Supplementary Material to the article

“Features of radiation heating kinetics of metal plates under fluctuation-electromagnetic friction”

We use the permittivity $\varepsilon(\omega)$ for both metal plates in the form of the Drude expression

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu(T))}, \quad (S1)$$

where ω_p is the plasma frequency, $\nu(T) = \omega_p^2\rho(T)/4\pi$ is the relaxation frequency of electrons, and $\rho(T)$ is the metal resistivity. Figure S1 shows the dependencies $\rho(T)$ corresponding to the Bloch–Grüneisen (BG) model [30] (dashed line in Fig. S1), modified Bloch–Grüneisen model (MBG) (circles in Fig. S1) and the model with a linear dependence of resistance on temperature (LR) (straight line). In BG model (T_D=175 K , ω_p= 9.03 eV),

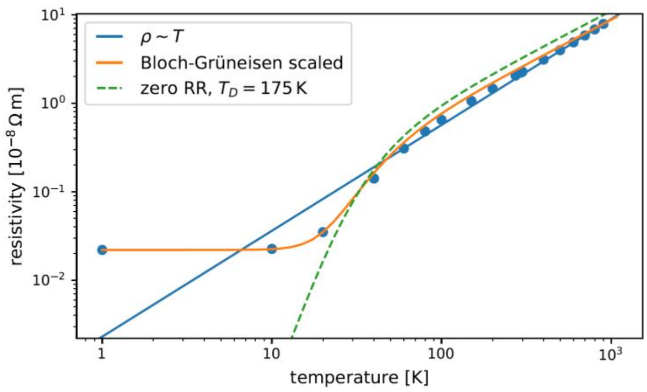

Fig. S1.(Color online). Resistivity of gold as a function of temperature (the data have been kindly supplied by C. Henkel).

$$\nu_{BG}(T) = 0.0212(T/T_D)^5 \int_0^{T_D/T} x^5 \mathrm{sh}(x/2)^{-2} dx, \text{ (eV)}. \quad (S2)$$

In addition, we also used a truncated BG model

$$\nu(T, T_0) = \nu(T), T > T_0;\ \nu(T) = \nu(T_0), T \le T_0. \quad (S3)$$

In MBG model, the residual resistance of gold is $\rho_0 = 2.3 \cdot 10^{-10}\ \Omega \cdot \mathrm{m}$. To obtain the resistivity in Gaussian units, one should use the relation $\Omega \cdot \mathrm{m} = (1/9)10^{-9}\mathrm{s}^{-1}$.

Because $\varepsilon(\omega) \gg 1$ for good conductors like gold, and the inequality gets stronger as $T \to 0$, the second terms in in (7)-(9), corresponding to electromagnetic P modes are negligible compared to the first terms with $\mu_{1,2}$, corresponding to the S modes. This is all the more true for the near-field range of distances a between the plates. Therefore, the contributions of P modes in the numerical calculations were neglected.

To compute the integrals in (7)–(9), a change of variable $\omega = \nu_m(T_1, T_2)t$ is performed, with $\nu_m(T_1, T_2) = \max\,(\nu_1(T_1), \nu_2(T_2))$ and $\nu_i(T_i)$ being the relaxation frequencies of plates 1 and 2 depending on their temperatures T_1 and T_2 $(i = 1,2)$. The 2D wave-vector modulus (in polar coordinates (k, ϕ) of the plane (k_x, k_y)) is written as $k = (\omega_p/c)\sqrt{y^2 + \beta_m^2 t^2}$ in the evanescent sector $k > \omega/c$ and $k = (\omega_p/c)\sqrt{\beta_m^2 t^2 - y^2}$ in the radiation sector $k < \omega/c$.

Additional parameters are $\beta_m = \nu_m/\omega_p$, $\alpha_i = \hbar\nu_i/T_i$, $\gamma_i = \nu_i/\nu_m$, $\lambda = \omega_p a/c$, $\zeta = (V/c)\beta_m^{-1}$ and $K = \hbar\nu_m^2(\omega_p/c)^4/\pi^3$. For $k > \omega/c$, formulas (7)–(9) take the form

$$P_1 = K\int_0^\infty dt\int_0^\infty dyy^3 f_1(t,y), \tag{S4}$$

$$P_2 = -K\int_0^\infty dt\int_0^\infty dyy^3 f_2(t,y), \tag{S5}$$

$$F_x V = K\int_0^\infty dt\int_0^\infty dyy^3 f_3(t,y), \tag{S6}$$

$$f_1(t,y) = t\int_0^\pi d\phi\,\frac{\mathrm{Im}w_1\mathrm{Im}w_2}{|D|^2}\left(\coth\left(\frac{\alpha_1 t}{2}\right) - \coth\left(\frac{\alpha_2 t^-}{2}\right)\right), \tag{S7}$$

$$f_2(t,y) = t\int_0^\pi d\phi\frac{\mathrm{Im}w_1\mathrm{Im}w_2}{|D|^2}\left(\coth\left(\frac{\alpha_1 t^-}{2}\right) - \coth\left(\frac{\alpha_2 t}{2}\right)\right), \tag{S8}$$

$$f_3(t,y) = \eta\int_0^\pi d\phi\cos\phi\sqrt{y^2+\beta_m^2 t^2}\frac{\mathrm{Im}w_1\mathrm{Im}w_2}{|D|^2}\left(\coth\left(\frac{\alpha_1 t}{2}\right) - \coth\left(\frac{\alpha_2 t^-}{2}\right)\right). \tag{S9}$$

$$w_1 = \sqrt{y^2+\frac{t}{t+\mathrm{i}\cdot\gamma_1}}, w_2 = \sqrt{y^2+\frac{t^-}{t^-+\mathrm{i}\cdot\gamma_2}},\ \ t^- = t-\zeta\cos\phi\sqrt{y^2+\beta_m^2 t^2}, \tag{S10}$$

$$D = (y+w_1)(y+w_2)\exp(\lambda y) - (y-w_1)(y-w_2)\exp(-\lambda y). \tag{S11}$$

For $k < \omega/c$, expressions (S10), (S11) should be used by replacing $y \to \mathrm{i}\cdot y$, and substituting $\beta_m t$ for ∞ in the integrals over y in (S4)–(S6).

Figure S2 shows the velocity dependencies of P_1 and P_2 calculated using the truncated BG model (S3) at $T_0 = 7$ K, fixed $\Delta T = 3$ K and a constant ratio $\gamma_1/\gamma_2 = 2$. Note that $P_{1,2} < 0$ corresponds to cooling. Increasing the speed of the moving plate changes the "normal" direction of heat transfer from the hot plate to the cold one (cf. two upper curves marked as (6,3) and (3,6)). The numerical values indicate the temperatures of plate 1 and plate 2. For the two lower curves, the heat flow direction is “normal”: plates 1 and 2 heat up at a lower temperature, and heat exchange occurs faster with increasing velocity of plate 2.

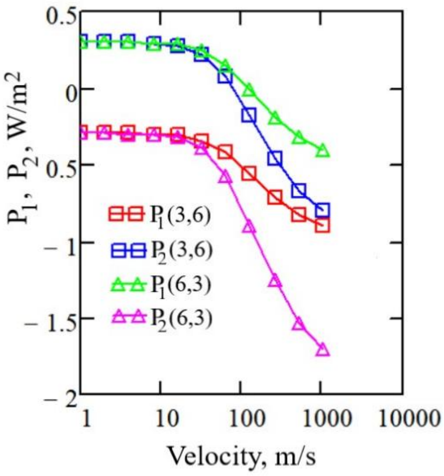

Fig. S2. (Color online). Heat transfer rates of gold plates under Casimir-Lifshitz friction as functions of velocity of plate 2. The truncated BG model (S3) is used with $T_0 = 7$ K. The temperature values and their order in the brackets correspond to plates 1 and 2.