HYBRIDIZATION OF ACOUSTIC TAMM STATES AND DEFECTIVE MODES OF ONE-DIMENSIONAL PHONON CRYSTAL

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Abstract. The spectral properties of a one-dimensional phonon crystal bounded by a reflector in the form of an air layer are studied. The presence of a defect in a phonon crystal with a reflector at the edge leads to a connection between the detective mode and the acoustic Tamm state. This connection of modes of different nature manifests itself in the form of hybridization of modes, and the pushing apart of dips in the reflection spectrum is explained by avoided crossing of modes.

Keywords: phonon crystal, acoustic Tamm state, defect mode

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INTRODUCTION

Wave propagation in layered media began to be considered more than fifty years ago [1], applying various mathematical methods to describe elastic and electromagnetic waves. At present, a new field of science has been formed to study the properties of photonic crystals (PCs) and devices based on them [2,3]. From the general point of view, a PC is a superlattice in which an additional period with a characteristic scale of periodicity of dielectric permittivity of the order of the light wavelength has been artificially created. A few years later, the concept of FC was transferred to the case of elastic waves [4,5] for various studies and the concept of phonon crystal (PhnC) appeared. The aim of these studies was to investigate the acoustic properties of two-dimensional and three-dimensional periodic medium, in order to find the so-called full forbidden zones. Like any periodic structure, the propagation of acoustic waves in a phonon crystal is described by Bloch's theorem, which can be used to calculate the zone structure. The periodic structures defining the Brillouin zone can be in one (1D), two (2D) or three dimensions (3D). Dispersion curves show forbidden zones (ZZs), at frequencies at which wave propagation is forbidden in a periodic structure. Such gaps can occur for certain wave vector directions, but they can also cover the entire 2D [6] or 3D [7] Brillouin zone, where elastic wave propagation becomes forbidden for any polarization and any angle of incidence. Such a structure behaves like a perfect mirror at any angle of incidence, thus prohibiting the transmission of sound waves.

The application of FnC, as well as FC [8], is not limited to its use as an ideal mirror. By analogy with the localization of an electronic state near the surface of a solid, it is possible to localize a light wave at the boundary of a PC and a metal or other PC [9-11]. Such localization is referred to as an optical Tamms state (OTS). By analogy with the OTS, elastic wave localization is also possible in FnC with a defect or in FnC with a violation of the periodicity of the structure [12-14].

We should also note recent work on acousto-optical effects in composite materials, the simplest example of which is layered media. These results have applications in the radio-frequency range [15].

This paper studies the propagation of longitudinal acoustic waves in a phonon crystal consisting of alternating layers of ED-10 epoxy-diane resin and water. The effect due to the peculiarities of the structure is discussed: FnC with an air reflector and a defect in the structure in the form of a water layer with a thickness different from the thickness of the water layer in the FnC volume.

MODEL DESCRIPTION AND CALCULATION METHOD

A periodic structure consisting of resin layers and water layers between them, the number of periods is 5 (Fig. *1a*) served as FnC. The thickness of the water layer between the neighboring resin layers $d_{water}$is 1 cm, the thickness of the resin $d_{epoxy}$is 0.3 cm.

The acoustic impedance (acoustic impedance, wave impedance) of a material normalized to the speed of sound and density of water is calculated using the following formula:

$$Z = \frac{\rho c}{c_0 \rho_0}, \quad (1)$$

where , c_0 – скорость звука в воде 1500 $\frac{м}{с}$ с – is the sound speed of the material, ρ_0 is the density of water 1000 ,$\frac{кг}{м^3}$ ρ is the density of the material.

The refractive index of sound was calculated by analogy with the refractive index in optics, normalized by the speed of sound in water:

$$n = \frac{c}{c_0} \tag{2}$$

To break the periodicity of the structure, a reflector in the form of a layer of air is added on the right side. This layer reflects radiation in a wide wavelength range, so the manifestation of resonances in the transmission spectrum is not noticeable. To manifest resonances in the reflection spectrum, a layer of sound absorber (aqueous glycerol solution) with refractive indexn$_{abs} = 1.27 + 0.057i$, and thickness $d_{abs} =$ 0.4 см is added between the FnC and the reflector (Fig. *1b*). $N_1$is the number of periods between the boundary and the defect, i.e. the distance between them.

The transfer matrix method was used to calculate the acoustic wave passage [16,17]. The acoustic field in a homogeneous plane-parallel layer is defined by two complex amplitudes of displacement and pressure vectors in the form of plane waves:

$$U(z) = U_r \cdot e^{ikz} + U_l \cdot e^{-ikz}; \tag{3}$$

$$p(z) = \frac{C}{ik}\frac{dU}{dz}; \tag{4}$$

Substituting (3) into (4), we obtain:

$$p(z) = C \cdot U_r \cdot e^{ikz} - C \cdot U_l \cdot e^{-ikz}, \tag{5}$$

where$U(z)$ is displacement,$p(z)$ is acoustic pressure,C has dimension .$\text{Па}/\text{м}$

The change of the acoustic field when passing through each layer of the structure is determined by the second-order transfer matrix (dimensionality2×2). The transfer matrix of the whole structure relates the complex amplitudes at the right and left boundaries of the structure. It is defined by the product of the matrices of adjacent layers:

$$\widehat{M} = \widehat{D}_{outN} \cdot \widehat{P}_{N} \cdot \widehat{D}_{N(N-1)} \cdot \widehat{P}_{(N-1)} \dots \widehat{P}_{1} \cdot \widehat{D}_{1in} \qquad (6)$$

Here $\widehat{D}_{ji}-$ is a matrix that shows the change in amplitudes when the wave crosses the boundary of the ii and ji layers, :$j = i + 1$

$$\widehat{D}_{ij} = \frac{1}{t_{ij}}\begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix}, \qquad (7)$$

where $r_{ji}$and $t_{ji}$are the amplitude reflection and transmission coefficients at the boundary between the *jth* and *i-th* layers:

$$r_{ji} = \frac{Z_i - Z_j}{Z_i + Z_j}; t_{ji} = \frac{2Z_i}{Z_i + Z_j} \qquad (8)$$

whereZ_i andZ_j are the acoustic impedance of the i-th and *j-th* layers, respectively.

The matrix$\widehat{P}_i$is called the propagation matrix, it shows the change in amplitudes as the wave propagates in layer *i*:

$$\widehat{P}_i = \begin{bmatrix} e^{ik_{iz}d_i} & 0 \\ 0 & e^{-ik_{iz}d_i} \end{bmatrix} \qquad (9)$$

whered_i andk_{iz} are the thickness and wave vector of the *i-th* layer.

The energy coefficients of reflection, transmission and absorption of the whole structure are determined by expressions, respectively:

$$R = \left|\frac{\widehat{M}_{21}}{\widehat{M}_{11}}\right|^2, \ T = \left|\frac{1}{\widehat{M}_{11}}\right|^2, \ A = 1 - R - T, \qquad (10)$$

where$\widehat{M}_{21}, \widehat{M}_{11}$ are the elements of the matrix .$\widehat{M}$

RESULTS AND DISCUSSION

Localization at the FnC defect was previously considered in [17]. In this paragraph, hybridization of the defect mode and ATC is investigated.

The parameters of the system are chosen in such a way that both modes (states corresponding to the eigenstate solutions of the wave equation) appear at frequencies in the first forbidden zone. Let us construct the spectrum of reflection of this system to determine the frequencies of localized states (Fig. 2). As can be seen from the reflection spectrum, there are two resonance frequencies in the forbidden zone, which appear as reflection dips.

From Fig. 3 shows that the resonance frequencies correspond to two localized modes. At the same time, localization can occur both at the boundary between the FnC and the reflector and at the defect. The localization of the field more at the FnC-reflector boundary corresponds to Fig. *3b*, whereas the field localization in the defect corresponds more to Fig. *3*a. Mode hybridization is the coupling and spatial superposition of two modes of different nature. The hybridization is manifested in the spatial distribution of the field in the two kinds of localization, which is found in Figs. *3a* and *3b*. Since the partial modes, i.e., the normal modes of the subsystems [19], are of different nature, their superposition is of hybrid nature and the coupling is called hybridization.

Let us study the hybridization of modes depending on the distance between the defect and the reflector (Fig. 4). We construct the reflection spectrum by varying N_1in the frequency range from 35 kHz to 60 kHz, i.e., considering the first ZZ.

Figure 4 shows that with increasing N_1, i.e., increasing the distance between the defect and the reflector, the two resonance frequencies converge. In this case, the mode with a lower frequency, corresponding to a greater degree of localization on the defect FnC, starting from the distance $N_1 = 4$, ceases to appear in the reflection spectrum. At the same time, the mode with a higher frequency, corresponding to a greater degree to ATC, is preserved.

Let us fix the position of the defect at the middle of the FnC ($N_1 = 2$) and change the thickness of the sound absorber layer adjacent to the reflector. Let us plot the reflection spectrum for this case (Fig. 5).

Analyzing the resulting spectrum, it can be seen that the frequencies of the two modes change as the thickness of the defect layer changes. Two curves with hyperbolic shape in the region of their convergence can be compared to the areas of reflection dips, blue color. This picture is similar to the quasi-crossing (avoided crossing) of the defect and Tamm modes in a photonic crystal [18]. The quasi-crossing phenomenon is explained in the language of normal and partial modes introduced in the discussion of Fig. 3. In Fig. 5, the two reflection dips correspond to the normal modes depicted in Fig. 3. Each of these modes is a superposition of two partial modes: the ATC on the right and the defective mode on the left. Since the partial modes, i.e., the normal modes of the subsystems [19], are of different nature, their superposition is of hybrid nature and the relation is called hybridization. The frequency dependencies of the normal modes on the thickness of the sound absorber layerd_{abs} overlap, but this is not seen in the reflection spectrum of the whole structure. Instead, two characteristic curves of the normal modes are seen, whose common asymptotes are the dashed lines of the partial modes. The more strongly coupled the partial modes are, the greater the disentanglement, i.e., the frequency distance between the normal mode curves. It can also be seen that in the frequency region of 30-35 kHz the mode goodness drops. This is due to the fact that the mode frequency has gone beyond the ZZ boundaries and the FnC-ATS coupling is broken.

Thus, it is possible to control the spectral position of the modes by varying the thickness of the water layer adjacent to the reflector and by varying the distance between the two layers under consideration.

CONCLUSION

The spectral properties of a one-dimensional FnC with a defect bounded on one side by a reflector have been investigated. The results are obtained using the transfer matrix method.

The phenomenon of mode coupling is common for various vibrational systems, starting from coupled pendulums. At the same time, the peculiarities of the manifestation of this coupling in the defect and at the phonon crystal boundary are of interest. As far as we know from the literature, such a manifestation of mode coupling in phonon crystals has not been considered by anyone so far. We have found a characteristic for pendulums mismatch of normal frequencies, which is explained by their quasi-crossing. The change in the spatial profile of hybrid modes is described (Fig. 3). In addition, the dependences of the mastalking on the distance between the defect and the reflector (Fig. 4), as well as on the thickness of the sound absorber layer (Fig. 5), are revealed. This effect suggests the principle of operation of a new acoustic filter with tunable frequency.

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FIGURE CAPTIONS

Fig. 1. Structure of FnC (a) and FnC with defect and reflector (b). The green line is the dependence of the real part of the refractive index of the material on the depth of the FnC. System parameters: $d_{water} = 1$ см, $n_{water} = 1$; $d_{epoxy} = 0.3$ см, $n_{epoxy} = 1.75$; $d_{water1} = 1.4$ см, $n_{water1} = 1$; $d_{air} = 2.5$ см, $n_{air} = 0.2$; $d_{abs} = 0.4$ см, $n_{abs} = 1.27 + 0.057i$.

Fig. 2. Reflection spectra of FnC with a defect in the middle of the structure and a reflector on the boundary (blue line) and inoculum FnC (crimson line). The forbidden zone of the inoculum FnC corresponds to the frequency region where the reflection coefficient is close to unity. Reflection failures - frequencies on the spectrum of FnC with defect and reflector, where reflection coefficients are minimal in the ZZ. Reflection failures correspond to resonance frequencies at 36.78 and 43.32 kHz.

Fig. 3. Hybridization of the defect and Tamma modes in the spatial field distribution for the smaller (a) and larger (b) of the two resonance frequencies. The black line is the dependence of the square of the acoustic pressure modulus *p* on the FnC depth. Green line - dependence of the real part of the refractive index of the material on the FnK depth. Fg - frequency of the incident acoustic wave. The maximum of the defect mode appears near$z = 5$ см, the maximum of the ATS - near .$z = 8.7$ см

Fig. 4. Removal of mode degeneracy in the reflection spectrum of FnC with a defect and a reflector under variation of the distance between them. The *x-axis* is the frequency of the incident wave. The *y-axis* is the distance between the defect and the reflector, expressed in the number of periods N_1. The *z-axis* is the reflection coefficient.

Fig. 5. Quasi-crossing of dips in the reflection spectrum of FnC with defect and reflector at variation of the thickness of the sound absorber layerd_{abs} . The *x-axis* is the frequency of the incident wave. The *y-axis* is the thickness of the water sound absorber layerd_{abs} . The *z-axis* is the reflection coefficient.

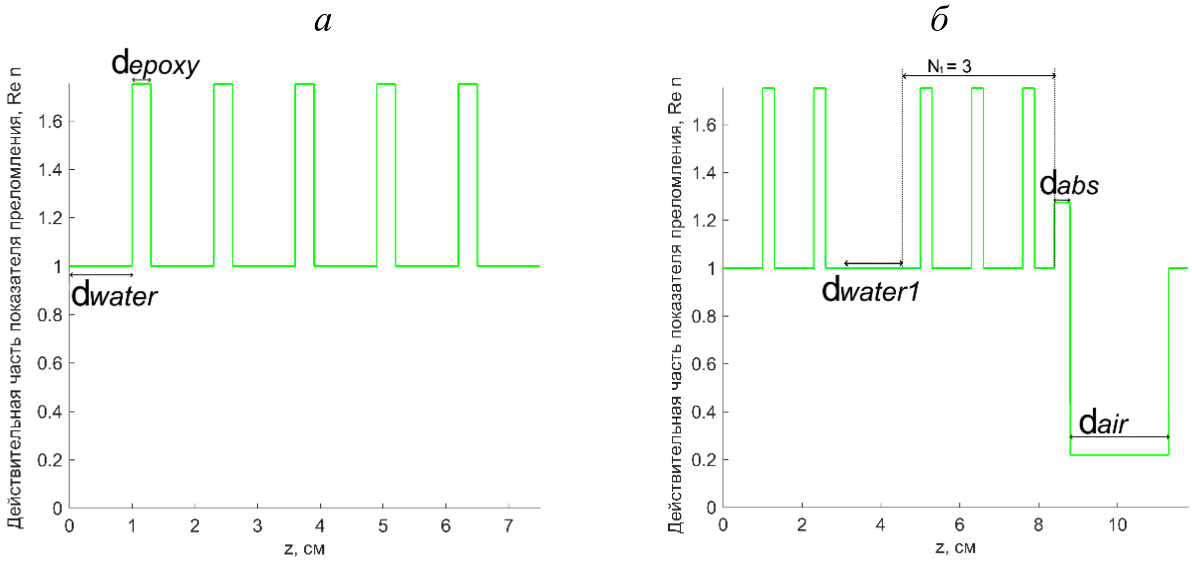

а
б
depoxy
dwater
Действительная часть показателя преломления, Re n
z, см
N1 = 3
dabs
dwater1
dair

Fig. 1.

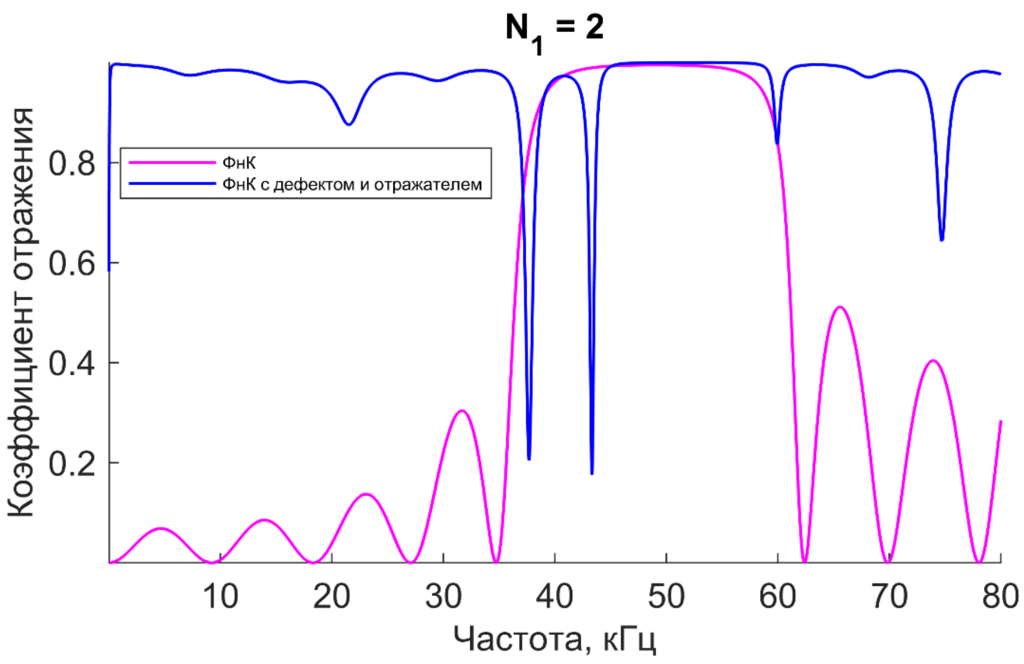
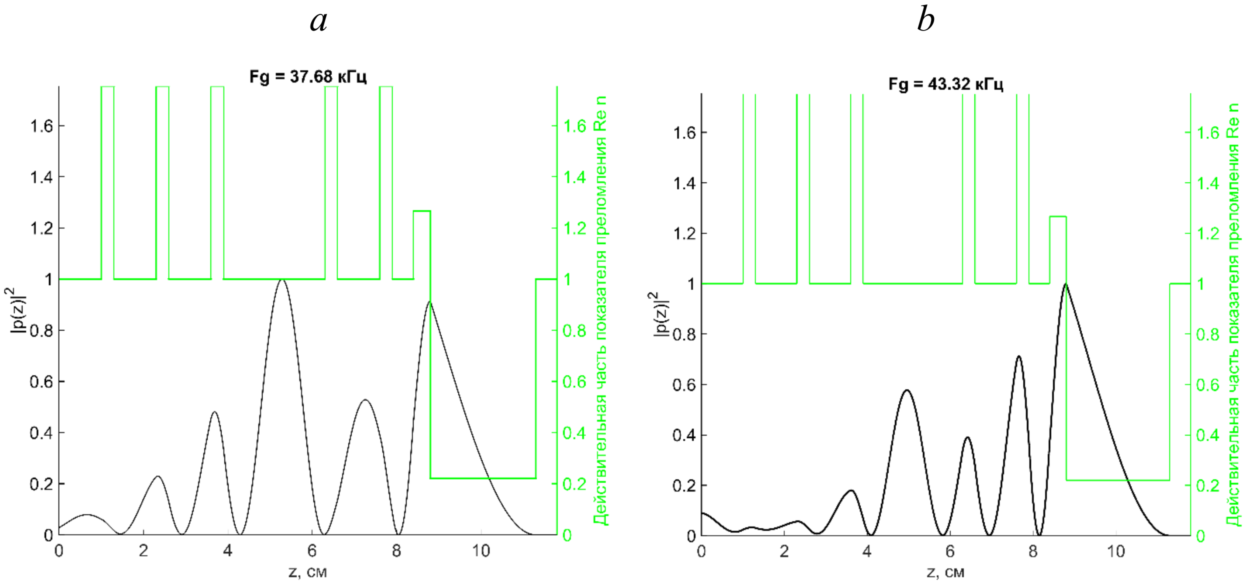
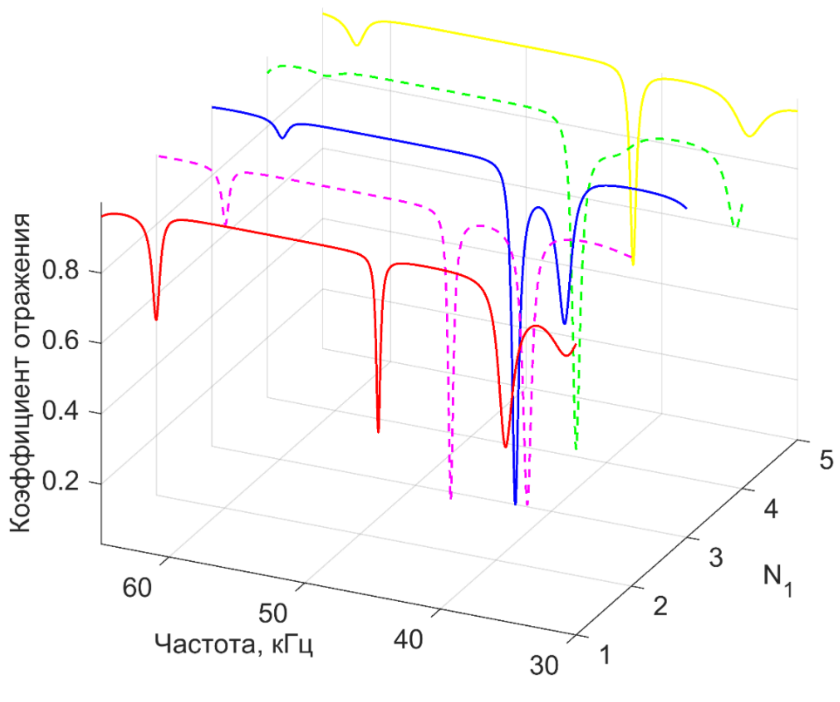

N1 = 2
ФнК
ФнК с дефектом и отражателем
Коэффициент отражения
Частота, кГц

Fig. 2.



a
b
Fg = 37.68 кГц
Fg = 43.32 кГц
|p(z)|^2
z, см
Действительная часть показателя преломления Re n

Fig. 3.



Коэффициент отражения
Частота, кГц
N$_1$

Fig. 4.

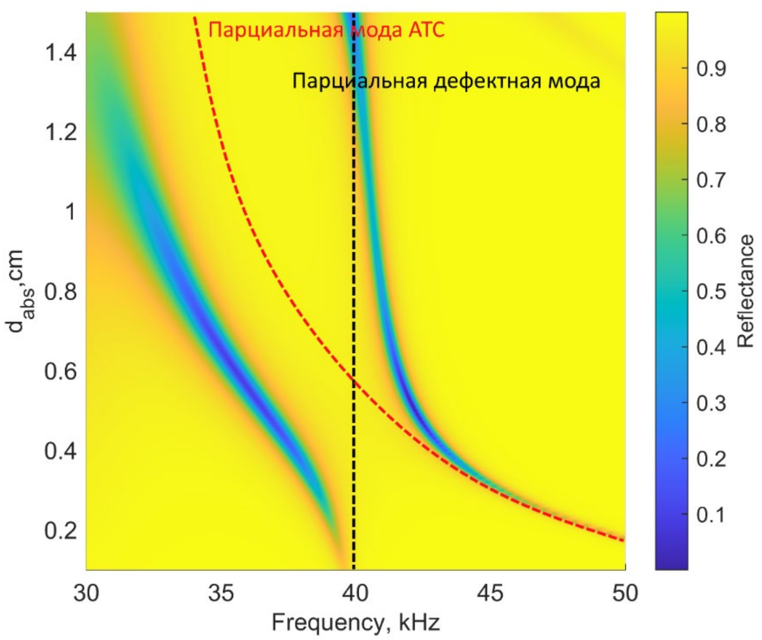

Парциальная мода АТС
Парциальная дефектная мода
d_abs,cm
Frequency, kHz
Reflectance
30
35
40
45
50
0.2
0.4
0.6
0.8
1
1.2
1.4
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Fig. 5.