cos\,\theta)) +$$

$$+ cos\,\theta\,sin(\phi\,cos\,\theta)\,(1 + \beta\,cos(\phi\,sin\,\theta)) = 0. \qquad (10)$$

In the case when$\theta = 0$ or$\theta = \pi/2$, equation (10) corresponds to the sine-Gordon equation, and when the condition$\theta = \pi/4$ is satisfied, equation (10) takes the form of the double sine-Gordon equation [14]. Thus, equation (10) describes all possible cases of propagation of plane solitary waves in a 2D symmetric square GSR (generalized sine-Gordon equation).

We will search for the solution of equation (10) in the form of a traveling wave by introducing the variable$\xi = (\eta - \frac{u}{V}\tau)/\sqrt{1 - \frac{u^2}{V^2}}$, where *u* is the kink velocity. After transformations we have

$$\phi''_{\xi\xi} = sin\,\theta\,sin(\phi\,sin\,\theta)\,(1 + \beta\,cos(\phi\,cos\,\theta)) + + cos\,\theta\,sin(\phi\,cos\,\theta)\,(1 + \beta\,cos(\phi\,sin\,\theta)). \qquad (11)$$

Multiplying both parts of the equation by the derivativeϕ'_ξ and further integrating once, we obtain

$$\frac{d\phi}{d\xi} = \sqrt{2\big(C - cos(\phi \sin\theta) - cos(\phi \cos\theta)\,(1 + \beta\, cos(\phi \sin\theta))\big)}. \qquad (12)$$

The constant C *is* determined from the considerations of equality to zero of the derivative of the potential at $\varphi = 0$.

The phase portraits of the system at different values of the angle θ are shown in Figs. 2 and 3. The bold lines correspond to the separatrices. We see that for the angles $\theta = 0$ and $\theta = \pi/4$, the phase trajectories of the system corresponding to the separatrices separating the regions of the finite (oscillatory, if we refer to the analogy of a pendulum) and infinite (rotational) motions, correspond to the existence of solitary waves [19]. In the case of angles θ, different from $0, \pi/4$ and $\pi/2$, the corresponding phase trajectories do not have a periodic appearance, as shown in Fig. 3. The bold lines in Fig. 3 show the phase curves corresponding to equation (9) under the initial condition $\phi'_{\xi=0} = 0$. The analysis of the phase portrait shows that these phase curves do not separate the regions of finite and infinite motions and, therefore, are not separatrices and, therefore, as one would expect, cannot set the conditions for the existence of plane solitary waves along the corresponding directions relative to the GSR axes. Thus, plane UEWs in a square 2D GSR can propagate only in the directions of the principal axes or diagonal of the square GSR.

CONCLUSION

The problem of the possibility of propagation of plane UEBs in the plane of a 2D GSR at different angles to its axes is investigated. A generalized sine-Gordon equation describing the propagation of a solitary electromagnetic pulse along certain directions in the plane of a square 2D GSR with weak nonadditivity of the energy spectrum in the column-free approximation is obtained. The phase portraits corresponding to its solution are investigated and the conditions for the existence of the UEB in the considered GSR are analyzed. On the basis of the obtained results, it is concluded that the planar UEB is able to propagate only at angles 0° and 45° to the axes of the square GSR.

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FIGURE CAPTIONS

Figure 1. Geometry of the problem.

Figure 2. Phase portrait of the system at $\theta = 0$ (solid lines) and $\theta = \pi/4$ (dashed lines).

Bold lines correspond to separatrices.

Figure 3. Phase portrait of the system at $\theta = \pi/8$ (solid lines) and $\theta = \pi/6$ (dashed lines).

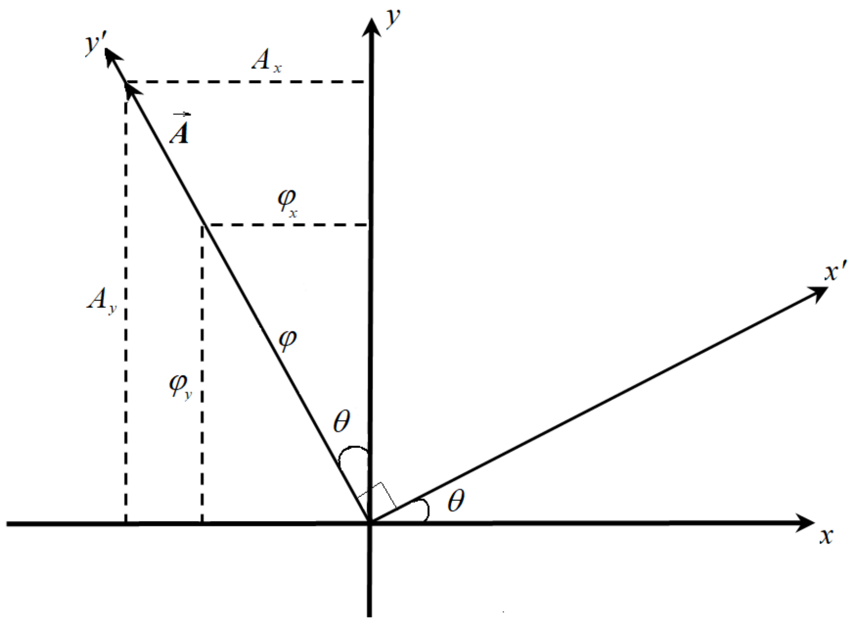
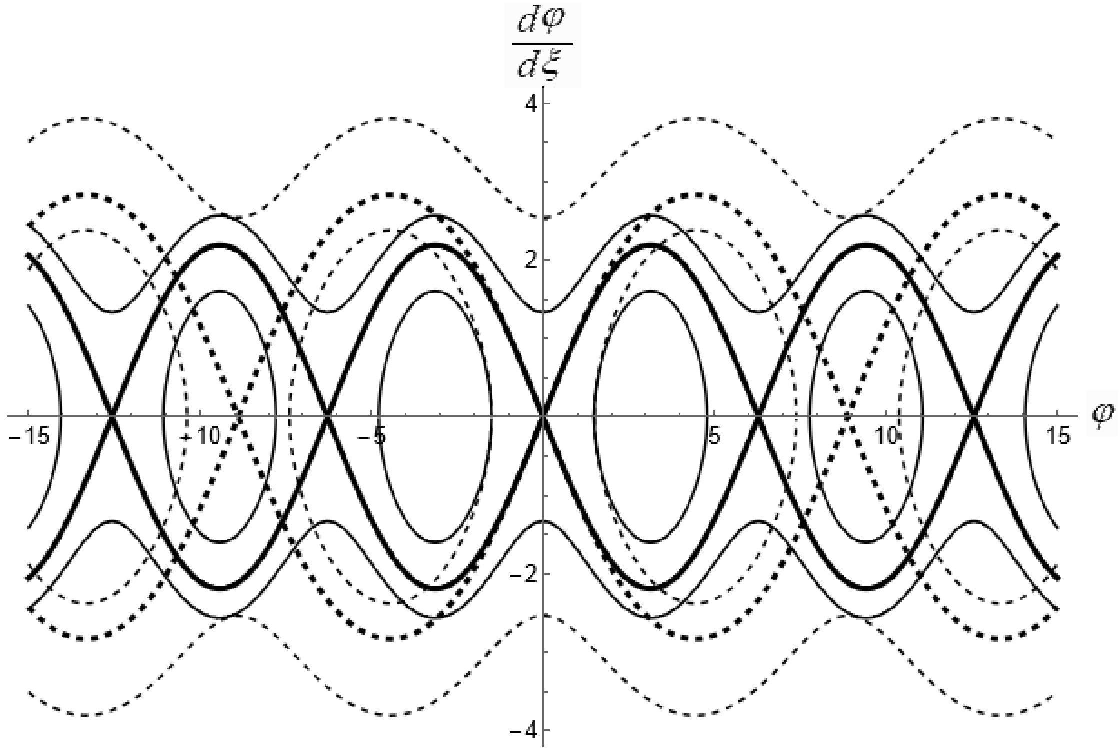
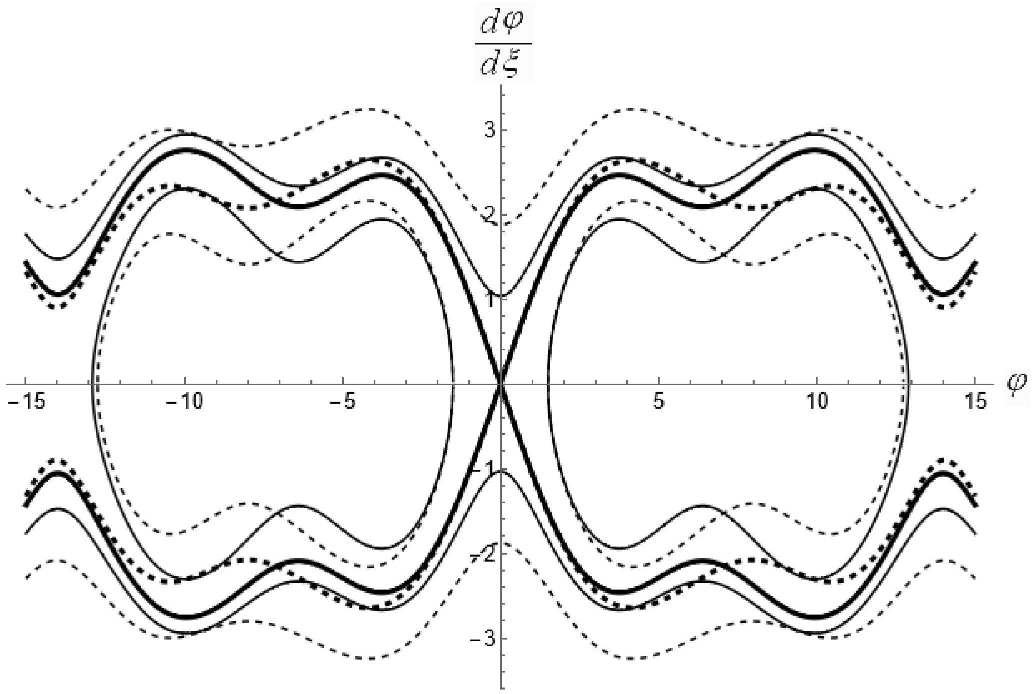


Fig. 1.



$\frac{d\varphi}{d\xi}$
4
2
−2
−4
−15
−10
−5
5
10
15
φ

Fig. 2.



$\frac{d\varphi}{d\xi}$
3
2
1
−1
−2
−3
−15
−10
−5
5
10
15
φ

Fig. 3.