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De Sitter local thermodynamics in $f(R)$ gravity

G. E. Volovik[1)]

Low Temperature Laboratory, Aalto University, P.O. Box 15100, FI-00076 Aalto, Finland

Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia

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The $f(\mathcal{R})$ gravity in terms of the Ricci scalar $\mathcal{R}$ is one of the simplest geometrical models, which describes the dark energy and de Sitter expansion of the Universe [1–7]. It was used to construct an inflationary model of the early Universe – the Starobinsky inflation, which is controlled by the $\mathcal{R}^2$ contribution to the effective action. This class of models was also reproduced in the so-called q-theory [8], where q is the 4-form field introduced by Hawking [9] for the phenomenological description of the physics of the deep (ultraviolet) vacuum. The Starobinsky model is in good agreement with the observations, although it is difficult to embed the model into a ultraviolet (UV) complete theory [10–12].

In this paper we consider the de Sitter stage of the expansion of the Universe, and use the $f(\mathcal{R})$ gravity for the general consideration of the local thermodynamics of the de Sitter state. The term "local" means that we consider the de Sitter vacuum as the thermal state, which is characterized by the local temperature. This consideration is based on observation, that matter immersed in the de Sitter vacuum feels this vacuum as the heat bath with the local temperature $T = H/\pi$, where H is the Hubble parameter. This temperature is twice larger than the Gibbons–Hawking one, and it has no relation to the cosmological horizon. The existence of the local temperature suggests the existence of the other local thermodynamic quantities, which participate in the local thermodynamics of the de Sitter state. They include the entropy density s, the vacuum energy density ϵ and the thermodynamic variables related to the gravitational degrees of freedom. In the $f(\mathcal{R})$ theory these are the thermodynamically conjugate variables [13, 14] – the scalar curvature $\mathcal{R}$ and the effective gravitational coupling $K = df/d\mathcal{R}$ (the inverse Newton constant, $K = 1/16\pi G$).

Example of the influence of the de Sitter vacuum to the matter immersed into this vacuum is provided by an atom in the de Sitter environment. As distinct from the atom in the flat space, the atom in the de Sitter vacuum has a certain probability of ionization. The rate of ionization is similar to the rate of ionization in the Minkowski spacetime in the presence of the thermal bath with temperature $T = H/\pi$ [15–17]. The same temperature determines the other activation processes, which are energetically forbidden in the Minkowski vacuum, but are allowed in the de Sitter background, see also [18, 19]. That is why it is natural to consider the temperature $T = H/\pi$ as the local temperature of the de Sitter vacuum. Although the local temperature is twice larger than the Gibbons–Hawking temperature assigned to the horizon, $T_{\rm GH} = H/2\pi$, there is the certain connection between the local thermodynamics and the thermodynamics of the event horizon. Using the local thermodynamics with $T = H/\pi$, we obtained the general result for the total entropy inside the horizon. The total entropy of the volume V_H bounded by the cosmological horizon coincides with the Gibbons–Hawking entropy assigned to the horizon, $S_{\rm bulk} = sV_H = 4\pi KA = S_{\rm GH}$, where $K = df/d\mathcal{R}$. The connection between the bulk entropy of the Hubble volume, $S_{\rm bulk}$, and the surface entropy of the cosmological horizon, $S_{\rm GH}$, suggests the bulk-surface correspondence of the holographic origin [20–22].

It is not excluded that our Universe is finite. Its volume V might be comparatively small, not much larger than the currently observed Hubble volume V_H [23]. Then, if the de Sitter state represents the excited thermal state of the quantum vacuum, the thermal fluctuations of the deep quantum vacuum become important. The local thermodynamics of the de Sitter state demonstrates that the relative magnitude of thermal fluctuations of the vacuum energy density $\epsilon_{\rm vac}$ is determined

[1)]e-mail: grigori.volovik@aalto.fi

by the ratio of the Hubble volume to the volume of the Universe:

$$\frac{\left\langle(\Delta\epsilon_{\rm vac})^2\right\rangle}{\left\langle\epsilon_{\rm vac}\right\rangle^2} \sim \frac{V_H}{V}. \qquad (1)$$

Since the de Sitter state serves as the thermal bath for matter, the de Sitter quantum vacuum may have its own temperature [24], which can be different from the temperature of the matter degrees of freedom [25]. The initially empty pure de Sitter state becomes locally unstable towards the creation of thermal matter from the vacuum by thermal activation. The process of the thermalization of matter transforms the de Sitter state to the quasi- equilibrium state of the expanding Universe, which is characterized by two different temperatures: the temperature of the gravitational vacuum and the temperature of matter. To describe the decay of the vacuum due to activation and thermalization of matter, the extension of the Starobinsky analysis of the vacuum decay [26–29] is needed. We provide the simple phenomenological model of the thermal exchange between the vacuum and the excited matter, which leads to the following time dependence of the energy densities of vacuum and matter:

$$\epsilon_{\rm vac} \sim M_{\rm P}^4 \left(\frac{t_{\rm P}}{t+t_0}\right)^{2/3}, \ \epsilon_M \sim H^4 \sim M_{\rm P}^4 \left(\frac{t_{\rm P}}{t+t_0}\right)^{4/3}. \quad (2)$$

Here $M_{\rm P}$ is the Planck mass, $M_{\rm P}^2 = K$, and $t_{\rm P} = 1/M_{\rm P}$ is Planck time. The obtained power law decay of H agrees with that found in [30–34]. The parameter t_0 is related to the initial value of the Hubble parameter at the beginning of inflation, $t_0 \sim E_{\rm P}^2/H_{t=0}^3$. It is called the quantum breaking time of space-times with positive cosmological constant [35, 36].

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