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Anisotropic Josephson diode effect in the topological hybrid junctions with the hexagonal warping

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One of the most promising platforms for the implementation of the SDE is topological insulator based diodes [1]. The surface of TI provides strong spin-orbit coupling (SOC) which makes it possible to demonstrate a substantial magneto-electric effect [2]. Special attention has been paid to the magneto-electric effect in the TI based Josephson junctions, where it reveals itself in the form of the anomalous ground state phase shift [3, 4]. Recently, it has been demonstrated that in the TI hybrid structures where superconductivity and ferromagnetism are spatially separated, the ground state is also modified [5, 6]. In this case the ground state corresponds to the spatially inhomogeneous superconducting order parameter. This superconducting state is commonly referred as the helical state [7].

The superconducting helical state became one of the options in achieving the SDE [8]. Described by finite Cooper pair momentum, the helical state can be realized in systems with broken inversion and time reversal symmetries. The former is connected to the appearance of the SOC term in the Hamiltonian, while the latter can be introduced by the magnetic field. In this case the direction of the Cooper pair momentum depends on the direction of the magnetic field. The finite momentum of the Cooper pair, which is locked to the direction of the magnetic field leads the nonreciprocal depairing current in various systems.

Here we discuss the consequences of the hexagonal warping of the TI surface states on the Josephson critical current and nonreciprocal transport in the S/TI/S system with an in-plane Zeeman field. The effect of hexagonal warping is important in the TI based devices, since it can significantly change some of the transport properties. For example, it is well known that the interference effects near the defects are strongly enhanced due to the deformation of the Fermi surface [9]. The warping term also leads to the anisotropy of the spin conductivity in the TI materials [10]. The influence of the snowflake Fermi surface on superconducting properties in the hybrid structures has also been questioned [11–13]. However, the impact of the warping term on the superconducting transport and especially nonreciprocal transport have not been well studied.

We formulate the model of the S/TI/S hybrid structure (Fig. 1) in the framework of the tight-binding

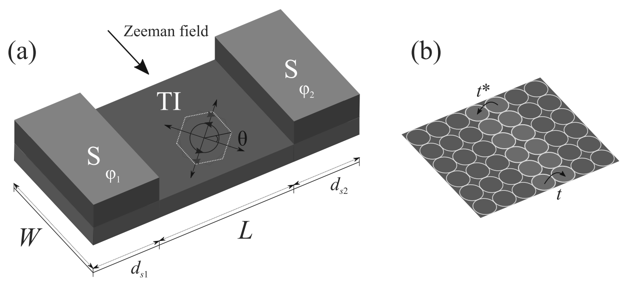

Fig. 1. (Color online) (a) – Schematic geometry of the Josephson diode based on the TI weak link. The superconducting layers S are assumed to be thin so that the system is effectively two-dimensional. $\phi = \phi_2 - \phi_1$ corresponds to the phase difference between the superconducting parts. θ represents the angle of the Fermi surface rotation in the momentum space. (b) – The tight-binding representation of the middle normal region. Here two-layer region with blue sites corresponds to the virtual self energy lead with hopping term t from the left to the right site

Bogoliubov–de Gennes (BdG) Hamiltonian and assume the nearest neighbour hopping approximation. Such system can be described by the following effective Hamiltonian [9, 14]:

$$H(\mathbf{k}) = H_0 + H_{SOC} + H_W + H_S + H_Z = \\ = \left(\frac{\hbar^2}{2m_{\text{eff}}}(k_y^2 + k_x^2) - \mu_x\right)\hat{s}_0 \otimes \hat{\sigma}_z + \\ + \alpha\left(k_y \hat{s}_x - k_x \hat{s}_y\right) \otimes \hat{\sigma}_z + \lambda k_x\left(k_x^2 - 3k_y^2\right)\hat{s}_z \otimes \hat{\sigma}_z + \\ + g\mu_B \mathbf{B} \cdot \hat{\mathbf{s}} \otimes \hat{\sigma}_z + \Delta \hat{s}_0 \otimes \hat{\sigma}_x. \quad (1)$$

Here μ_x is the chemical potential in the corresponding

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region ($x = S$ for the superconducting region S and $x = N$ for the bare topological insulator surface TI), α is the strength of the spin-orbit coupling and λ is the warping coefficient. μ_B is the Bohr magneton and $\mathbf{B} = (0, B_y, 0)$ is the Zeeman field. The Zeeman field $\mathbf{B}$ is assumed to be finite in all the regions of the hybrid structure. The order parameter $\Delta(x) = \Delta e^{i\phi_2}\theta(x - d_n/2)\theta(x - d_n/2 - d_s) + \Delta e^{i\phi_1}\theta(x + d_n)\theta(x + d_n/2 + d_s)$, where Δ is a real-valued constant, $\phi = \phi_2 - \phi_1$ is the phase difference between the superconducting islands and $\theta(x)$ is the Heaviside step function. The matrices $\hat{s}$ and $\hat{\sigma}$ are the Pauli matrices in the spin and particle-hole spaces respectively. These spaces are combined by the Kronecker product $\otimes$.

The Hamiltonian in Eq. (1) can describe both the surface and quasi-two-dimensional bulk states of the topological insulator [14]. The case of the surface states with a robust Dirac cone dispersion can be realized in the limit of $m_{\text{eff}} \to \infty$. On the other hand, when the quadratic term is not sufficiently small the model coincides with the two-dimensional electron gas with the Rashba spin-orbit coupling.

Within the model we calculate the superconducting current-phase relations (CPR) and the Josephson critical current. First we show that the warping term causes the Josephson current anisotropy, i.e. the supercurrent depends on the orientation of the warped Fermi surface. We demonstrate that in the presence of the finite in-plane Zeeman field there is a Josephson current nonreciprocity between the two superconducting islands. This phenomenon is known as the superconducting Josephson diode effect [8]. As a next step we take into account the hexagonal warping of the TI Fermi surface which is inherent in certain 3D topological insulators [9]. We claim that the presence of the hexagonal warping leads to the anisotropy of the supercurrent nonreciprocity in the system under consideration. This is a direct consequence of the rotational symmetry breaking caused by the presence of the hexagonal warping.

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