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Control of gaps in technical structures during ground vibration testing

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ABSTRACT

Introduction. A fair number of technical structures have gaps (backlashes) which can be contingently divided into two types. One of them is the gaps in connections between substructures which are introduced so that the connections may operate correctly. Sizes of such gaps are usually normalized. Another type is the backlashes which occur during operation. Due to the normalized gaps usually expand while operating, both of the types may lead to increased loading and wear of mechanical parts, an alteration in dynamical characteristics and a deterioration in a technical state of mechanical structures. It explains the necessity to control the gaps. When the ground vibration testing of the structures is performed, it seems appropriate to use these tests to detect such gaps. **Research Objective:** developing the method to control the gaps in the technical structures during the ground vibration testing based on distortions of portraits of forced oscillations. **Research Technique.** The steady-state forced oscillations of the technical structures, which were measured by acceleration sensors, are excited by means of shakers. The sensor signals are represented as the portraits: the vertical scanning is proportional to the signal and the horizontal scanning – to its first harmonic with the phase shift of $\pi/2$. In case of a linear system, the portraits are circles. The presence of the gaps distorts the portraits of oscillations specifically. To estimate the distortions numerically, the first harmonic is subtracted from the Fourier series of the portrait of oscillations, the absolute maximum of the residue is calculated over the oscillation period and used subsequently as the distortion parameter Ψ. The value of the parameter Ψ is normalized and denoted as ξ. The ξ distributions are plotted on controlled objects. The locations of the gaps are determined through the positions of the local maxima of the distortions. While calculating the parameter ξ, the two types of normalization, which were conditionally named the global and local ones, are being used. In case of the global normalization, the value of Ψ is related to the amplitude of the first harmonic at the control point of the structure. The local normalization means that the magnitude of Ψ is related to the amplitude of the first harmonic of the sensor where that parameter was previously calculated. The global normalization is required to analyze the distortion distribution of the portraits of oscillations of the entire technical structure. The local normalization of the distortions of the portraits of oscillations is utilized to establish the locations of the gaps in the mechanical parts and structural connections. The ground vibration tests were carried out via *Test.Lab softw*are. The subprogram is integrated into the software interface in order to analyze the portraits of oscillations. It enabled one to calculate the distortions of the portraits of oscillations, plot the distortion distributions of the structure and save it for further use. It allowed one to control the gaps during vibration strength tests, as well as while the structures being used, by means of comparing the distortion distributions of the parameter ξ related to different states of the structure. Additionally, the plotting of the distortion distributions of the portraits of oscillations for each structural component is added to the subprogram so as to control the defects subsequently. Not only the locations of the gaps are determined in the force-displacement application systems but also the equation is given to calculate its magnitudes. The practical recommendations on using that equation are presented. **Results and Discussion.** The possibility of detecting the gaps by the distortions of the portraits of oscillations is illustrated with the example of the diagnostics of the layout of the control wiring and the airplanes during the ground vibration testing as well as the open-type spacecraft structures. It is shown that the developed method enables one to detect all the gaps in the testing object which distort the portraits of oscillations.

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Introduction

A fair number of technical structures contain gaps (backlashes) which can be contingently divided into two types. The first type includes gaps in connections between substructures which are introduced so that the connections may operate correctly. Sizes of such gaps are usually normalized. Another type is the backlashes, which occur during operation. Due to the normalized gaps usually expand while operating, both of the types may lead to increased loading and wear of mechanical parts, an alteration in dynamical characteristics and a deterioration in a technical state of mechanical structures. It explains the necessity to control the gaps. When ground vibration testing (GVT) of the structures is performed, it seems expedient to use the tests to detect such gaps.

Vibrodiagnostics of machinery is widely used to evaluate a technical condition of mechanical transmissions, couplings and bearings [1–5]. These rotating parts of machines tend to generate oscillations when rotating parts are imbalanced, contain the backlashes or a shaft misalignment has occurred. Oscillations, which are diagnosed as vibrations of chassis, contain information about dynamic processes in the working machine. One has to select a specific piece of data in order to detect structural damage and make decisions on structural health [6–8].

Methods of vibration-based damage detection can be divided into three categories. The first one consists of methods to detect mechanical damage by changes in modal properties [9–15]. It is important to notice that the modal properties do not alter drastically, even though the mechanical structure is damaged considerably. Additionally, the conclusive damage identification is limited due to the fact that modal properties are integral characteristics, and the location and size of the defect are differential ones [16].

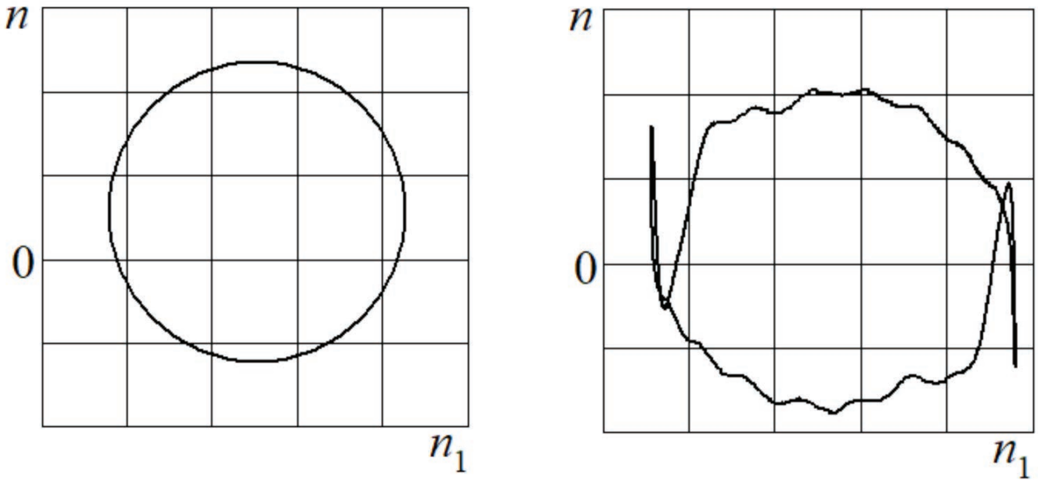
The second category is represented by the methods of the wave-based damage detection [17–21]. However, the application of these methods is considered to be challenging when the structures have discontinuities such as holes and notches.

When design parameters of a mechanical structure are close to ones of a linear dynamical system, portraits of oscillations distort and vibrational subharmonic and superharmonic resonances occur. The methods for the damage identification based on these features may be combined into the third group [22–30].

The paper [29] shows that one can determine and evaluate the gaps in the control wiring of the deflectable surfaces by using nonlinear distortions of portraits of oscillations. In this way, results of the GVT are utilized to calculate the distortions. The current paper is aimed at creating a vibration-based technique to monitor the gaps by means of the distortions of the portraits of oscillations. So, the computer program has been developed and integrated into the vibration testing software to compute and plot the aforementioned distortions. Also, to detect the structural damage sequentially, the novel approach has been proposed. This allows not only to identify gaps, but also to estimate its size.

Research Technique

The identification of the gaps in the mechanical structures by the means of the portraits of oscillations is similar to the vibration-based cracks detection [30]. The steady-state forced oscillations of the technical structures, which have been measured by acceleration sensors, were excited by means of shakers. The sensors are located near moving connections and attachment points of mechanical parts and apparatus. The sensor signals are represented as the portraits: the vertical scanning is proportional to the signal and the horizontal scanning – to its first harmonic with the phase shift of $\pi/2$. In case of a linear system, the portraits are circles. The presence of the gaps distorts the portraits of oscillations specifically (Fig. 1, n – the sensor signal, n_1 – its first harmonic with the phase shift of $\pi/2$). To estimate the distortions numerically, the first harmonic was subtracted from the Fourier series of the portrait of oscillations; the absolute maximum of the residue was calculated over the oscillation period and used subsequently as the distortion parameter Ψ. The value of the parameter Ψ was normalized and denoted as ξ. The ξ distributions were plotted on controlled objects. The locations of the gaps were determined through the positions of the local maxima of the distortions.



a *b*

Fig. 1. The portraits of oscillations of a structure

a – without a gap; *b* – with a gap

The ground vibration tests were carried out via Test.Lab software. The subprogram was integrated into the software interface in order to analyze the portraits of oscillations. It allowed to calculate the distortions of the portraits of oscillations for all the measurement channels simultaneously, plot the distortion distributions of the structure and save it for further use. It enabled monitoring the gaps during vibration strength tests, as well as while the structures being used, by means of comparing the distortion distributions of the parameter ξ related to different states of the structure. Additionally, the plotting of the distortion distributions of the portraits of oscillations for each structural component was added to the subprogram so as to control the defects sequentially.

While calculating the parameter ξ, the two types of normalization, which were conditionally named the global and local ones, were being used. In case of the global normalization, the value of Ψ was related to the amplitude of the first harmonic at the control point of the structure. The point where the amplitude of the first harmonic is maximal among the signals should be used as the control one.

In case of the local normalization:

$$\xi_i = \frac{\max|\Psi_i|}{(A_1)_i}, \quad (1)$$

where i – the measurement channel index; $\max|\Psi_i|$ – the absolute maximum of the distortions of the portraits of oscillations; $(A_1)_i$ – the amplitude of the first harmonic.

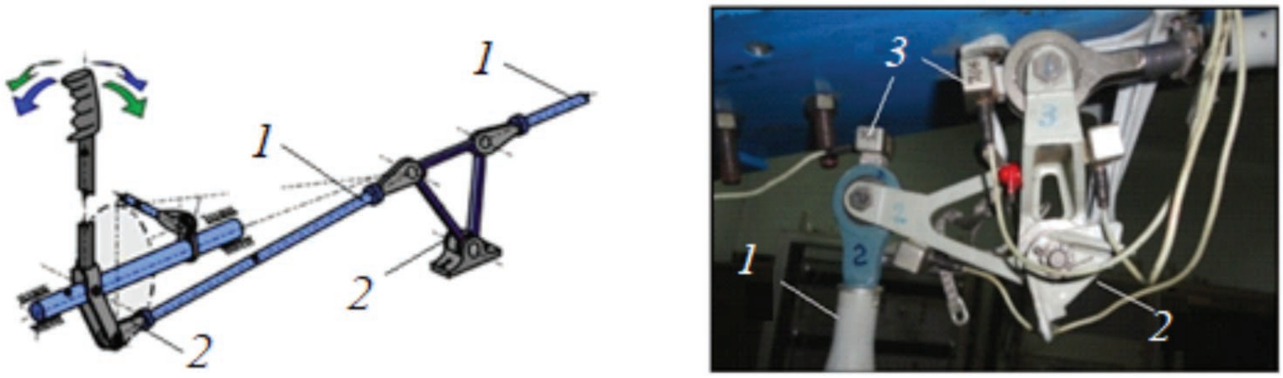
The global normalization is required to analyze the distortion distribution of the portraits of oscillations of the entire technical structure. A frequency-driven force usually acts near a natural frequency of the structure; that is why the false local maxima of the distortions have to be excluded. It occurs because of some acceleration sensors are situated close to mode shape nodes.

The local normalization of the distortions of the portraits of oscillations is utilized to establish the locations of the gaps in the mechanical parts and structural connections. Such normalization facilitates comparison between the different gaps and control of its dynamics while being tested or utilized.

Let us consider how one may make use of the proposed method to detect the gaps in force-displacement application systems. It is possible not only to identify gaps, but also to estimate its values based on experimental results.

The increased gaps in the moving connections of the force-displacement application systems become to exist due to operating for a long period of time and fabrication errors. The gaps in transmissions of helicopters and automobiles lead to self-oscillations and make difficult to control it. Existing methods usually require the partial dismantlement of the structures.

The papers [26, 28] demonstrate how the gaps in the moving connections of the force-displacement application systems might be identified on the basis of the GVT. It was suggested to measure overloads in the moving connections of its substructures and plot it as Lissajous figures. The paper [29] shows that the results acquired by means of the portraits of oscillations and the Lissajous figures appeared to be the same. Figure 2 shows an example of the displacement application system – a control wiring of deflectable surfaces.



a *b*

Fig. 2. A control wiring of deflectable surfaces

a – an exemplary control wiring scheme; *b* – cables and rockers with sensors; *1* – cables; *2* – rockers; *3* – acceleration sensors

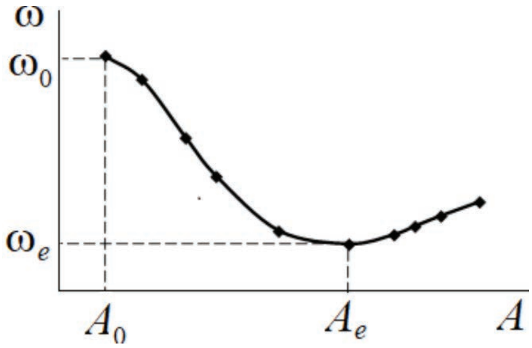


Fig. 3. The natural frequency as a function of the oscillation amplitude

The presence of the gaps in the moving connections of the control wiring results in the increased displacement of the deflectable surface. That is why the rotating frequency of the surface depends on the oscillation amplitude. Figure 3 illustrates the intrinsic natural frequency dependency on the oscillation amplitude of the deflectable surface which is neutrally imbalanced (there is a static force in the system).

As shown in Fig. 3, A – the oscillation amplitude of the control point of the surface; ω – the natural frequency of the deflectable surface (the resonance frequency); A_0 – the oscillation amplitude when the static force in the control wiring is achieved; ω_0 – the natural frequency of the system without the gap; ω_e – the minimal value of the natural frequency.

If the displacement of the deflectable surface exceeds a permissible value due to the increased gap in one of the moving connections, the damaged structural part is located by the ξ parameter and the size of the gap is estimated according to the equation ([28]):

$$\tau\delta = A_0\left[1-\left(\frac{\omega_e}{\omega_0}\right)^2\right]\left\{4,2\left[1-\left(\frac{\omega_e}{\omega_0}\right)^2\right]+1,39\right\}. \qquad (2)$$

Where δ – the size of the gap; τ – the fraction of the displacements of the sensors located near the damaged and the control points.

It is worth to be noticed that if the decrease in the rotating resonance frequency of the deflectable surface due to the gap is less than 12 % and the oscillation amplitude is not higher than 50 % of the A_0 value, the size of the gap (2) is calculated with the precision error no more than 10 %.

The equation (2) should be used specifically. For instance, to determine the ω_e frequency of the deflectable surface at the moment when the gap is disclosing, one has to descend the frequency of the driving force. The oscillation amplitude A_0 is calculated when the distortions of the portraits of oscillations emerge, as a rule, discontinuously. Measuring the A_0 value is complicated because of its small magnitude. It is possible to artificially increase it by changing the balance of the deflectable surface or introducing the static component to the driving force. Moreover, there are filters in the measurement system to eliminate noise.

It needs to be highlighted that usage of the filters makes the signals smoother as well as changing the oscillation amplitude. When using a low-pass filter, nonlinear effects due to the presence of the gap may disappear. So, Figure 4 illustrates the results of applying the second-order low pass Butterworth filer with cutoff frequencies of 50 and 100 Hz to the portrait of oscillations of a system. The first natural frequency did not exceed 10 Hz.

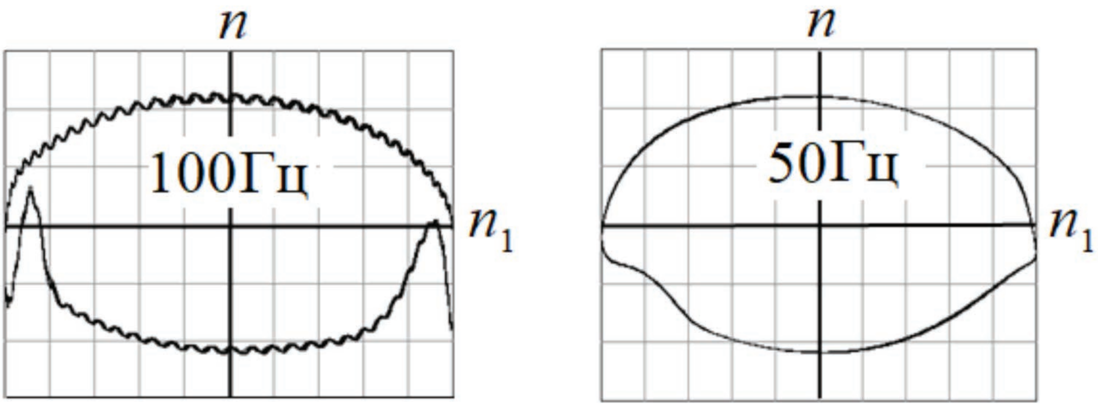


Fig. 4. The portraits of oscillations for the different cutoff frequencies of a filter

Even if there are several gaps in the control wiring, its overall value is evaluated by means of the second equation. The sizes of the gaps located at each damaged structural part are calculated according to the ξ parameter.

Results and Discussions

The method for detecting gaps based on the distortions of the portraits of oscillations was utilized to monitor the layout of the control wiring and the aeronautical vehicles in the course of the GVT.

The study [28] contains the results of detecting backlashes in the layout of the control wiring (Fig. 5). The example of identifying the backlash in the node 6 by means of the distortion parameter of the Lissajous figures is summarized in Table 1. Table 2 summarizes the numerical evaluations of the backlashes calculated by using the equation (2).

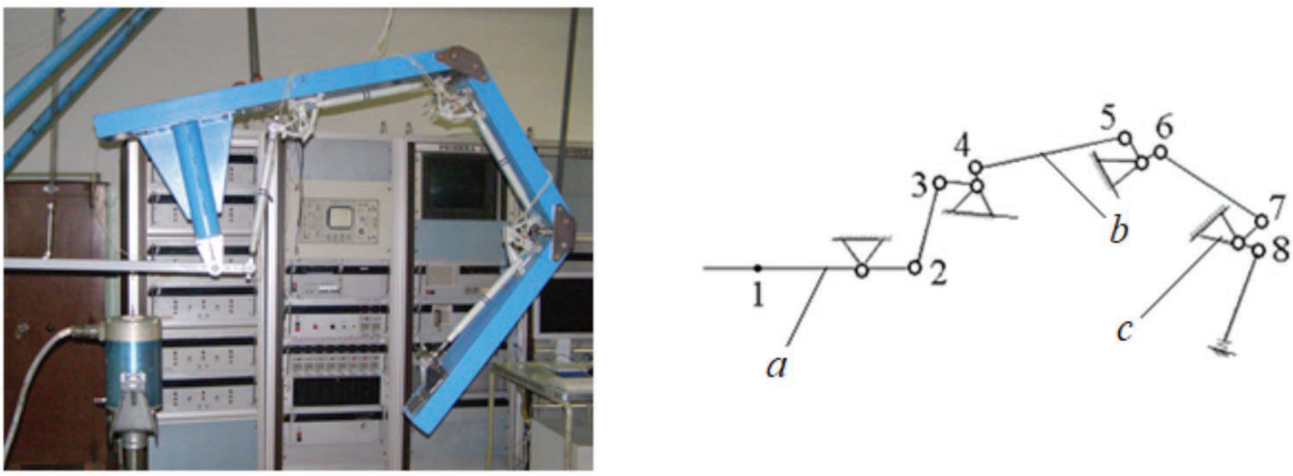


Fig. 5. A layout of a control wiring and its scheme:
a – a deflectable surface; *b* – a cable; *c* – a rocker

Table 1

Detecting the backlash

No sensor	1	2	3	4	5	**6**	7	8
ξ	10.42	8.82	5.99	26.28	9.83	**101.62**	59.67	43.64

Table 2

Calculating the sizes of the backlashes

No node	3	3	3	4	4	6	6	3, 6
True backlash, μm	50.0	43.0	20.0	50.0	35.0	50.0	35.0	30+50
Calculated backlash, μm	46.5	49.0	22.0	52.6	38.3	47.3	35.3	74.0

Figures 6–11 illustrate the distortion distributions of the portraits of oscillations which were computed throughout the GVT of several airplanes. In the figures the upper limit of the color scale corresponds to the fields contained the maxima of the distortions and the lower end – to the parts where the magnitudes are the smallest.

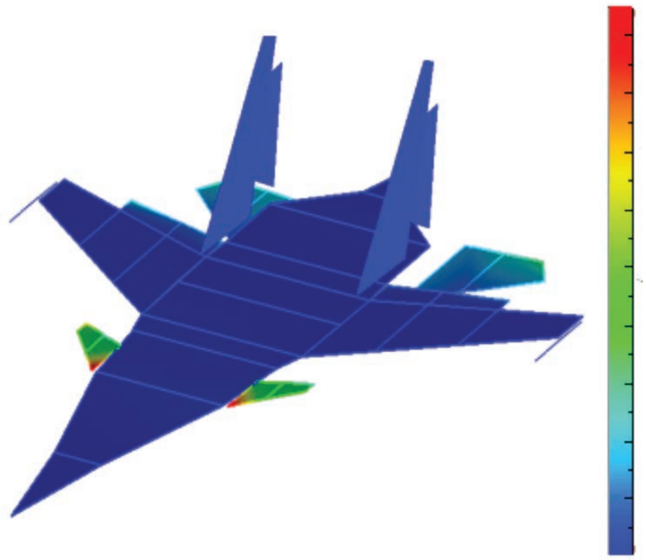


Fig. 6. Gaps in attachment points of a forewing. The global normalization of the distortions at the rotating frequency of the forewing.

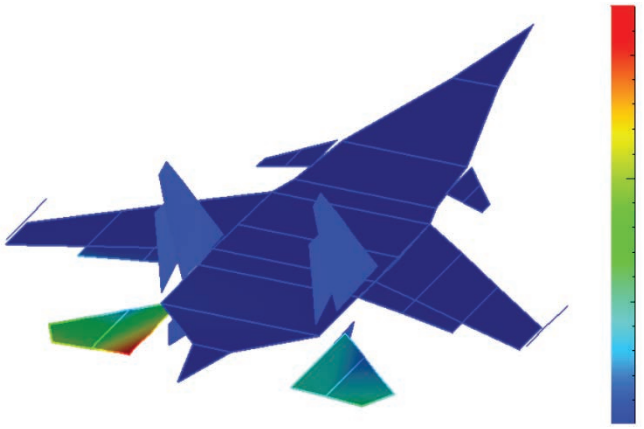


Fig. 7. The global normalization of the distortions at the rotating frequency of the stabilator

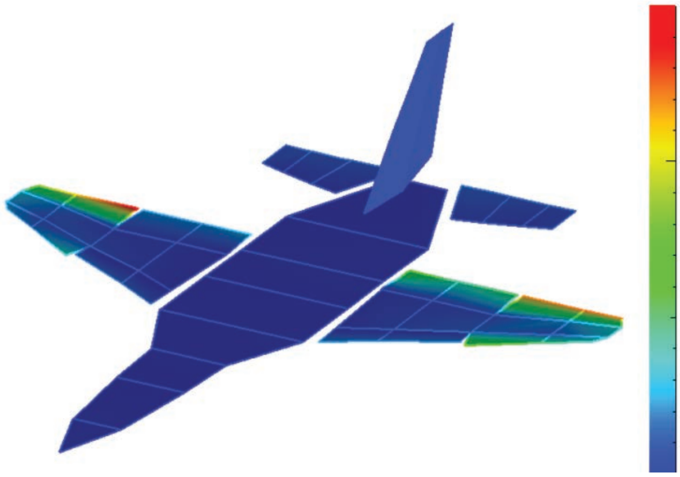


Fig. 8. Gaps in a control wiring of high-lift devices of an airplane. The global normalization of the distortions at the bending frequency of the wing.

The distortions of the portraits of oscillations distributed along the airplane, which control system is unboosted, are shown in Figure 9. The maxima of the distortions were discovered on the elevator and elevator tab because of the gaps in the control wiring. After exclusion of these distortions from the distribution, the maximum value was found at the connection between the control stick and control wiring, where the increased gap was detected (Fig. 10).

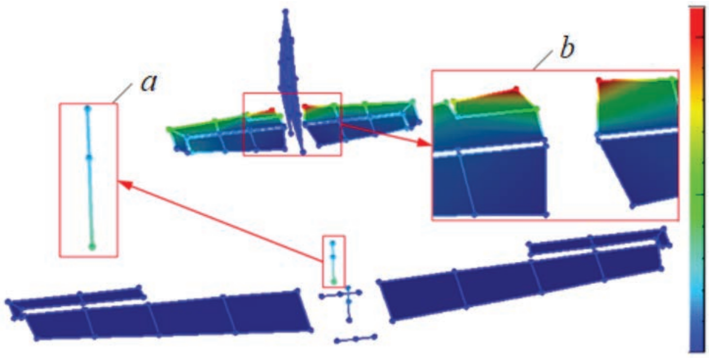


Fig. 9. Gaps in a control wiring of an elevator and elevator tab:

a – sensors located on the control stick; *b* – the distortions distributed along the elevator and elevator tab

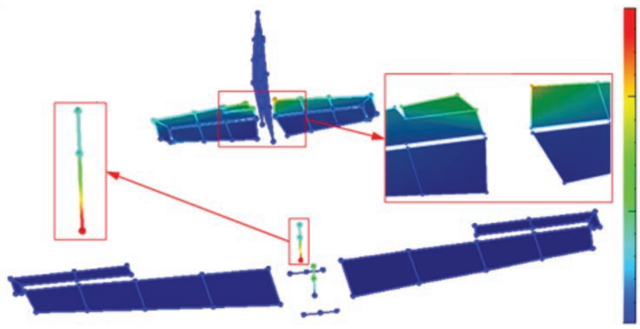


Fig. 10. The backlash in the connection between the control stick and control wiring. The global normalization of the distortions of the portraits of oscillations

Figure 11 shows the distortion distribution of the portraits of oscillations of a vertical stabilizer. The increased transverse gaps were identified in the front attachment points of the vertical stabilizer.

Spacecraft while designed are tested by means of the GVT. Results of the experiments are vital to prove quality of a fabricated vehicle and measure its vibration strength, including detecting manufacturing damage. Since the maximum vibrational loads occur during launching, the spacecraft are tested in a launch configuration. Figure 12 illustrates a design scheme of an unsealed satellite. Flat sandwich panels along with a carbon composite cylinder form a structural framework. Satellite equipment (antennas, solar batteries and etc.) as well as an astroplate with navigation sensors and orientation systems are located on the panels. To perform the GVT, the satellite is set on the adapter which connects the spacecraft with the launch vehicle.

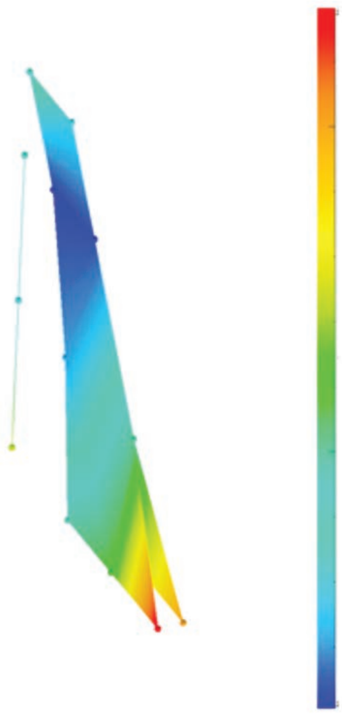


Fig. 11. Gaps in the front attachment points of a vertical stabilizer. The local normalization of the distortions

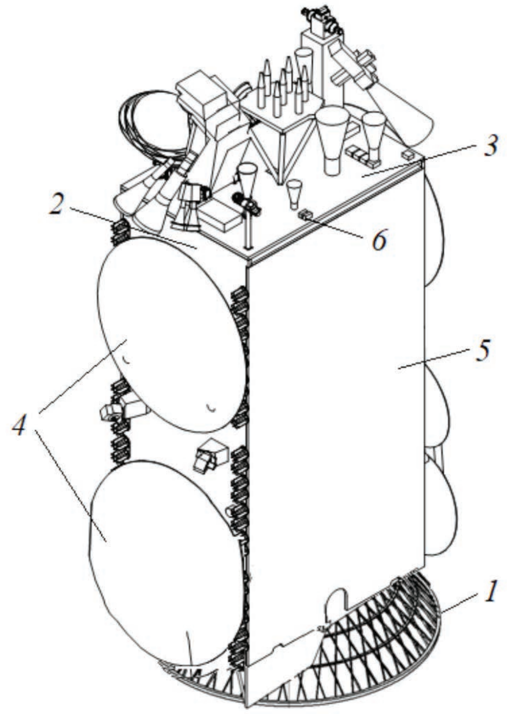


Fig. 12. A spacecraft structure design scheme:
1 – an adapter; *2* – a panel; *3* – an astroplate; *4* – antenna reflectors; *5* – panels of solar batteries; *6* – an attachment point of a solar battery

The several steps of the spacecraft GVT are carried out [29]. Firstly, the spacecraft is exposed to loading with low intensity to find out whether its dynamical properties are close to design ones or not. After that, normalized vibrational loads are applied to the spacecraft. During the loading, damage might occur and propagate, for example, interconnections become broken because of the gaps. The third step repeats the program of the first one. Taking into account how the vibrational parameters have changed: the resonance frequency and oscillation amplitude; high-frequency components occurrence and spectrum shift, one detects, locates and identifies the defects.

The unsealed spacecraft while the GVT are exposed to wide-band harmonic and random acoustic vibrations.

Figure 13 and Figure 15 illustrate how the gaps in the two spacecraft were detected by using the distortion distributions. It is essential to highlight that in the course of testing the driving vibration was sinusoidal and the frequency was altering logarithmically. Since the process is considered to be nonstationary, the time segments close to resonance frequencies were gathered to calculate the global distortions of the portraits of oscillations. The maximum distortion distributions were chosen among the calculated ones.

Figure 13 illustrates how the distortions of the portraits of oscillations distribute on the spacecraft surface. It is the only one case within the frequency range of 20 to 100 Hz when the distortions of the portraits of oscillations were higher than calculation errors. The maximum distortions occurred near the attachment points of the solar batteries which contain the design gaps.

Figures 14 and 15 illustrate the setup to perform the GVT of the different satellite and the distortion distributions of the portraits of oscillations of the antenna reflector. The points in Figure 15 represent the locations of the acceleration sensors on the surface of the reflector. The arrow depicts the damage location (gap): the destroyed adhesive connection between the support and frame. That location is characterized by the maximum distortions of the portraits of oscillations.

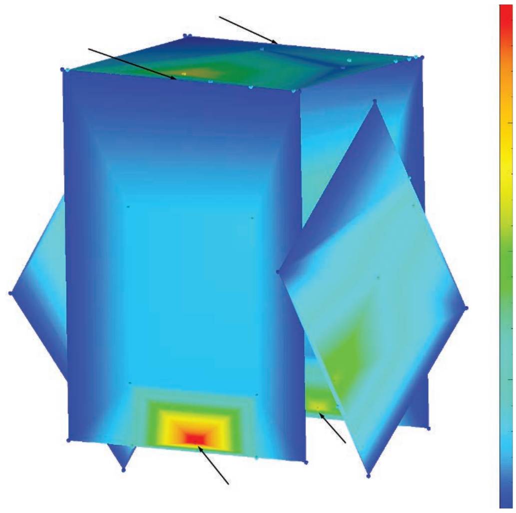


Fig. 13. Detecting gaps in attachment points of solar batteries

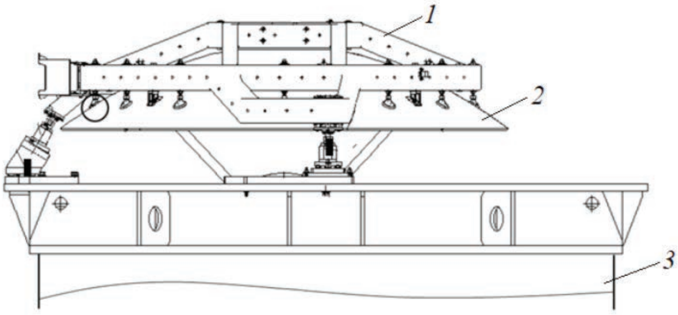


Fig. 14. A setup for testing an antenna reflector of a spacecraft structure

1 – a frame; *2* – a reflector; *3* – a shaker table

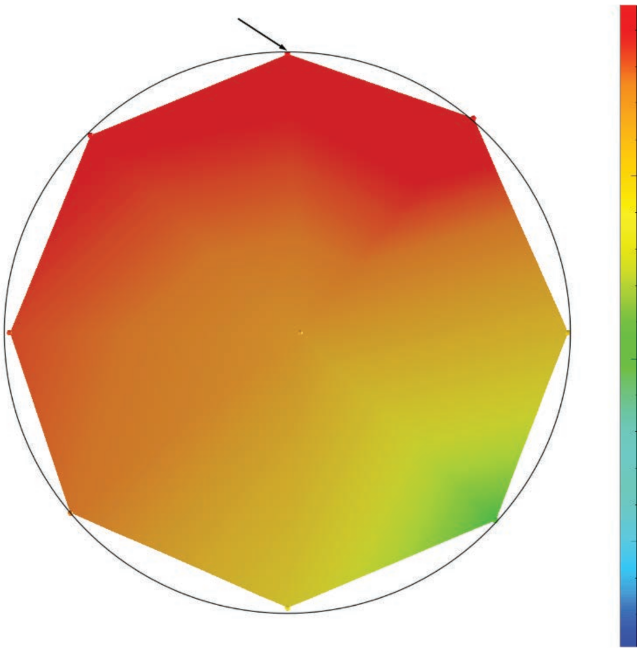


Fig. 15. The distortion distribution of the portraits of oscillations

Conclusions

The conducted research indicates that the nonlinear distortions of the portraits of oscillations may be utilized to detect and localize the gaps in a technical structure. In order to monitor the gaps during the GVT, the subprogram was integrated into the vibration testing software. The subprogram enables one to compute, normalize and plot the portraits of oscillations. In that way, the occurrence and propagation of the gaps may be detected in the course of operating and the GVT of the mechanical structure.

The proposed technique might be used to monitor a technical state and operational conditions of technical products if last ones are exposed to one-component harmonic vibrations. The sources of the vibrations are usually unbalanced rotating masses.

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Conflicts of Interest

The authors declare no conflict of interest.