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The effect of the relative vibrations of the abrasive tool and the workpiece on the probability of material removing during finishing grinding

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ABSTRACT

Introduction. Grinding remains the most efficient and effective method of final finishing that is indispensable in the production of high-precision parts. The characteristic features of grinding materials are that the removal of the material roughness of the workpiece surface occurs due to the stochastic interaction of the grains of the abrasive material with the surface of the workpiece, in the presence of mutual oscillatory movements of the abrasive tool and the workpiece being processed. During processing workpieces with abrasive tools, the material is removed by a large number of grains that do not have a regular geometry and are randomly located on the working surface. This makes it necessary to apply probability theory and the theory of random processes in mathematical simulation of operations. In real conditions, during grinding, the contact of the wheel with the workpiece is carried out with a periodically changing depth due to machine vibrations, tool shape deviations from roundness, unbalance of the wheel or insufficient rigidity of the workpiece. To eliminate the influence of vibrations in production, tools with soft ligaments are used, the value of longitudinal and transverse feeds is reduced, but all these measures lead to a decrease in the operation efficiency, which is extremely undesirable. To avoid cost losses, mathematical models are needed that adequately describe the process, taking into account the influence of vibrations on the output indicators of the grinding process. **The purpose of the work** is to create a theoretical and probabilistic model of material removing during finishing and fine grinding, which allows, taking into account the relative vibrations of the abrasive tool and the workpiece, to trace the patterns of its removal in the contact zone. **The research methods** are mathematical and physical simulation using the basic provisions of probability theory, the laws of distribution of random variables, as well as the theory of cutting and the theory of deformable solids. **Results and discussion**. The developed mathematical models allow tracing the effect on the removal of the material of the superimposition of single sections on each other during the final grinding of materials. The proposed dependencies show the regularity of the stock removal within the arc of contact of the grinding wheel with the workpiece. The considered features of the change in the probability of material removal when the treated surface comes into contact with an abrasive tool in the presence of vibrations, the proposed analytical dependences are valid for a wide range of grinding modes, wheel characteristics and a number of other technological factors. The expressions obtained allow finding the amount of material removal also for the schemes of end, profile, flat and round external and internal grinding, for which it is necessary to know the magnitude of relative vibrations. However, the parameters of the technological system do not remain constant, but change over time, for example, as a result of wear of the grinding wheel. To assess the state of the technological system, experimental studies are carried out taking into account the above changes over the period of durability of the grinding wheel.

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Introduction

The development of science and technology sets the task of wide application in the designs of products of electrical engineering, instrumentation engineering, nuclear energy, rocket science, aircraft engineering, space technology, medicine and, more recently, in general engineering, new materials, which are subject to increased requirements for heat resistance, wear resistance, corrosion resistance and chemical resistance. Industrial enterprises are faced with the challenges of efficient processing of the above materials.

In the context of the development of a market economy, the most important factor in the success of an enterprise is the creation of technological processes that ensure the satisfaction of consumer needs. These include reducing the cost of manufacturing range while ensuring high performance characteristics of products, increasing the productivity of creating products with desired properties, for example, in the production of a friction pair, it is necessary to technologically ensure the optimal structure of the surface layer of the working surfaces of parts in the shortest possible time, creating a surface microrelief at the stage of machining parts close to equilibrium. This approach will reduce the running-in stage of the friction pair and increase its service life [1].

An analysis of existing research in the field of material processing shows that, despite the presence of a large number of high-precision processing methods, such as ultrasonic, laser, high-speed milling, and others, grinding remains the most used and efficient method in the manufacture of high-precision parts [2–6].

Grinding remains the most efficient and effective method of final finishing that is indispensable in the production of high-precision parts.

The characteristic features of grinding materials are that the removal of the material roughness of the workpiece surface occurs due to the stochastic interaction of the grains of the abrasive material with the surface of the workpiece, in the presence of mutual oscillatory movements of the abrasive tool and the workpiece being processed.

Considerable attention is paid to the study of grinding processes in the works of *A.I. Grabchenko*, *V.L. Dobroskoka*, *V.I. Kalchenko*, *F.N. Novikova*, *M.D. Uzunyan*, *V.A. Fedorovich*, *L.N. Filimonova*, *A.V. Yakimov* and other authors who, using various statistical and probabilistic methods, obtained calculated dependencies in relation to specific grinding schemes and conditions. The authors have shown that any conclusions about the number of working grains, about its percentage with grains on the surface of the wheel can have real meaning only in relation to specific conditions inherent in this process, which is associated with the nonstationarity of grinding operations.

The first mathematical models of abrasive-diamond machining, reflecting the dynamic properties of processes, its stochastic nature, as well as the nonstationarity of the states of technological operations, were obtained and published by *Yu.K. Novoselov* in 1971. In 1975, publications of *A.V. Korolev* appeared, which used a similar approach.

The above works have made a significant contribution to the development of the theory of shaping of ground surfaces, however, it does not take into account the specifics of products processing in the presence of relative vibrations of the wheel and the workpiece on the output indicators of the grinding operation, therefore, it has a limited scope [7–10].

During processing workpieces with abrasive tools, the material is removed by a large number of grains that do not have a regular geometry and are randomly located on the working surface. This makes it necessary to apply probability theory and the theory of random processes in mathematical simulation of operations [11–14].

In real conditions, during grinding, the contact of the wheel with the workpiece is carried out with a periodically changing depth due to machine vibrations, tool shape deviations from roundness, unbalance of the wheel or insufficient rigidity of the workpiece. To eliminate the influence of vibrations in production, tools with soft ligaments are used, the value of longitudinal and transverse feeds is reduced, but all these measures lead to a decrease in the operation efficiency, which is extremely undesirable.

To avoid cost losses, mathematical models are needed that adequately describe the process, taking into account the influence of vibrations on the output indicators of the grinding process [15–19].

Based on the above, the purpose of this work is is to create a theoretical and probabilistic model of material removing during finishing and fine grinding, which allows, taking into account the relative vibrations of the abrasive tool and the workpiece, to trace the patterns of its removal in the contact zone.

The research methods

The presence of mutual oscillatory movements of the abrasive tool and the workpiece being processed is a characteristic feature of the grinding process. Oscillatory movements arise due to the imbalance of the rotating parts of the machine, vibrations coming from outside, self-oscillations that accompany the cutting process. The frequency of forced oscillations for grinding machines according to *P.I. Liashcheritsyn* is 150–350 Hz, the frequency of self-oscillations is 300–900 Hz [1].

The presence of relative oscillatory movements of the grinding wheel and the workpiece leads to a change in the size and shape of the contact zone, to a distortion of the trajectories of the relative movement of the tops of abrasive grains in the material being processed, to a change in the current depth of microcutting, Figure 1.

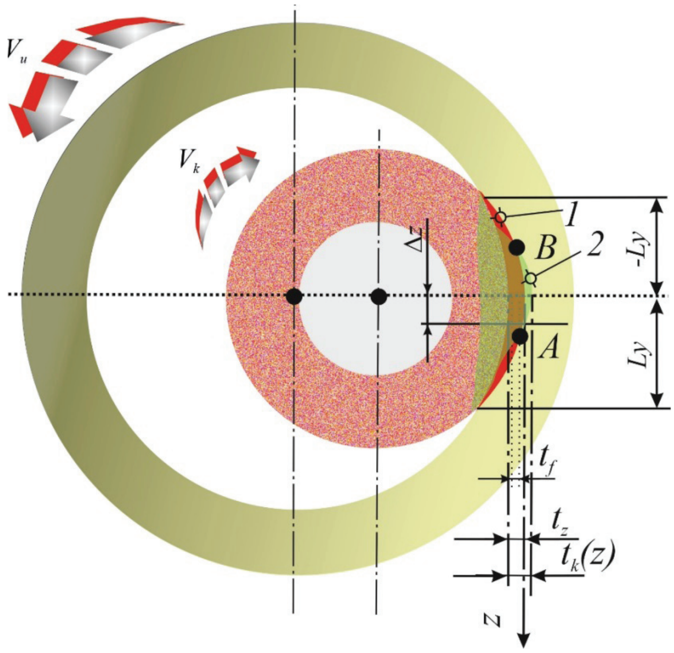

Fig.1. Influence of vibrations on the depth of microcutting during internal grinding

Relative displacements in the direction of the center line of the grinding head and the workpiece, regardless of the reasons that caused it, can be described by the equation:

$$Y = \sum_i A_{\omega i} \cdot \cos(\omega_i \tau + \psi_{yi}), \qquad (1)$$

where $A_{\omega i}$, ω_i, ψ_{yi} – amplitude, cyclic frequency and initial phase of deviations t_f; τ – contact time of the surface with the tool.

The current value of the microcutting depth t_z depends on the radius vectors of the workpiece *r* and a wheel *R*, center-to-center distance *A* (see Fig. 1). For the most protruding grains, it can be determined by the equation:

$$t(z) = t_f - \frac{z^2 \cdot D \cdot d}{(d - D)} = t_f - \frac{z^2}{D_e}, \qquad (2)$$

where *D*, *d* are the diameters of the tool and the workpiece, respectively, D_e is the equivalent diameter, z is the distance of the workpiece section to the main plane.

When the workpiece rotates, the section of the machined surface passes in the contact zone from point *A* to point *B*. The depth of cut in the absence of vibrations changes monotonously (line 1) from zero to t_f and from t_f to zero, the current contact time is determined as $\tau = \frac{z}{V_u}$.

For point A $z = -L$, $\tau = 0$, for point B $z = +L$, $\tau = \frac{2 \cdot L}{V_u}$.

For a surface section passing through the zone of contact between the workpiece and the wheel, the instantaneous depth of microcutting by single abrasive grains, taking into account (1) and (2), can be described by the function:

$$t(k)_i = t_f - \frac{z^2}{D_e} + \sum_i A_{\omega i} \cos\left(\frac{\omega_i \cdot z}{V_u} + \psi_{yi}\right). \tag{3}$$

To describe the patterns of material removing in the contact zone, the authors of [20] proposed the concepts of material removing probability $P(M)$ and material not removing probability $P(\overline{M})$. The first indicator $P(M)$ is determined by the probability of an event in which the material at the point of the treated surface is removed. The second indicator $P(\overline{M})$ is the probability of an event in which the material is not removed from the treated surface. The sum of the probabilities, as the probabilities of opposite events, is equal to one, and its values depend on the position of the point in the contact zone. For the processing of workpieces with abrasive tools, the probability of material removal is calculated from the dependence:

$$P(M) = 1 - \exp^{-a(y) - a(y,\tau)}, \tag{4}$$

where $a(y)$ is an indicator that determines the probability of material removal at the level y before the surface enters the zone of contact between the workpiece and the wheel; $a(y,\tau)$ is an indicator that characterizes the change in the areas of dimples formed by the sum of the profiles of abrasive grains passing through the considered section of the workpiece after the corresponding contacts of the grains with the surface of the workpiece.

During time $\Delta\tau$ turns through an angle $\Delta\varphi$ and a section passes through it with an arc length $(V_k \pm V_u) \cdot \Delta\tau$ or taking into account that $\tau = \frac{z}{V_u}$ we get $\frac{\Delta z \cdot (V_k \pm V_u)}{V_u}$. Of the total number of grains that have passed through the section, the width of the profile b_g will have grains whose vertices are located in the layer of the wheel $\frac{(V_k \pm V_u)}{V_u} \Delta z \Delta u$. The number of such vertices is calculated from the density of its distribution over the depth of the tool $f(u)$

$$\Delta\lambda = n_g \cdot f(u) \frac{(V_k \pm V_u)}{V_u} \Delta z \Delta u. \tag{5}$$

In the presence of vibrations, the width of the vertex contour corresponding to a given level is nonstationary, it does not remain constant, but changes over time. Its value can be described by a power dependence [1], which, taking into account the fact that $\tau = \frac{z}{V_u}$, is calculated by the equation:

$$b_g(y) = C_b h^m = C_b \left[t(k) - y - u\right]^m = C_b \left(t_f - y - u - \frac{z^2}{D_e} + A_\omega \cos\left(\frac{\omega z}{V_u} + \psi_y\right)\right)^m. \tag{6}$$

To describe the distribution density of the abrasive grains vertices, *O. Coyle* suggested using a following dependence [17]:

$$f(u) = C_h \cdot u^{\chi - 1} \; f(u) = C_h \cdot u^{\chi - 1}, \tag{7}$$

where C_h – distribution curve proportionality factor:

$$C_h = \frac{\chi}{H_u^{\chi}},$$

where H_u – thickness of the tool working surface layer in contact with the workpiece.

Taking into account the above, dependence (7) can be represented as:

$$f(u) = \frac{\chi}{H_u^{\chi}} \cdot u^{\chi-1}, \tag{8}$$

where χ – density function parameter.

The change in the parameter $a(y, \tau)$ is determined by the increment of the sum of the transverse dimensions of the abrasive grains profiles:

$$\Delta a(y, \tau) = K_c n_g b_g(\tau) f(u) \Delta u (V_k \pm V_u) \Delta \tau, \tag{9}$$

where K_c – chip formation coefficient, which takes into account that not all material is removed from the volume of the scratch mark, but part of it is displaced and forms piles along the edges of the scratch mark.

After integrating (9), we obtain an integral equation for calculating the parameter $a(y, z)$ in the contact zone

$$a(y, z) = \int_{-L_y}^{z} \int_{0}^{t(k)-y} K_c n_g b_g f(u) \frac{V_k \pm V_u}{V_u} du dz, \tag{10}$$

where L_y – the distance from the main plane to the intersection of the level with the conventional outer surface of the tool is determined from the equation

$$L_{yi} = \sqrt{(t_{ki} - y) D_e}. \tag{11}$$

The models of grain tops and densities of its distribution over depth considered above make it possible to establish functional relationships between the probability of material not removing with technological factors.

When substituting the obtained expressions b_g and $f(u)$ from equations (6) and (8) into equation (10), the last one takes the following form:

$$a(y, z) = \frac{K_c C_b (V_k \pm V_u) n_g \chi}{V_u H_u^{\chi}} \int_{-L_y}^{z} \int_{0}^{t(k)-y} \left(t_f - y - u - \frac{z^2}{D_e} + A_{\omega} \cos\left(\frac{\omega z}{V_u} + \psi_y \right) \right)^m u^{\chi-1} du dz. \tag{12}$$

After integrating the resulting equation with respect to *u*, we obtain

$$a(y, z) = \frac{\Gamma(m+1)\Gamma(\chi) K_c C_b (V_k \pm V_u) n_g \chi}{\Gamma(m+\chi+1) V_u H_u^{\chi}} \int_{-L_y}^{z} \left(t_f - \frac{z^2}{D_e} - y + A_w \cos\left(\frac{\omega z}{V_u} + \psi_y \right) \right)^{m+\chi} dz, \tag{13}$$

where $\Gamma(m+1)$, $\Gamma(\chi)$, $\Gamma(m+\chi+1)$ – corresponding gamma-functions.

Дальнейшее интегрирование уравнения (13) возможно только при известных значениях показателей χ, m и значениях ψ_y характеризующих начальную фазу отклонений. Вид зависимостей определяется их суммой. При $\chi = 1{,}5$, $m = 0{,}5$ и $C_b = 2\sqrt{2\rho_g}$.

Further integration of equation (13) is possible only when values of indicators χ, m, and values ψ_y characterizing the initial phase of deviations are known. The type of dependencies is determined by its sum. When For $\chi = 1.5$, $m = 0.5$ and $C_b = 2\sqrt{2 p_g}$:

$$a(y,z)=\frac{3\pi n_g K_c\sqrt{2\rho_g}\,(V_k \pm V_u)\Upsilon_{\psi_y}}{8V_u H_u^{3/2}}, \quad (14)$$

where $\Upsilon_{\psi_y}=\int_{-L_y}^{z}\left(t_f-\frac{z^2}{D_e}-y+A_\omega\cos\left(\frac{\omega z}{V_u}+\psi_y\right)\right)^2 dz.$

Denote , $\beta=\frac{\omega z}{V_u}+\psi_y$ and perform the integration for cases when the initial phase is equal to: 1– $\psi_y=0(2\pi)$; 2–$\psi_y=\pi$; 3.– $\psi_y=\frac{\pi}{2}$; 4.– $\psi_y=\frac{3\pi}{2}$

$$\Upsilon_{2\pi}=\frac{L_y^5+z^5}{5D_e^2}-\frac{2(t_f-y)\left(L_y^3+z^3\right)}{3D_e}+\frac{(z+L_y)\left(A^2+2(t_f-y)^2\right)}{2}+$$
$$+\frac{A^2V_u(\sin 2\alpha+\sin 2\beta)}{4\omega}-\frac{4AV_u^2(L_y\cos\alpha+z\cos\beta)}{D_e\omega^2}+$$
$$+\frac{2AV_u\left(2V_u^2+D_e\omega^2(t_f-y)\right)(\sin\alpha+\sin\beta)}{D_e\omega^3}-\frac{2AV_u\left(L_y^2\sin\alpha+z^2\sin\beta\right)}{D_e\omega}, \quad (15)$$

$$\Upsilon_{\pi}=\frac{L_y^5+z^5}{5D_e^2}-\frac{2(t_f-y)\left(L_y^3+z^3\right)}{3D_e}+\frac{\left(z+L_y\right)\left(A^2+2(t_f-y)^2\right)}{2}+$$
$$+\frac{A^2V_u(\sin 2\alpha+\sin 2\beta)}{4\omega}+\frac{4AV_u^2\left(L_y\cos\alpha+z\cos\beta\right)}{D_e\omega^2}-$$
$$-\frac{2AV_u\left(2V_u^2+D_e\omega^2(t_f-y)\right)(\sin\alpha+\sin\beta)}{D_e\omega^3}+\frac{2AV_u\left(L_y^2\sin\alpha+z^2\sin\beta\right)}{D_e\omega}, \quad (16)$$

$$\Upsilon_{\frac{\pi}{2}}=\frac{L_y^5+z^5}{5D_e^2}-\frac{2(t_f-y)\left(L_y^3+z^3\right)}{3D_e}+\frac{\left(z+L_y\right)\left(A^2+2(t_f-y)^2\right)}{2}-$$
$$-\frac{A^2V_u(\sin 2\alpha+\sin 2\beta)}{4\omega}-\frac{4AV_u^2\left(L_y\sin\alpha-z\sin\beta\right)}{D_e\omega^2}-$$
$$-\frac{2AV_u\left(2V_u^2+D_e\omega^2(t_f-y)\right)(\cos\alpha-\cos\beta)}{D_e\omega^3}+\frac{2AV_u\left(L_y^2\cos\alpha-z^2\cos\beta\right)}{D_e\omega} \quad (17)$$

$$\Upsilon_{\frac{3\pi}{2}}=\frac{L_y^5+z^5}{5D_e^2}-\frac{2(t_f-y)\left(L_y^3+z^3\right)}{3D_e}+\frac{(z+L_y)\left(A^2+2(t_f-y)^2\right)}{2}-$$
$$-\frac{A^2V_u(\sin 2\alpha+\sin 2\beta)}{4\omega}+\frac{4AV_u^2(L_y\sin\alpha-z\sin\beta)}{D_e\omega^2}-$$
$$-\frac{2AV_u\left(2V_u^2+D_e\omega^2(t_f-y)\right)(\cos\alpha-\cos\beta)}{D_e\omega^3}-\frac{2AV_u\left(L_y^2\cos\alpha-z^2\cos\beta\right)}{D_e\omega}. \quad (18)$$

Results and Discussion

The calculation of the probability of material removing in the presence of vibrations in any area of the contact zone with a known initial state of the surface is calculated by substituting the indicator $a(y,\tau) = a$ from expression (14) into equation (4) taking into account the parameter Y, for each of the cases when the initial phase is: $\psi_y = 0(2\pi)$ (15); $\psi_y = \pi$ (16); $\psi_y = 0(2\pi)$ (17); $\psi = 0(2\pi)$ (18). For clarity of the calculation procedure, let's consider a numerical example.

Let's calculate the probability of not removing and the probability of removing the material when grinding holes with a diameter of 150 mm in workpieces made of titanium alloy *VT3-1* with a tool *AW 60 × 25 × 13 63C F90 M 7 BA 35 m/s* (at a wheel speed of 35 m/s, a workpiece speed of 0.25 m/s, longitudinal feed – 33 mm/s, transverse feed – 0.005 mm/stroke). From the calculation of the balance of displacements [20], we determine that for the given processing conditions $t_f = 11.54 \cdot 10^{-6}$ m. Based on the research data [20, 21, 22], we accept: $K_c = 0.9$ m; $\rho_z = 7.31 \cdot 10^{-6}$ m; $n_z = 15.86$ grains/mm^2. For the considered conditions $L_y = 3.397 \cdot 10^{-4}$ m, $\omega = 628$ rad/s, $\nu = 100$ Hz. The calculation is performed according to equations (2), (3), (4) for the level $y = 10.38 \cdot 10^{-6}$ m at $z = -0.8\dfrac{L_y}{2}$, $A = 0.2t_f$.

Let's calculate the parameters $\alpha = \dfrac{\omega z}{V_u} + \psi_y$ and $\beta = \dfrac{\omega L_y}{V_u} + \psi_y$ for the cases when the initial phase is equal to $\psi_y = 0$:

$$\alpha = \frac{628(-0.136 \cdot 10^{-4})}{0.25} + 0 = -0.341, \quad \beta = \frac{628 \cdot 3.397 \cdot 10^{-4}}{0.25} + 0 = 0.853$$

Отсюда получим $\sin\alpha = -0.3344$, $\cos\alpha = 0.942$, $\sin\beta = 0.753$, $\cos\beta = 0.658$, $\sin 2\alpha = -0.63$, $\sin 2\beta = 0.991$.

After substituting the numerical values of the parameters in (15), we obtain:

$$Y_0 = \frac{(3.397 \cdot 10^{-4})^5 + (-0.136 \cdot 10^{-3})^5}{5 \cdot 0.1^2} - \frac{2(11.54 \cdot 10^{-6} - 10.36 \cdot 10^{-6})\left((3.397 \cdot 10^{-4})^3 + (-0.136 \cdot 10^3)^3\right)}{3 \cdot 0.1} +$$

$$+ \frac{\left(-0.136 \cdot 10^{-3} + 3.397 \cdot 10^{-4}\right)\left(\left(2.308 \cdot 10^{-6}\right)^2 + 2\left(11.54 \cdot 10^{-6} - 10.38 \cdot 10^{-6}\right)^2\right)}{2} +$$

$$+ \frac{(2.308 \cdot 10^{-6})^2 \cdot 0.25(-0.63 + 0.991)}{4 \cdot 628} - \frac{4 \cdot 2.308 \cdot 10^{-6} \cdot 0.25^2 (3.397 \cdot 10^{-4} \cdot 0.942 \pm 0.136 \cdot 10^{-3} \cdot 0.658)}{0.1 \cdot 628^2} +$$

$$+ \frac{2 \cdot 2.308 \cdot 10^{-6} \cdot 0.25\left(2 \cdot 0.25^2 + 0.1 \cdot 628^2 (11.54 \cdot 10^6 - 10.38 \cdot 10^{-6})\right)(-0.3344 + 0.753)}{0.1 \cdot 628^3} -$$

$$- \frac{2 \cdot 2.308 \cdot 10^{-6} \cdot 0.25\left((3.397 \cdot 10^{-4})^2 (-0.3344) + (-0.136 \cdot 10^{-3})^2 \cdot 0.753\right)}{0.1 \cdot 628} = 1.22 \cdot 10^{-15}.$$

Then, according to equation (14), we calculate the value of the indicator, taking into account vibrations $a(y,\tau) = a(y,z)$:

$$a(y,z) = \frac{3 \cdot 3.14 \cdot 15.866 \cdot 10^6 \cdot 1\sqrt{2 \cdot 7.31 \cdot 10^{-6}}(35 \pm 0.25) 1.22 \cdot 10^{-15}}{8 \cdot 0.25 \cdot (11.54 \cdot 10^{-6})^{3/2}} = 0.282.$$

In the absence of vibrations, the indicator $a_{cm}(y,z)$ can be calculated according to the dependence [21]:

$$a_{cm}(y,z)=\frac{3\pi n_g K_c\sqrt{2\rho_g}(V_k\pm V_u)(t_f-y)^2}{8V_u H_u^{3/2}}\left(z-\frac{2\cdot z^3}{\sqrt[3]{L_y}}+\frac{z^5}{5\cdot L_y}+\frac{8}{15}L_y\right) \quad (19)$$

When substituting the above values of the parameters into expression (19), we obtain the value of the indicator in the absence of vibrations:

$$a_{\text{cm}}(y,z)=\frac{3\cdot 3.14\cdot 15.86\cdot 10^6\cdot 1\sqrt{2\cdot 7.31\cdot 10^{-6}}\cdot(35\pm 0.25)(11.54\cdot 10^{-6}-10.38\cdot 10^{-6})^2}{8\cdot 0.25(11.54\cdot 10^{-6})^{3/2}}\times$$

$$\times\left(-0.136\cdot 10^{-3}-\frac{2\left(-0.136\cdot 10^{-3}\right)^3}{\sqrt[3]{3.397\cdot 10^{-4}}}+\frac{\left(-0.136\cdot 10^{-3}\right)^5}{5\cdot 3.397\cdot 10^{-4}}+\frac{8}{15}\cdot 3.397\cdot 10^{-4}\right)=0.014.$$

The absolute error ΔA of calculations is

$$\Delta A=\left|a_{\text{cm}}(y,z)-a(y,z)\right|=\left|0.014-0.313\right|=0.299,$$

and relative error δ_A of calculations is:

$$\delta_A=\frac{\Delta A}{a(y,z)}100\ \%=\frac{0.299}{0.313}100\ \%=95.5\ \%.$$

Experimental studies were carried out to verify the calculation results. On a *Knuth RSM 500* CNC machine, holes with a diameter of 150 mm in workpieces made of titanium alloy *VT3-1* with a tool *AW 60 × 25 × 13 63C F90 M 7 BA 35 m/s* (at a wheel speed of 35 m/s, a workpiece speed of 0.25 m/s, longitudinal feed – 33 mm/s, transverse feed – 0.005 mm/stroke). Profile diagrams were taken from the prototypes after the grinding operation, according to which the value of the indicator $a_{\text{experiment}}(y,z)$ was estimated (Table 1) and the relative error was determined (Table 2).

The probability of an event characterizing removing the surface layer at the level $y = 0.004$ mm at the value of the indicator $a_0 = 0.545$ is calculated according to the equation by substituting the calculation results $a(y,\tau)$ into expression (4):

$$P(M)=1-\exp^{-a_0-a(y,\tau)}=1-\exp^{-0.545-0.282}=0.576.$$

The probability of no material removal, as an opposite event, can be determined from the total probability formula:

$$P(\bar{M})=1-P(M)=1-0.576=0.424.$$

Table 1

The values of the indicator *a* characterizing the change in the probability of material removing at the considered level

The value of the indicator a characterizing the change in the probability of material removing at the considered level	Номер опыта				
	1	2	3	4	5
$a_{\text{experiment}}(y,z)$	0.392	0.313	0.264	0.4	0.305
$a_{\text{cm}}(y,z)$	0.014				
$a(y,z)$	0.282				

Table 2

Relative error of calculations

Experimental (actual) values	Relative error $\delta_A = \frac{\Delta A}{a_{\text{experiment}}(y,z)} 100\ \%,\ \%$	
$a_{\text{experiment}}(y,z)$	$a_{\text{cm}}(y,z)$	$a(y,z)$
0.392	96.4	28.06
0.313	91.05	9.9
0.264	94.6	6.82
0.4	96.5	29.5
0.305	95.4	7.54

The obtained calculations show that the probability of removing at values $A = 0.2t_f$, $z = -0.8\frac{L_y}{2}$, $L_y = 3.397 \cdot 10^{-4}$ m, $y = 10.38 \cdot 10^{-6} m$, $t_f = 11{,}54 \cdot 10^{-6}$ m, $\omega = 628$ rad/s, $\nu = 100$ Hz is 0.424. This means that 42 % of the material will be removed and 58 % of the processed material will stay on the surface in the form of microroughness.

For other levels and values of oscillation frequencies of the considered example, the calculated data on the probability of material removing are shown in Fig. 2 and 3 and in table 3.

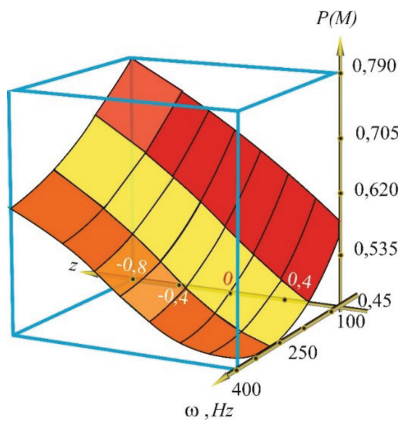

Fig. 2. Change in the probability of material removing along the contact zone from the value of relative vibrations during internal grinding

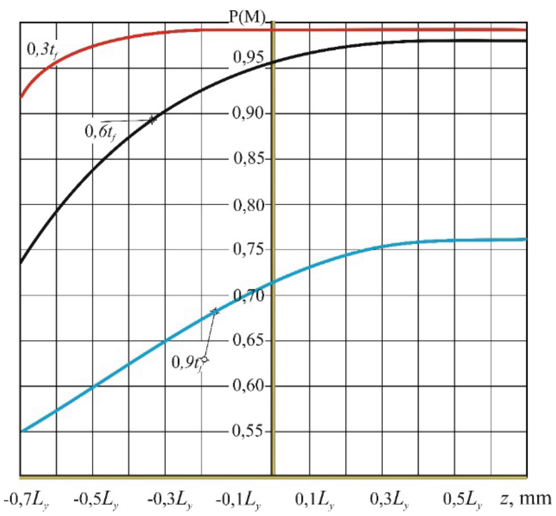

Fig. 3. Change in the probability of material removing along the contact zone from the level of grain penetration in the workpiece material during internal grinding at a relative vibration value of 100 Hz

Table 3

The change in the probability of material removing along the contact zone at different levels depends on the values of the relative vibrations of the grinding wheel and the workpiece during grinding holes

	z, m	$P(M)$			
		ν, Hz			
		100	200	300	400
$y = 0.3 \cdot t_f$, m	–0.36	0.925	0.816	0.984	0.973
	–0.27	0.977	0.931	0.994	0.992
	–0.18	0.995	0.958	0.998	0.997
	–0.09	0.999	0.987	0.999	0.999
	0	1	0.997	1	1
	0.09	1	0.999	1	1
	0.18	1	1	1	1
	0.27	1	1	1	1
	0.36	1	1	1	1
$y = 0.6 \cdot t_f$, m	–0.27	0.769	0.477	0.623	0.844
	–0.20	0.853	0.548	0.652	0.876
	–0.14	0.914	0.652	0.699	0.899
	–0.07	0.953	0.763	0.769	0.922
	0	0.976	0.855	0.843	0.947
	0.07	0.988	0.917	0.901	0.963
	0.014	0.994	0.954	0.929	0.977
	0.20	0.997	0.974	0.961	0.984
	0.27	0.999	0.985	0.974	0.989
$y = 0.9 \cdot t_f$, m	–0.14	0.576	0.488	0.45	0.465
	–0.11	0.608	0.515	0.463	0.466
	–0.07	0.639	0.546	0.485	0.475
	–0.03	0.669	0.579	0.514	0.497
	0	0.698	0.612	0.548	0.526
	0.03	0.724	0.644	0.581	0.558
	0.07	0.749	0.673	0.613	0.587
	0.11	0.770	0.699	0.64	0.61
	0.14	0.788	0.721	0.661	0.629

The analysis of the data obtained gives a clear illustration of the patterns of material removal along the contact zone at different levels at different frequencies of the relative oscillations of the grinding wheel and the workpiece.

The data obtained show that when passing the surface of the contact zone of the wheel with the workpiece, the probability of metal removing increases within the actual depth of cut, and decreases with an increase in the frequency of relative vibrations of the tool and the workpiece at all levels. The probability increases most intensively at the value when the abrasive grains pass through the main plane. This is explained by the fact that during this period the depth of cut is maximum and the largest number of abrasive grains is involved in cutting. Due to the presence of vibrations, the removal still grows intensively even after the grains have passed the level of the main plane.

Conclusions

The developed mathematical models allow tracing the effect on the removal of the material of the superimposition of single sections on each other during the final grinding of materials. The proposed dependencies show the regularity of the stock removal within the arc of contact of the grinding wheel with the workpiece. The considered features of the change in the probability of material removal when the treated surface comes into contact with an abrasive tool in the presence of vibrations, the proposed analytical dependences are valid for a wide range of grinding modes, wheel characteristics and a number of other technological factors [20–22].

The expressions obtained allow finding the amount of material removal also for the schemes of end, profile, flat and round external and internal grinding, for which it is necessary to know the magnitude of relative vibrations. However, the parameters of the technological system do not remain constant, but change over time, for example, as a result of wear of the grinding wheel. To assess the state of the technological system, experimental studies are carried out taking into account the above changes over the period of durability of the grinding wheel. One of the ways to determine the parameters of a technological system is a full-scale experiment.

Experimental confirmation of the results was carried out on a CNC grinding machine *Knuth RSM 500 CNC* in the Common Use Center "Engineering and industrial design" SevGU when processing elements of the experimental system – a pump developed at Sevastopol State University. The design of this product includes parts (leading rotor) made of *VT3-1* titanium alloy, the quality parameters of which are ensured only during grinding operations.

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Conflicts of Interest

The authors declare no conflict of interest.