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Fermionic quartet and vestigial gravity

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Experiments on superfluid ^{3}He-B demonstrated the anomalous behaviour, which suggested the existence of a new state above the superfluid transition but below the normal state [1]. The possible interpretation was that when the disorder suppresses the anisotropic Cooper pairing, the 4-fermion state with reduced anisotropy is formed. Such intertwined states are called the vestigial order, see review paper [2].

There can be many different realizations of the fermionic quartets, see, e.g., Eqs. (10.2)–(10.7) in [3] and Eqs. (1) and (2) in [4]. These states have different types of the topological objects. For example, if the intertwined state has the form $<\Psi\Psi\Psi\Psi>$, the mass of the effective boson is $4m$, which gives rise to the fractional vortices with circulation quantum $2\pi\hbar/4m$.

An interesting example of the quartet states is the combination of the $S=1$ pairing in the s-wave channel and the $S=0$ pairing in the p-wave channel, both being forbidden separately [5]. One may also expect the quartet order parameter, which combines the p-wave pairing with ferromagnetism, while both the Cooper pairing and ferromagnetism are absent. The four-fermion condensate naturally appears in the core of vortices [6]. More on the $4e$ condensates can be found in [7–17]. The sextuplets of fermions in the $2+1$ systems give the fractional values (1/3 or 1/6) of the intrinsic Quantum Hall Effect [18].

We consider the vestigial order on example of the spin-triplet p-wave superfluid phases of liquid ^{3}He, where the order parameter is the complex 3×3 matrix $A_{\alpha i}$ (α is the spin vector index, and i is the orbital vector index). $A_{\alpha i}$ breaks the symmetry $G = U(1)\times SO(3)_L\times SO(3)_S$ of the normal liquid, where S and L denote the spin and orbital rotations (we ignore the discrete symmetries). For the polar phase the order parameter is $A_{\alpha i}\propto e^{i\Phi}d_\alpha m_i$, where $\mathbf{d}$ is the unit vector in the spin space; and $\mathbf{m}$ is the unit vector in the orbital space.

The many-component order parameter is the typical source of the intertwined states. It is not excluded that there can be different intertwined vestigial states, which separate the polar phase from the normal state. Example is the state in which the pair order parameter is absent, $<A_{\alpha i}> = 0$, but the 4-fermionic order parameter is the same as in the nematic liquid crystals, $<A_{\alpha i}A^*_{\beta i} + A_{\beta i}A^*_{\alpha i}> \propto d_\alpha d_\beta$. The role of the director is played by the vector $\mathbf{d}$ in the spin space. This quartet represents the non-superfluid spin nematic, where the symmetry, which is broken in the polar phase, is partially restored. The polar phase symmetry $SO(2)_L\times SO(2)_S$ is enlarged to $U(1)\times SO(3)_L\times SO(2)_S$. So there is the following sequence of symmetry breaking transitions starting with the normal state:

$$G = U(1)\times SO(3)_L\times SO(3)_S \rightarrow \tag{1}$$

$$\rightarrow H_{\rm nematic} = U(1)\times SO(3)_L\times SO(2)_S \rightarrow \tag{2}$$

$$\rightarrow H_{\rm polar} = SO(2)_L\times SO(2)_S\,. \tag{3}$$

The symmetry breaking scenarios determine the behaviour of the topological defects in transition from the Cooper pairs to quartets. Example is the monopole in the planar phase of superfluid ^{3}He and in its 4-fermion partners. The monopole is the combined object: it is the monopole in spin space, which is accompanied by the monopole in the orbital vector [19]. If one tries to split the two monopoles, there appears the analog of the Nambu string which connects the spin and orbital monopoles [20]. In the 4-fermion phases the symmetry is partially restored, the orbital vector is absent, and the combined monopole in the planar phase transforms to the isolated monopole in the spin nematics vector.

In the two-step transitions one may expect the appearance of the hybrid defects, composed of two different types of topological defects with different dimensions. Such combined objects are described by the rela-

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tive homotopy groups [21]. These include walls bounded by strings and strings terminated by monopoles [22–25].

In superfluid ^{3}He-B, the order parameter $A_{\alpha i}$ plays the role of gravitational triads [26, 27]. This is the analog of the Akama–Diakonov–Wetterich (ADW) gravity [28–31], where tetrads are formed as the bilinear combinations of fermionic operators (see also [32]):

$$e^a_\mu = <\hat{E}^a_\mu> \ , \ \hat{E}^a_\mu = \frac{1}{2}\left(\Psi^\dagger\gamma^a\partial_\mu\Psi - \Psi^\dagger\overleftarrow{\partial_\mu}\gamma^a\Psi\right) . \quad (4)$$

The emergent quantum gravity here is of the type of Einstein–Cartan–Sciama–Kibble (ECSK) tetrad gravity, see review in [33]. The metric, which is the bilinear combination of tetrads, represents the fermionic quartet.

The analogy with the quartet phases in superfluid ^{3}He suggests that the ADW scenario can be extended to incorporate the vestigial states of quantum gravity, where the bilinear order parameter (tetrad) vanishes, while the quartet order parameter (metric) is nonzero:

$$<\hat{E}^a_\mu> = 0 \ , \ g_{\mu\nu} = \eta_{ab}<\hat{E}^a_\mu\hat{E}^b_\nu> . \quad (5)$$

The vestigial order in Eq. (5) describes the emergence of the Einstein general relativity in terms of metric fields $g_{\mu\nu}$. So, this symmetry breaking gives rise to the gravity for bosons, although it emerges in the fermionic vacuum.

The further spontaneous symmetry breaking is the breaking of the spin rotation symmetry by the tetrad order parameter e^a_μ in Eq. (4). This gives rise to the Weyl–Dirac action for fermions and to the Einstein–Cartan–Sciama–Kibble (ECSK) tetrad gravity, which interacts also with fermions. The sequence of symmetry breaking phase transitions is: disorder → GR → ECSK.

Due to the quartic correlators, the ECSK gravity may have the memory on the vestigial gravity:

$$g_{\mu\nu} = \eta_{ab}e^a_\mu e^b_\nu + \tilde{g}_{\mu\nu} \, , \quad (6)$$

$$\tilde{g}_{\mu\nu} = \eta_{ab}(<\hat{E}^a_\mu\hat{E}^b_\nu> - <\hat{E}^a_\mu><\hat{E}^b_\nu>) . \quad (7)$$

Fermions interact with tetradic part of the metric, while bosons interact with the full metric. Thus the Equivalence Principle can be violated on the level of particles, i.e. a boson and a fermion in a given gravitational field do not follow the same trajectories.

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