Two-Parametric / Game-Theory Model of a Service Concession in the Communal Heating Sector[1]

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Abstract

In this study, we propose to model the operation of a service concession arrangement in the economic area of municipal heat supply utilities. We offer a scheme of interaction between the concedent and concessionaire in this concessionary arrangement. Currently, the existing regulations regarding the temperature of coolant focusses on the daily average outdoor temperature, and the determination of a "normative" demand for heat energy. On any day of the heating period, this demand is a random variable, whose distribution can be described through the distribution of daily average air temperature.

In our model, heat energy is paid for at a fixed price, and the concessionaire pays a penalty for each unit of unsatisfied normative demand. The price and penalty values are the concession parameters, and are determined by the concedent. The concedent's goal is to minimise the thermal energy cost; the concessionaire's purpose is to maximise profit. The interaction is formalised as a two-move game model. First, the concedent determines the price and the value of the penalty. Then the concessionaire selects the capacity to be created. The concession's parameters should be set so that the individual rationality and incentive compatibility conditions are met.

Our results prove the existence of Stackelberg equilibrium, and we derive the relevant formulas for computing its parameters. In equilibrium, the optimum capacity for the concessionaire provides a sufficient probability of meeting demand. The price of thermal energy is minimal under this condition. We also formulate a one-parameter model (thermal energy price as a parameter), which is based on a typical concession scheme. In the two-parameter model, the equilibrium capacity and price do not exceed the corresponding parameters of the one-parameter model. The main advantage of the two-parameter model is an "embedded" economic mechanism that prevents the concessionaire's opportunistic behaviour. By contrast, in the one-parameter model there is no such mechanism.

The proposed approach can be applied to a concession for the production of any good or service, provided the concerned parties are interested in the availability and reliability of meeting a corresponding need, which may be described as a random variable. However, typical concession schemes do not penalise unsatisfied demand, so the implementation of our two-parametric model is possible only after modification of the pertinent concession legislation.

Keywords: concession, heating, game-theoretic model, economic mechanism, Stackelberg equilibrium, opportunistic behaviour

JEL classification: C62, G38, L32, L38, Q41

Introduction

In public-private partnership (PPP) agreements the choice of the regulation type (contractual and (or) administrative) exerts a strong influence on the probability of the project implementation and its efficiency [1; 2]. This is particularly so in the case of government service concessions, that is, concessionary deals in regulated infrastructure sectors. Where the regulation of heat supply is concerned, such projects may combine the tariff (or restriction of price), requirements to capacity and process characteristics of the concession subject, and the investment returns norm, etc. The experience of projects focusing on modernising the heat supply system in the Novosibirsk Region (implemented in accordance with concession agreements) shows that such service concession arrangements can be effective in small municipal entities. The project participants noted specific risks arising when preparing and implementing such projects. Significant risks are related to inconformity in terms of a contrast of budgetary, tax-related, and concession-related regulatory documents, with the practice of tariff regulation - which does not prompt power-generating companies to reduce expenses and increase energy efficiency [3, p. 54]. To a great extent a project's success is predetermined at the stage of contract preparation when the legal and financial obligations of the parties are defined.

In this paper, we will consider a building, operation, and transfer (BOT) concession for the development and operation of heat producing capacity. When entering into such a concession, one of the main risks is related to the probability of the concessionaire's opportunistic behaviour. The concessionaire may (in order to reduce capital costs) develop a capacity insufficient for a consistent heat supply. Also, if the capacity is sufficient, the concessionaire may (in order to reduce current costs) decrease delivery of heat energy. We therefore seek to propose an economic mechanism which makes opportunistic behaviour disadvantageous for the concessionaire.

Our proposed model features the following assumptions and special characteristics:

1) In order to provide for a high quality heat supply, the concedent establishes the norms of the coolant's temperature, depending on the daily average outdoor temperature. In this way the "normative demand" for heat energy is created. This demand is a discrete random variable, the distribution for which is easy to define, knowing the distribution of average daily temperature of outdoor air.
2) The concedent can calculate the minimal power z_1, which ensures meeting the norm demand at an admissible probability. In order to incite creation of sufficient power and its use to satisfy the demand – that is the incentive compatibility condition – the concedent determines the price of heat energy p and penalty q for each unit of unsatisfied demand.
3) Customers pay for heat energy. Serving their interests, the concedent strives to minimise the price of heat energy. However, this price should provide to the concessionaire a non-negative operating profit (taking into consideration heat losses in the supply system) and, for the whole term of the concession, a non-negative cumulative net present value (NPV) – the individual rationality condition.
4) We formalise the interaction of the parties as a two-move game. The concedent (leader according to Stackelberg) makes the first move selecting the values of parameters p and q. Knowing these parameters, the concessionaire makes the second move choosing the target capacity z in a way that maximises the revenue.
5) Our proposed model provides for the opportunity of the concessionaire's participation in financing of modernisation of the heat-transmitting system, related to the concession subject (hereinafter referred to as the "transmitting system project", TSP). For this reason, unlike article [13], we do not postulate the non-negative characteristic of q. If the concessionaire's expenses for modernisation of the heat-transmitting system are significant, the parameter of q in equilibrium is negative. Additionally, as other characteristic features of equilibrium it helps to calculate the concedent's payments which ensure the concessionaire's guaranteed minimum income.

The novelty and main advantage of the offered model consists in the fact that an economic mechanism which makes the concessionaire's opportunistic behaviour unprofitable for it is "embedded" in the model (there is no such mechanism in the typical model of concession). With the recommended choice of p and q (price and penalty) parameters, the concessionaire will earn the maximum profit if it develops exactly the amount of power preferable for the concedent, and uses it to satisfy the normative demand to the maximum extent. The distinctive feature of this model is the fact that the concessionaire does not maximise some "governmental" benefit, but rather minimises the heat energy price. As far as we know, such concession models have not been described in the previous literature.

The article is presented as follows. Section 2 comprises our literature review. In Section 3 we set a problem and substantiate the main assumptions. In Section 4, we formalise the problem: we describe the concession game-theoretic model (a two-parameter model). In Section 5 we prove the existence and define the parameters of Stackelberg equilibrium. In Section 6, we develop and analyse the simplified model, close to the generally accepted concession schemes (one-parameter model). Section 7 compares the abovementioned models, and outlines the advantages of the two-parameter model. Finally, the article concludes with the presentation of our results and conclusions.

Literature Review

Game theory, and the economic mechanisms theory, provide an adequate language for modelling economic entities' interaction while making and fulfilling concession agreements. In particular, paper [4] is dedicated to the opportunities of applying game theory to public-private partnership problems. Papers [5; 6] describe the game-theoretic model of choice between delivery of a service by a state enterprise (regulated monopoly), concession and procurements in the free market. Papers [7; 8] offer models which substantiate the choice of the winner in a tender for participation in PPP and a reasonable reimbursement to those who fail to win the contract. Publications have also analysed the following problem: when a concessionaire suffers financial difficulties (for example, there is a threat of bankruptcy) under which conditions it is reasonable (from the concedent's point of view) to provide additional financing of the project, and under conditions it is reasonable to renew the agreement (see [5; 7]). In paper [9] it is observed that in construction, opportunistic bidding often causes financial difficulties for the contractor. In such cases, when striving to win the contract, the contractor undervalues the estimated cost, expecting that in future the customer will be reimbursed for the losses (because it needs to complete the project or on the basis of a legal action). A standard BOT concession contract has little protection from such opportunistic behaviour.

Paper [10] describes the model of the closed first-price auction for a public contract. A tenderer (agent) is interested in an increase of the contract value, while the customer (principal) strives to minimise it. An economic mechanism which reconciles the interests of the customer and the executor was created. Under these parameters, it is unprofitable for the agent to undervalue the announced expenses (in comparison with the genuine ones) for the sake of winning the tender, as in this case the victory will result in losses. Taking into consideration some additional assumptions, an equation was derived which is satisfied by the value of the key parameter of the optimal contract, which minimises the expected expenses of the customer. The customer (as the leader according to Stackelberg) fixes this value of this parameter, thus "switching on" the sought economic mechanism.

Papers [11; 12] are dedicated to concessions for natural resource utilisation with the assumption that both concedent and concessionaire strive to maximise NPV. Paper [11] considers the possibility of the concessionaire's withdrawal from a BOT concession as a real option, which influences the duration of the contract term. The parties' interests are reconciled as follows: in each single period the concedent (leader according to Stackelberg) chooses the values of the concession payment, and the concessionaire chooses the term of the concession. A corresponding Stackelberg equilibrium is derived. Paper [12] considers manufacturing projects from a certain selection defined by the government as the concession subject. The government also makes public the list of infrastructure and environmental projects, which support manufacturing projects. It is anticipated that infrastructure projects are implemented at the expense of the budget and environmental ones – through PPP. On the basis of this information an investor chooses the projects to participate in. The search for Stackelberg equilibrium is formalised as a problem of double level mixed integer linear programming. An approximate solution algorithm was offered.

The model of concession for the manufacturing of a public good is offered in article [13]. The authors presume that the good is free of charge for the end-use customers, and the demand is described as a random variable with the differentiable distribution function. The concedent pays for the satisfied demand at the price of p, and the concessionaire pays the penalty q for each unit of unsatisfied demand. The concessionaire maximises the expected NPV during the concession term. The concedent chooses p and q in such a way that the optimal capacity for the concessionaire (which manufactures the public good) provides a sufficient expected level of demand satisfaction. With this provision, it minimises the expected discounted costs. The existence of Stackelberg equilibrium is therefore proven, and its parameters are defined.

Statement of the Problem

Let us assume that the concession subject is heat generating capacity in the municipal heat supply system. It may be a new facility or an operating enterprise. In the first case, design and construction are usually assigned to the concessionaire. In the second case, the terms of the concession may contemplate modernisation (e.g. reconstruction, expansion) of the enterprise. In any event, the concedent is a competent authority (central or municipal).

The concedent is interested in the development and support by the concessionaire of a heat generating capacity which provides a consistent heat supply to consumers in accordance with norms and at a minimum price. The normative demand for heat energy on each day of the heating season depends on the average daily temperature on such day and, consequently, is a random variable, which distribution may be derived out of meteorological statistics and rules of calculation of the normative demand for heat energy.

In the pre-plan period, the concessionaire, at its own expense (or with partial budgetary financing) designs and expands (e.g. modernises, reconstructs) the enterprise capacity up to the chosen level within the planned period (term of the concession), and bears corresponding operational and productive costs. The concessionaire's costs should be reimbursed (in case of normal revenue) by the consumers' payments, the concedent may subsidise these payments (for all or some consumers).

The concession agreement may provide for modernisation (at the concessionaire's expense or with partial budgetary financing) of the heat-transmitting system related to the concession subject. In accordance with the Federal Law of 17.08.1995 No. 147-FZ Concerning Natural Monopolies (art. 4), the market of heat energy transmitting services belongs to the natural monopoly sphere. The actual generation of heat energy, on the other hand, is not a monopoly type of activity either legislatively, or *ad rem*. Managing a heat generating enterprise and a heat supply system, the concessionaire would have achieved significant power in the market. The possible consequence, however, is an increase in the cost of heat energy for the consumers, and this contradicts the concedent's purpose. In light of the above, we presume that the heat supply system is not the concession subject.

The higher the heat energy losses during transmission, the greater should be the generating power able to satisfy consistently the normative demand. As this trend develops, capital costs increase correspondingly. The necessity to reimburse for the heat energy losses increases current costs as well, and as a result, the cost of heat energy for consumers rises, which contradicts the concedent's purpose. If the metering station is located on the boundary of balance sheet attribution of the heat supply system, the consumers pay only for the delivered heat energy. In such a situation, the generator's current costs will be proportionate to the generated energy, and therefore implementation of the TSP is beneficial for the concessionaire as well. The concedent, in acting in the interests of consumers, may initiate the development of the TSP. In doing so, the concedent may therefore determine a transfer to the concession of the generating capacity, where the concessionaire participates in the financing of the TSP.

In order to ensure alignment of interests of the concession parties, we offer to introduce a penalty which the concessionaire has to pay to the concedent for each unit of unsatisfied demand for heat energy. That is to say, we presume that the concessionaire gets from the consumers the payment $p \geq 0$ for each unit of satisfied demand, and pays to the concedent the penalty q (probably, negative) for each unit of unsatisfied demand (payments are made at the end of the year). With this provision, the concessionaire chooses the capacity z in order to maximise the expected NPV within the planned period.

The concedent chooses the value of the concession parameters p and q. The concedent's purpose at the minimal price for heat energy p is to ensure sufficient consistency of satisfaction of daily demand for heat energy. First, the generated capacity should be sufficient to make the probability of satisfaction of demand on each day of the planned period not less than the specified value. Second, at any average daily temperature within the considered range, the concessionaire's operating profit should be positive in order to eliminate the latter's motivation to stop heat generation (the lower the temperature, the larger the losses in heat energy transmission, and so generation may become unprofitable).

Below we will formalise the described situation as a dynamic game with perfect information, prove the existence of equilibrium in this game, and state the method of its calculation.

Formalisation of the Problem

Demand for Heat Energy: Satisfied and Unsatisfied

For each local heat supply system, a heat supplier develops a 'Schedule of Qualitative Regulation' of water temperature in the heating system[1]. This document indicates specified values of the temperature of the supplied and returned water in the heating system at the boundary of balance sheet attribution of the heat supplier and consumers, depending on the average daily outdoor temperature. With knowledge of the temperature of the cold water added to the system and the weight of the heat carrier medium, one can[2] calculate the normative daily demand for heat energy at an average daily outdoor temperature t. Inasmuch as the average daily temperatures are random variables, the daily demand for heat energy each day should be considered a discrete random variable with known distribution.

Designation and Definitions

τ_i – average daily temperature at the day i of the heating season, a random variable.

$t_1 > \ldots > t_n$ – values of the average daily temperature t for which the 'Schedule of Qualitative Regulation' of the considered heat supply system defines the normative temperature of the heat carrier medium.

d_j – normative daily demand for heat energy corresponding to the temperature of t_j, $d_1 < d_2 < \ldots < d_n$.

η_i – normative demand for heat energy on day i of the heating season, a discrete random variable.

$t(\tau)$ – the value t_j nearest to τ on the right, if $\tau \leq t_1$, otherwise $t(\tau) = t_1$.

We introduce a discrete random variable $\xi_i = t(\tau_i)$ and assume that $a_{ij} = \mathrm{P}(\xi_i = t_j)$.

$\lambda(\tau) \in (0, 1)$ – the share of heat energy losses in the system (up to the boundary of balance sheet attribution of the heat supplier and consumers) at the ambient temperature τ. We suppose that $\lambda_j = \lambda(t_j)$ and replace the function $\lambda(\tau)$ with the step approximation: $\lambda(\tau) = \lambda_j$, if $t(\tau) = t_j$.

c – cost of production of one gigacalorie of heat energy.

z_0 and z (Gcal per day) – enterprise capacity before and after reconstruction.

$x_i(z)$ – quantity of heat energy generated per one day i of the heating season at the capacity of z.

$D_k = d_k / (1 - \lambda_k)$ – minimal capacity which at the average daily temperature τ, ensures the normative demand for heat energy d_k provided $t(\tau) = t_k$.

[1] Rules and Standards of Operation of Housing Resources MDK 2-03.2003: approved by order of the State Committee for Construction of the Russian Federation of 27.09.2003 No.170). M.: State Unitary Enterprise Centre of Construction Design Products, 2004.

[2] Method of defining the quantity of heat energy and heat carrier medium in water systems of municipal heat supply MDS 41-4.2000: approved by order of the State Committee for Construction of the Russian Federation of 06.05.2000 No. 105. M.: State Unitary Enterprise Centre of Construction Design Products, 2000.

Previous definitions have taken into account the possibility that during the heating season the average daily temperature will go beyond the range of $[t_n, t_1]$: if $\tau < t_n$, $\xi_i = t_n$ and $\eta_i = d_n$; if $\tau > t_1$, $\xi_i = t_1$ and $\eta_i = d_1$. It is clear that $\xi_i \in \{t_1, \ldots, t_n\}$ and $\eta_i \in \{d_1, \ldots, d_n\}$, and also $P(\eta_i = d_j) = P(\xi_i = t_j) = a_{ij}$.

When the ambient temperature decreases the normative demand and heat losses increase, therefore $\lambda_k < \lambda_{k+1}$ and $D_k < D_{k+1}$ for all $k < n$. With respect to z and z_0 we assume the following natural assumptions: $z \geq z_0 \geq 0$ and $z \in [D_1, D_n]$.

The concessionaire chooses the value of $x_i(z)$ on each day i of the heating season, maximising the current profit. It is clear that $0 \leq x_i(z) \leq \min\{z, D_j\}$, if $t(\tau_i) = t_j$. It is desirable that the concessionaire uses to the maximum extent the existing capacity to satisfy the demand and this is equivalent to the choice of $x_i(z) = \min\{z, D_j\}$. We will prove that the concedent can incite such choice with admissible values of parameters p and q.

Theorem 1. Let us assume that $t(\tau_i) = t_j$ at the heat producing capacity z. in this case the concessionaire will choose $x_i(z) = \min\{z, D_j\} > 0$, if $(p + q)(1 - \lambda_j) \geq c$, and otherwise $x_i(z) = 0$.

Proof. Under the hypotheses of the theorem $\xi_i = t_j$, $\lambda(\tau_i) = \lambda_j$ and $\eta_i = d_j$. The concessionaire's profit at the day i at the production value of x_i will be written as: $\pi(x_i) = p\min\{x_i(1 - \lambda_j), d_j\} - q\max\{0, d_j - x_i(1 - \lambda_j)\} - cx_i$. If $x_i \in [D_j, z]$ then $x_i(1 - \lambda_j) \geq d_j$, from which $\pi(x_i) = p(1 - \lambda_j)D_j - cx_i$ and $\max\{\pi(x_i) \mid x_i \in [D_j, z]\} = [p(1 - \lambda_j) - c]D_j = \pi(D_j) = \pi(\min\{z, D_j\})$. If $x_i \in [0, D_j]$ then $\pi(x_i) = [(p + q)(1 - \lambda_j) - c]x_i - qd_j$. If $(p + q)(1 - \lambda_j) < c$ then $\max\{\pi(x) \mid x \in [0, D_j], x \leq z\} = \pi(0) = -qd_j$; if in this case $z \geq D_j$, then $\max\{\pi(x_i) \mid x_i \in [D_j, z]\} = [p(1 - \lambda_j) - c]D_j < -q(1 - \lambda_j)D_j = -qd_j$; consequently, $x_i(z) = 0$. If however $(p + q)(1 - \lambda_j) \geq c$, then $\max\{\pi(x) \mid x \in [0, D_j], x \leq z\} = \pi(\min\{z, D_j\})$. (1)

The hypothesis $(p + q)(1 - \lambda_j) \geq c$ means that at the average daily temperature t_j the heat energy unit generated "for expected demand" will give the consumer $1 - \lambda_j$ of energy and the concessionaire will get payment for delivery and will "save" on the penalty for short-delivery of this quantity; wherein the concessionaire's total benefit should be less than the cost of production. Inasmuch as values of λ_j do not diminish as j it is necessary and sufficient to ensure the following inequation:

$$(p + q)(1 - \lambda_n) \geq c. \qquad (1)$$

If this hypothesis is not met, the concessionaire is incited to decrease the heat energy at low temperatures.

Designations. $\mu_i(z)$ and $\nu_i(z)$ – satisfied and, respectively, unsatisfied normative demand for heat energy on the day i of the heating season at the capacity of z.

The value of $x_i(z)$ depends on the average daily temperature; the values of $\mu_i(z)$ and $\nu_i(z)$ depend on $x_i(z)$ and normative demand η_i. Consequently, $x_i(z)$, $\mu_i(z)$ and $\nu_i(z)$ are random variables. The following lemma indicates their expected values.

Lemma 1. If the hypothesis (1) is met and $z \in [D_k, D_{k+1})$ then

$$\mathrm{E}x_i(z) = \sum_{j=1}^{k} D_j a_{ij} + z \sum_{j=k+1}^{n} a_{ij}, \quad \mathrm{E}\mu_i(z) =$$

$$= \sum_{j=1}^{k} d_j a_{ij} + z \sum_{j=k+1}^{n} (1 - \lambda_j) a_{ij},$$

$$\mathrm{E}\nu_i(z) = \sum_{j=k+1}^{n} \left[d_j - z(1 - \lambda_j) \right] a_{ij}$$

Proof. Granted that $\xi_i = t_j$. Then $\lambda_j = \lambda(\xi_i)$, $d_j = D_j(1 - \lambda_j) = \eta_i$, theorem 1 implies that $x_i(z) = \min\{z, D_j\} = \min\{z, \eta_i / [1 - \lambda(\xi_i)]\}$. Hence $\mu_i(z) = \min\{d_j, x_i(z)(1 - \lambda_j)\} = \min\{d_j, z(1 - \lambda_j)\}$ and $\nu_i(z) = \max\{0, d_j - \mu_i(z)\} = \max\{0, d_j - z(1 - \lambda_j)\}$. If $\eta_i = d_j \leq z(1 - \lambda_j)$ then $D_j \leq z$, $x_i(z) = D_j$, $\mu_i(z) = d_j$ and $\nu_i(z) = 0$. However, if $\eta_i = d_j > z(1 - \lambda_j)$ then $D_j > z$, $x_i(z) = z$, $\mu_i(z) = z(1 - \lambda_j)$ and $\nu_i(z) = d_j - z(1 - \lambda_j)$. The lemma statement follows from the fact that $P(\xi_i = t_j) = a_{ij}$ and while $z \in [D_k, D_{k+1})$ the inequation $D_j \leq z$ is equivalent to $j \leq k$.

Lemma 2. In the interval $[D_1, D_n]$ functions $\mathrm{E}x_i(z)$, $\mathrm{E}\mu_i(z)$ and $\mathrm{E}\nu_i(z)$ are continuous, whereby $\mathrm{E}\mu_i(z)$ and $\mathrm{E}x_i(z)$ do not decrease, and $\mathrm{E}\nu_i(z)$ does not increase.

Proof. Continuity of functions $\mathrm{E}x_i(z)$, $\mathrm{E}\mu_i(z)$ and $\mathrm{E}\nu_i(z)$ follows easily from the formulas of lemma 1. Therein $\mathrm{E}\mu_i(x)$ and $\mathrm{E}x_i(z)$ do not decrease, and $\mathrm{E}\nu_i(x)$ does not increase at each interval of $[D_k, D_{k+1})$ and, consequently, at $[D_1, D_n]$.

Concessionaire's Profit

The concessionaire gets payment p from the consumers for each unit of satisfied demand and pays to the concedent the penalty q (probably, negative) for each unit of unsatisfied demand. Payments are made at the end of the year.

Designation and Definition

$u = (p, q)$ – vector of parameters defined by the concedent.

$\pi_i(u, z) = p\mathrm{E}\mu_i(z) - c\mathrm{E}x_i(z) - q\mathrm{E}\nu_i(z)$ – expected operating profit of the concessionaire for the day i at the capacity of z (without operational costs, they will be accounted for in the annual profit).

Let us assume that $z \in [D_k, D_{k+1}]$. It follows from lemma 2 that formulas of lemma 1 are valid for $z = D_{k+1}$. Using lemma 1, we arrive at

$$\pi_i(z) = \sum_{j=1}^{k} \left[p(1 - \lambda_j) - c \right] D_j a_{ij} +$$

$$+ \sum_{j=k+1}^{n} \left[z\left((p + q)(1 - \lambda_j) - c\right) - qD_j(1 - \lambda_j) a_{ij} \right]. \qquad (2)$$

Our immediate goal is to prove the concavity of function $\pi_i(u, z)$ in z.

Designation. For $r \in \{1, \ldots, n\}$ suppose

$$L(u,r,z)=\sum_{j=1}^{r}\left[p\left(1-\lambda_j\right)-c\right)]D_j <_{ij} +$$

$$+\sum_{j=r+1}^{n}\left[z((p+q)\left(1-\lambda_j\right)-c\right)-qD_j\left(1-\lambda_j\right)a_{ij}].$$

Theorem 2. Function $\pi_i(u, z)$ is continuous in z in the interval of $[D_1, D_n]$ and concave in z within this interval if hypothesis (1) is met.

Proof. Continuity of $\pi_i(u, z)$ follows from lemma 2. Let us assume that hypothesis (1) is met and $z \in [D_k, D_{k+1}]$. Then $\pi_i(u, z) = L_i(u, k, z)$. If $r < k$ then

$$L(u,r,z)-L(u,k,z)=-\sum_{j=r+1}^{k}\left[p\left(1-\lambda_j\right)-c\right)]D_j a_{ij} +$$

$$+\sum_{j=r+1}^{k}\left[z((p+q)\left(1-\lambda_j\right)-c\right)-qD_j\left(1-\lambda_j\right)a_{\equiv j}]=$$

$$=\sum_{j=r+1}^{k}\left[(p+q)\left(1-\lambda_j\right)-c\right]a_{ij}(z-D_j)\geq 0,$$

as far as $(p + q)(1 - \lambda_j) - c \geq 0$ in (1) and $z \geq D_j$ when $j \leq k$. Similarly we prove that $L(u,r,z)-L(u,k,z)\geq 0$ when $r > k$. Consequently, $\min_r\{L_i(u, r, z)\} = \pi_i(u, z)$ for $z \in [D_1, D_n]$, and concavity of $\pi_i(z)$ follows from [15, theorem 5.5].

Designations

$\Pi_t(u, z) = \sum_i \pi_i(u,z)$ –

expected operating profit of the concessionaire for the year t of the planned period at the capacity of z.

$\mathrm{E}x(z) = \sum_i \mathrm{E}x_i(z)$, $\mathrm{E}\mu(z) = \sum_i \mathrm{E}\mu_i(z)$,

$\mathrm{E}\nu(z) = \sum_i \mathrm{E}\nu_i(z)$.

Using the introduced designations and definition $\pi_i(u, z)$, we can write that:

$\Pi_t(u, z) = p\mathrm{E}\mu(z) - c\mathrm{E}x(z) - q\mathrm{E}\nu(z)$. (3)

We get a more detailed representation of $\Pi_t(u, z)$ using lemma 1.

Designations

$$b_j=\sum_i a_{ij};\ M_1(\mathrm{k})=\sum_{j=k+1}^{n}\left(1-\lambda_j\right)b_j;$$

$$M_2(\mathrm{k})=\sum_{j=k+1}^{n}b_j;M_3(\mathrm{k})=\sum_{j=1}^{k}\left(1-\lambda_j\right)b_jD_j;$$

$$M_4(\mathrm{k})=\sum_{j=1}^{k}b_jD_j;M_5(\mathrm{k})=\sum_{j=k+1}^{n}\left(1-\lambda_j\right)b_jD_j.$$

When $z \in [D_k, D_{k+1}]$, taking into consideration (2),

$t(u, z) = z[(p + q)M_1(k) - cM_2(k)] + pM_3(k) - cM_4(k) - qM_5(k)$. (4)

If hypothesis (1) is met, from continuity and concavity of $\pi_i(u, z)$ (theorem 2) continuity and concavity of $\Pi_t(u, z)$ in z follow in the interval of $[D_1, D_n]$.

Designations

K_1 – concessionaire's contribution into TSP.

$K_2(z)$ – capital costs necessary to achieve the capacity of $z \geq z_0$, $K_2(z_0) = 0$.

$C(z)$ – annual operational costs at the capacity of $z \geq 0$ (including the costs necessary to maintain the capacity).

T (лет) – duration of the concession term, $[0, T]$ – planned period.

$\alpha = \sum_{t=1}^{T}(1+\delta)^{-t}$, where δ – discount rate per annum.

$A(z) = K_1 + K_2(z) + \alpha C(z)$ – the concessionaire's discounted costs for the TSP, generation of the capacity $z \geq z_0$ and its operation within the planned period.

$F(u, z)$ – expected NPV of the concessionaire for the planned period with the concession vector of parameters $u = (p, q)$ and capacity z.

Inasmuch as the value of $\Pi_t(u, z)$ does not depend on t,

$$F(u,z)=-A(z)+\sum_{t-1}^{T}\Pi_t(u,z)=-A(z)+\alpha\Pi_t(u,z). \quad (5)$$

If $z \geq z_0$ and $z \in [D_k, D_{k+1}]$ then from (4) and (5) we obtain:

$F(u, z) = -A(z) + \alpha[z((p + q)M_1(k) - cM_2(k)) + pM_3(k) - cM_4(k) - qM_5(k)]$. (6)

The Concedent's Purpose

Let us assume that the concedent fixes the value of $\sigma \in (0, 1)$ and tries to define p and q which satisfy the following conditions. First, inequality (1) should be satisfied in order to eliminate the concessionaire's impetus to stop generating energy at low average daily temperatures. Second, the capacity of z chosen by the concessionaire should ensure sufficient consistency of heat supply: $P(\eta_i \leq z) \geq \sigma$ for all i.Let us assume that condition (1) is satisfied. Then at the capacity of $z \in [D_k, D_{k+1})$ the normative demand for heat energy on the day i of the heating season is satisfied with the probability of

$P(\eta_i = d_j \leq z(1 - \lambda_j)) = P(\eta_i = d_j$ and $D_j \leq z) = P(\xi_i = t_j$ and $j \leq k) = \sum_{j=1}^{k} a_{ij}$.

Designations. $k(i) = \min\{k \mid \sum_{j=1}^{k} a_{i析} \geq \sigma\}$;

$z_1 = \max_i\{D_{k(i)}\} = D_m$.

On day i the normative demand for heat energy is satisfied with the probability not less than σ at the minimal capacity of $D_{k(i)}$, and on any day of the heating season – at the minimum capacity of z_1. Assume the capacity which satisfies the minimal demand does not provide for sufficient consistency of heat supply ($D_1 < z_1$), and a decrease of the existing capacity is unacceptable ($z \geq z_0$). At the specified conditions the concedent minimises p.

Game-Theoretic Model

This is a two-move game. First, the concedent (leader according to Stackelberg [14, p. 56]) chooses the strategy $u = (p, q) \in R_+ \times R$ (further we will see that with big values of K_1 the value of q should be negative in order to repay to the concessionaire the investments with the contractual profitability of δ). Second, knowing u the concessionaire selects $z \in \{0\} \cup [z_0, D_n]$. Choosing $z = 0$ equals relinquishment of concession and choosing $z \geq z_0$ means a consent to enter into the concession, invest the amount of K_1 in TSP and to develop the capacity of z.

If $z < z_1$ (in particular, when $z = 0$) the concedent will fail to achieve its goal. Assume in this case its gain function takes on the of value of $-\infty$. The concedent's gain function (maximised) appears as follows:

$H_1(u, z) = -p$, if $(p + q)(1 - \lambda_n) \geq c$ and $z \in [z_1, D_n]$, otherwise $H_1(u, z) = -\infty$.

The concessionaire's gain function appears as follows: $H_2(u, z) = F(u, z)$ when $z \in [z_0, D_n]$ and $H_2(u, 0) = 0$.

The concessionaire's strategy is function $z(\cdot)$: $R_+ \times R \to \{0\} \cup [z_0, D_n]$ which indicates the concessionaire's response to any concedent's strategy $u = (p, q)$.

Designation. $R(u) = \text{Argmax}\{H_2(u, z) \mid z \in \{0\} \cup [z_0, D_n]\}$ – representation of the concessionaire's response.

It is clear that the concessionaire will choose $z(u) \in R(u)$. As a matter of fact, sets $R(u)$ are not one-element sets but strategies $z(u)$ have the property that $z(u) \in R(u)$ with all $u \in R_+ \times R$ they are equivalent for the concessionaire. On this basis, let us assume that the concession parties can agree upon implementation of the capacity from the set of $R(u)$ which is preferable for the concedent, and let us accept the following definition.

Definition. Strategies $\bar{u} = (\bar{p}, \bar{q})$ and $\bar{z}(u)$ of the concedent and concessionaire respectively create *equilibrium* in the considered game if $\bar{z}(u) \in R(u)$ for all $u \in R_+ \times R$ and

$$H_1(\bar{u}, \bar{z}(\bar{u})) = \max\{H_1(u, \bar{z}(u)) | u \in \mathbb{R}_+ \times \mathbb{R}\}.$$

We call the proposed model a 'two-parametric' model, because the concedent defines two parameters p and q.

Analysis of the Two-Parametric Model

Let us phrase the assumptions of the cost functions used in the model.

$K_2(z)$ – does not decrease and is differentiable in $[z_0, D_n]$, $K_2(z_0) = 0$, $K_2(z) > 0$ when $z > z_0$.

$C(z)$ – does not decrease, is convex and is differentiable within the interval of $[0, D_n]$, $C(0) = 0$.

$A(z)$ – does not decrease, is convex and is twice differentiable within the interval of $[z_0, D_n]$.

Concessionaire's Response

For each concedent's strategy u the concessionaire obtains response $R(u)$ as a set of all solutions of the following problem: $H_2(u, z) \to \max$ when $z \in \{0\} \cup [z_0, D_n]$.

Designations. $Z(u) = \text{Argmax}\{F(u, z) \mid z \in [z_0, D_n]\}$; $\pi^*(u) = \max\{F(u, z) \mid z \in [z_0, D_n]\}$.

Lemma 3. Let us assume that $u = (p, q) \in R_+ \times R$ and condition (1) is met. Then:

(a) function $F(u, z)$ is continuous and concave in z within the interval of $[z_0, D_n]$;

(b) function $F(u, z)$ is differentiable in z within $[z_0, D_n]$ everywhere, except for the points of D_k, and in these points is has one-sided derivatives in z;

(c) in point $D_k \in (z_0, D_n)$ the derivative of the function on the left $F(u, z)$ is not less than the derivative on the right.

Proof. (a) The continuity and concavity of function $\Pi_t(u, z)$ in $[z_0, D_n]$ follow from theorem 2 and the definition of this function. Then, assertion (a) follows from (5) and the assumption of characteristics of function $A(z)$. Differentiability of $F(u, z)$ within the interval of $(D_k, D_{k+1}) \cap [z_0, D_n]$ follows from (6). The existence of one-sided derivatives in point D_k and the correlation between them follow from [15, theorem 23.1 and 24.1]. Assertions (b) and (c) are proven.

It is clear that:

$R(u) = Z(u)$, if $\pi^*(u) > 0$; $R(u) = Z(u) \cup \{0\}$, if $\pi^*(u) = 0$; $R(u) = \{0\}$, if $\pi^*(u) < 0$. (7)

In the latter case, the concession cannot take place.

Designation. For $z \in [z_0, D_n] \cap [D_k, D_{k+1}]$ and $u = (p, q)$ assume

$g_k(u, z) = -A'(z) + \alpha[(p + q)M_1(k) - cM_2(k)]$.

The value of $g_k(u, z)$ is derivative of function $F(u, z)$ in z when $z \in (D_k, D_{k+1})$, derivative on the left in point D_{k+1} when $z = D_{k+1}$ and derivative on the right in point D_k when $z = D_k$ $(k < n)$.

Lemma 4. Let us assume that $u \in R_+ \times R$ and condition (1) is met. Then $z \in Z(u)$, if and only if one of the following conditions is met:

(a) $z = z_0$ and $g_l(u, z_0) \leq 0$, where l is defined by condition $z_0 \in [D_l, D_{l+1})$;

(b) $z \in (z_0, D_n] \cap (D_k, D_{k+1})$ and $g_k(u, z) = 0$;

(c) $z = D_k \in (z_0, D_n)$, $g_{k-1}(u, D_k) \geq 0$ and $g_k(u, D_k) \leq 0$;

(d) $z = D_n$ and $g_{n-1}(u, D_n) \geq 0$.

Proof. The concave function $f(x)$ reaches its maximum in point x, if and only if $0 \in \partial f(x)$, where $\partial f(x)$ – subdifferential of function $f(x)$ in point x (see the symmetric assertion for convex function in [15, p. 279]). When $u \in R_+ \times R$ the subdifferential $\partial F(u, z)$ equals: $[g_1(u, z_0), +\infty)$, if $z = z_0$ and $z_0 \in [D_l, D_{l+1})$; $\{g_k(u, z)\}$ when $z \in (D_k, D_{k+1})$; $[g_k(u, z), g_{k-1}(u, z)]$ when $z = D_k \in (z_0, D_n)$; $(-\infty, g_{n-1}(u, z)$ when $z = D_n$. Assertion of lemma follows from it.

Concedent's Problem

Designations. $S(u) = R(u) \cap [z_1, D_n]$, $U = \{u = (p, q) \in R_+ \times R \mid S(u) \neq \emptyset, (p + q)(1 - \lambda_n) \geq c\}$.

If $S(u) = \emptyset$ the concessionaire will choose $z < z_1$. When $(p + q)(1 - \lambda_n) < c$ condition (1) is not met and it is unprofitable for the concessionaire to generate heat energy at low temperatures. In both cases, the concedent's benefit equals $-\infty$. Consequently, U is a set of all strategies acceptable for the concedent. In the language of the economic mechanisms theory $z \in R(u)$ is the condition of individual rationality while (1) and $z \geq z_1$ are the conditions of incentive compatibility, see [14, p. 57 and further]. If $U = \emptyset$ the concedent's goal is unattainable. However, if $u \in U$ then $H_1(u, z) = -p$ for any $z \in S(u)$. Taking equivalence into consideration for the concessionaire of all $z \in R(u)$, assume that in equilibrium $z(u) \in S(u)$ for all $u \in U$. Consequently, if the concedent's goal can be achieved ($U \neq \emptyset$), then its equilibrium strategy is defined on the basis of the following problem: $p \to \min$ under the condition of $(p, q) \in U$.

Equilibrium

Designation. $A_1(z) = A(z) - K_1 = K_2(z) + C(z)$.

Lemma 5. If $0 \leq z_0 \leq z' \leq z$, then $-K_1 \leq z'A'(z') - A(z') \leq zA'(z) - A(z)$.

Proof. It is known[1] that if function $f(x)$ is convex and differentiable in $X \subseteq$ R, then $f'(x)$ does not decrease in X and $f(x_2) - f(x_1) \geq f'(x_1)(x_2 - x_1)$ for any x_1, x_2 out of X, i.e.

$$x_1 f'(x_1) - f(x_1) \geq x_2 f'(x_1) - f(x_2). \quad (8)$$

Assume $z_0 \leq z' \leq z$. Applying (8) to the convex non-decreasing function $A_1(\cdot)$ when $x_1 = z$ and $x_2 = z'$, we obtain: $zA_1'(z) - A_1(z) \geq z'A_1'(z) - A_1(z') \geq z'A_1'(z') - A_1(z')$. Similarly, when $x_1 = z'$ and $x_2 = z_0$, taking into consideration that $K_2(z_0) = 0$ and $K_2'(z_0) \geq 0$, we have

$$z'A_1'(z') - A_1(z') \geq z_0 A_1'(z_0) - A_1(z_0) =$$
$$z_0[K_2'(z_0) + C'(z_0)] - [K_2(z_0) + C(z_0)] \geq z_0 C'(z_0) - C(z_0).$$

By assuming the function $C(z)$ is convex in $[0, D_n]$ and $C(0) = 0$, therefore it follows from (8) when $x_1 = z_0$ and $x_2 = 0$ that $z_0 C'(z_0) - C(z_0) \geq 0$. Hence $0 \leq z'A_1'(z') - A_1(z') \leq zA_1'(z) - A_1(z)$. Deducting K_1 from both sides of inequation and taking into consideration that $A_1'(z) = A'(z)$, we arrive at the desired result.

Designation. Assume $\Delta_k = [z_0, D_n] \cap [D_k, D_{k+1}]$. When $k < n$ each $z \in \Delta_k$ let us compare the system of equations

$$(p + q)M_1(k) = cM_2(k) + A'(z)/\alpha, \quad (9)$$

$$pM_3(k) - qM_5(k) = cM_4(k) + [A(z) - zA'(z)]/\alpha. \quad (10)$$

Lemma 6. At any $z \in \Delta_k$ the system of equations (9), (10) has the unique solution:

$$p = \frac{\alpha c\left[M_2(k)M_5(k) + M_1(k)M_4(k)\right] + A'(z)M_5(k) + [A(z) - zA'(z)]M_1(k)}{áM_1(k)\left[M_3(k) + M_5(k)\right]}, \quad (11)$$

$$q = \frac{ác\left[M_2(k)M_3(k) - M_1(k)M_4(k)\right] + A'(z)M_3(k) + \left[zA'(z) - A(z)\right]M_1(k)}{áM_1(k)\left[M_3(k) + M_5(k)\right]}. \quad (12)$$

If $u = (p, q)$ is the system solution, then $F(u, z) = 0$. If, apart from that, condition (1) is met then $z \in R(u)$.

Proof. Let us calculate the system determinant (9), (10):

$$\Delta = -M_1(k)[M_3(k) + M_5(k)] = -M_1(k)\sum_{j=1}^{n}(1-\lambda_j)D_j b_j < 0.$$

Consequently, the system has a unique solution, which we designate as $u = (p, q)$. Applying Cramer's rule, we obtain formulas (11), (12).

Using formula (6) it is easy to verify that $F(u, z) = 0$, if p and q satisfy equations (9), (10). Assume p and q meet condition (1). Equation (9) is equivalent to $g_k(u, z) = 0$, therefore $z \in R(u)$ follows from lemma 4 using assertion (c) of lemma 3.

Designations. Assume $z \in \Delta_k$.

$u(z, k) = (p(z, k), q(z, k))$ – solution of the system of equations (9)–(10).

For $k < n$ assume $K_0(z, k) = \alpha c[M_2(k)M_3(k)/M_1(k) - M_4(k)] + A'(z)[z + M_3(k)/M_1(k)] - A_1(z)$.

Note that for $z = D_k \in (z_0, D_n)$ both $u(z, k)$ and $u(z, k-1)$ are defined.

Consequence 1. Assume $z \in \Delta_k$. Then: (a) $q(z, k) < 0$ if and only if $K_1 > K_0(z, k)$; (b) $q(z, k) \geq 0$ when $K_1 = 0$; (c) $p(z, k) \geq c$.

Proof. The denominator of formula (12) is positive. In the equation which reflects negativeness of the numerator, let us replace $A(z)$ with $A_1(z) + K_1$. Solving the obtained inequation as regards K_1, we are left with assertion (*a*).

Values of λ_j do not decrease in j, therefore

$$M_2(k)M_3(k) = \sum_{j=k+1}^{n} b_j \bullet \sum_{j=1}^{k}(1-\lambda_j)D_j b_j \geq (1-\lambda_k)\sum_{j=k+1}^{n} b_j \bullet$$
$$\bullet \sum_{j=1}^{k} D_j b_j \geq\geq \sum_{j=k+1}^{n}(1-\lambda_j)b_j \bullet \sum_{j=1}^{k} D_j b_j = M_1(k)M_4(k).$$

Besides, if $K_1 = 0$ then $A(z) = A_1(z)$ and $zA'(z) - A(z) \geq 0$ according to lemma 5, from which $K_0(z, k) \geq 0 = K_1$. Then $q(z, k) \geq 0$ according to assertion (*a*). Assertion (b) has been proven.

[1] See: Pshenichny B.N. Convex Analysis and Extremum Problem. M.: Nauka, 1980. Theorem 1.1.

Let us rewrite (11) as follows:

$$p = c\frac{M_2(k)M_5(k)+M_1(k)M_4(k)}{M_1(k)M_5(k)+M_1(k)M_5(k)}+\frac{A'(z)M_5(k)+\left[A(z)-zA'(z)\right]M_1(k)}{\alpha M_1(k)\left[M_3(k)+M_5(k)\right]}. \quad (13)$$

Note that $M_2(k) \geq M_1(k)$ and $M_4(k) \geq M_3(k)$, therefore the coefficient when c in the right side of the equation is (13) is not less than unity. It means that $p(z, k) \geq c$ if $A(z) - zA'(z) \geq 0$. Let us assume that $A(z) - zA'(z) = K_1 + A_1(z) - zA'(z) < 0$. Then $K_1 < zA_1'(z) - A_1(z) \leq K_0(z, k)$ and $q(z, k) \geq 0$ according to assertion (a). The quantity of generated heat energy cannot be less than the satisfied demand, therefore $E\mu_i(z) \leq Ex_i(z)$ for all i and z (it can be easily proved formally, using lemma 1). If $p(z, k) \leq c$ then $\Pi_t(u(z, k), z) \leq 0$ in (3) and $F(u(z, k), z) < 0$ in (5), and this contradicts lemma 6. Consequently, $p(z, k) \geq c$ in all cases and assertion (c) has been proven.

Designation

For $z \in \Delta_k \setminus \{D_k\}$ assume $k(z) = k$ and $h(z) = p(z, k) + q(z, k)$.

Let us assume that $B = \{z \geq z_1 \mid h(z)(1 - \lambda_n) \geq c\}$ and z^* is the greatest lower bound of set B.

Lemma 7

(a) Function $h(z)$ is continuous in the interval of $\Delta_k \setminus \{D_k\}$ and does not decrease in $[z_0, D_n]$.

(b) $(z^*, D_n] \subseteq B \subseteq [z^*, D_n]$.

(c) If $u \in U$ and $z \in S(u)$ then $z \geq z^*$.

Proof. For $z \in \Delta_k \setminus \{D_k\}$ we obtain the following out of (9)

$h(z) = cM_2(k) / M_1(k) + A'(z) / \alpha M_1(k)$. (14)

Continuity of $h(z)$ in $\Delta_k \setminus \{D_k\}$ follows from (14). Function $k(z)$ does not decrease in z and $M_1(k)$ does not increase in k, therefore $M_1(k(z))$ does not increase in z. According to the assumption that $A(z)$ is a convex function, therefore $A'(z)$ does not decrease. Consequently, the addend in the expression for $h(z)$ does not decrease. The augend does not decrease either. In point of fact,

$M_2(k - 1) / M_1(k - 1) \leq M_2(k) / M_1(k) \leftrightarrow M_2(k - 1)M_1(k) \leq M_1(k - 1)\, M_2(k) \leftrightarrow$

$$\sum_{j=k}^{n} b_j \cdot \sum_{j=k+1}^{n}\left(1-\lambda_j\right)b_j \leq \sum_{j=k+1}^{n} b_j \cdot \sum_{j=k}^{n}\left(1-\lambda_j\right)b_j.$$

$$\sum_{j=k+1}^{n}\left(1-\lambda_j\right)b_j \leq \left(1-\lambda_k\right)\sum_{j=k+1}^{n} b_j.$$

The last inequality holds because $\lambda_j \geq \lambda_k$ for $j > k$. Consequently, $h(z)$ is a nondecreasing function within the interval of $[z_0, D_n]$ and assertion (a) has been proven.

By convention $k(D_n) = n - 1$ and according to (14)

$h(D_n) = cM_2(n - 1) / M_1(n - 1) + A'(D_n) / \alpha M_1(n - 1) = c / (1 - \lambda_n) + A'(D_n) / \alpha b_n \geq c / (1 - \lambda_n)$.

Hence, $D_n \in B$. Then assertion (b) follows from assertion (a).

Let us assume that $u = (p, q) \in U$ and $z \in S(u)$. Then $p + q \geq c / (1 - \lambda_n)$. If $z \in (D_k, D_{k+1})$ then according to lemma 4 $g_k(u, z) = 0$ follows from $z \in S(u) \subseteq R(u)$, and taking into consideration (9) it equals $h(z) = p + q$. Hence $h(z) \geq c / (1 - \lambda_n)$ and $z \geq z^*$. Assume $z = D_k$. Inasmuch as $D_n \geq z^*$ it is sufficient to consider the case of $k < n$. In this case according to lemma 4 it follows out of $z \in S(u)$ that $g_k(u, z) \leq 0$. Using (9) we obtain $p(z, k) + q(z, k) \geq p + q \geq c / (1 - \lambda_n)$. But $p(z, k) + q(z, k) = \lim_{y \to z+0} h(y)$, therefore taking into consideration assertion (a), $h(y) \geq c / (1 - \lambda_n)$ for $y > z$. Then it follows from the definition of z^* that $z \geq z^*$. Assertion (c) has been proven.

It follows from lemmas 6 and 7 that the concedent may, by choosing $u = u(z, k(z))$, stimulate generation of any capacity $z \in B$. Assertion (b) of lemma 7 describes set $B \setminus \{z^*\}$. Exogenous parameters of the model define whether z^* belongs to set B. Let us show that the concedent's optimal strategy $u^* = (p^*, q^*)$ makes strategy z^*, in particular, optimal for the concessionaire and let us find the values of p^* and q^*. The next lemma shows some properties of point z^*.

Lemma 8

(a) If $z^* \in (D_r, D_{r+1})$, then $p(z^*, r) + q(z^*, r) = c / (1 - \lambda_n)$ and $z^* \in B$.

(b) If $z^* = D_r \in B$ and $z^* > z_1$, then $p(z^*, r - 1) + q(z^*, r - 1) = c / (1 - \lambda_n)$.

(c) If $z^* = D_r \notin B$, then $1 < r < n$ and there exists number $\gamma \in (0, 1]$ with the property that

$(1 - \gamma)[p(z^*, r - 1) + q(z^*, r - 1)] + \gamma[p(z^*, r) + q(z^*, r)] = c / (1 - \lambda_n)$.

Proof. According to the choice of z^* we have $h(z) < c / (1 - \lambda_n)$ for $z < z^*$. Assume $z^* \in (D_r, D_{r+1})$. According to assertion (b) of lemma 7 $h(z) \geq c / (1 - \lambda_n)$ for $z \in (z^*, D_{r+1})$. Then $h(z^*) = c / (1 - \lambda_n)$ follows from assertion (a) of lemma 7. Now assume $z^* = D_r$. By assumption $z_1 = D_m > D_1$, therefore $r > 1$. If $D_r \in B$ and $D_r > z_1$, then $h(z^*) \geq c / (1 - \lambda_n)$ and $h(z) < c / (1 - \lambda_n)$ for $z < z^*$. Hence, taking into consideration assertion (a) of lemma 7 it follows that $h(z^*) = c / (1 - \lambda_n)$. If $D_r \notin B$, then $r < n$, as far as $D_n \in B$ according to assertion (b) of lemma 7. Therein $h(z^*) = p(z^*, r - 1) + q(z^*, r - 1) < c / (1 - \lambda_n)$, and it follows from (9) and assertion (b) of lemma 7 that $p(z^*, r) + q(z^*, r) = \lim_{z \to z^*+0}[p(z,r)+q(z,r)] \geq c / (1 - \lambda_n)$. Consequently, the equality indicated in assertion (c) holds for some number γ, which is easily calculated out of this equality.

Lemma 9. Let us assume that $z \in \Delta_k$, $u = (p, q) \in R_+ \times R$, $F(u, z) \geq 0$ and $g_k(u, z) \geq 0$. Then $p \geq p(z, k)$.

Proof. If it is granted that conditions of lemma are met and $p < p(z, k)$. Then $q > q(z, k)$, as far as $g_k(u, z) \geq 0$ implies $p + q \geq cM_2(k) / M_1(k) + A'(z) / \alpha M_1(k) = p(z, k) + q(z, k)$ in (9). Hence, using (5) and (3) we obtain $F(u, z) - F(u(z, k), z) = \alpha[(p - p(z, k)) \cdot E\mu(z) - (q - q(z, k)) \cdot E\nu(z)] < 0$.

But $F(u(z, k), z) = 0$ according to lemma 6, it means that $F(u, z) < 0$, contradiction.

Lemma 10. (a) Function $p(z, k)$ does not decrease in Δ_k.
(b) If $1 < s < n$ and for the concedent's strategy $u(D_s, s)$ condition (1) is met then $p(D_s, s - 1) \le p(D_s, s)$.

Proof. In accordance with (11), minimisation of $p(z, k)$ by Δ_k is equal to minimisation of function $f(z) = A'(z)M_5(k) + [A(z) - zA'(z)]M_1(k)$. Let us find $f'(z) = A''(z)[M_5(k) - zM_1(k)]$. From convexity $A(z)$ it follows that $A''(z) \ge 0$. If $z \le D_{k+1}$ then

$$M_5(k) - zM_1(k) = \sum_{j=k+1}^{n} (1-\lambda_j) b_j (D_j - z) \ge 0.$$

It means that $f'(z) \ge 0$ in Δ_k and assertion (a) has been proven.

Let us assume that the set $u = u(D_s, s)$ meets condition (1). Then $F(u, D_s) = 0$ and $D_s \in R(u)$ according to lemma 6. Hence, $g_{s-1}(u, D_s) \ge 0$ according to condition (c) of lemma 4. It means that when $k = s - 1$ and $p' = p(D_s, s)$ conditions of lemma 9 and $p(D_s, s) \ge p(D_s, s - 1)$ are met.

It follows from lemma 10 that function $p(z, k(z))$ does not decrease in the interval of $(D_1, D_n]$. Now let us define the concedent's strategy $u^* = (p^*, q^*)$, the optimality of which will be proven below.

Designations and definition. Assume $u^* = u(z^*, r)$, if $z^* \in (D_r, D_{r+1})$, and $u^* = u(z^*, r - 1)$, if $z^* = D_r \in B$ (in particular, if $z^* = D_n$). However, if $z^* = D_r \notin B$, then $u^* = (1 - \gamma) u(z^*, r - 1) + \gamma u(z^*, r)$, where γ is the number indicated in assertion (c) of lemma 8.

Theorem 3. Assume $z(\cdot)$ is the concessionaire's strategy such that $z(u) \in R(u)$ for all $u \in R_+ \times R$ and $z(u^*) = z^*$. Then $z(\cdot)$ and the concedent's strategy u^* create equilibrium in the considered game.

Proof. Assume $z^* \in \Delta_r$ and $u^* = (p^*, q^*)$. On the basis of (7) and lemma 3 we make the conclusion that $R(u) \ne \emptyset$ at all $u \in R_+ \times R$. The definition of u^*, taking into consideration assertion (c) of consequence 1, guarantees that $u^* \in R_+ \times R$ and condition (1) is met for u^*. If $u^* = u(z^*, r)$, then $z^* \in R(u^*)$ according to lemma 6. However, if $u^* \ne u(z^*, r)$, then $z^* = D_r < D_n$. In this case it follows from (9) that $g_{r-1}(u(z^*, r - 1), z^*) = g_r(u(z^*, r), z^*) = 0$, it means that $g_r(u(z^*, r - 1), z^*) \le 0$ and $g_{r-1}(u(z^*, r), z^*) \ge 0$ according to assertion (c) of lemma 3. Function $g_k(u, z)$ with fixed z and k is linear in u, therefore $g_{r-1}(u^*, z^*) \ge 0$, $g_r(u^*, z^*) \le 0$ and $z^* \in R(u^*)$ according to condition (c) of lemma 4. So, $z^* \in R(u^*)$ in all cases, therefore $u^* \in U$, and therefore concessionaire's strategies with the properties indicated in the theorem exist, and we assume $z(\cdot)$ is one of them.

If $u \notin U$, then $H_1(u, z(u)) = -\infty < -p^* = H_1(u^*, z^*)$. Let us assume that $u = (p, q) \in U$. Then $S(u) \ne \emptyset$ and $(p + q)(1 - \lambda_n) \ge c$. Let us show that $p^* \le p$. Assume $z(u) = z \in \Delta_k$. As long as all strategies from $R(u)$ are equivalent for the concessionaire we can think that $z \in S(u)$. Then $F(u, z) \ge 0$ in (7) and $z \ge z^*$ according to assertion (c) of lemma 7. At the same time if $z \in [z^*, D_{r+1}]$, then it follows from $z \in S(u) \subseteq R(u)$ according to lemma 4 that $g_r(u, z) \ge 0$. Then $p \ge p(z, r) \ge p(z^*, r)$ according to lemmas 9 and 10.

Granted that $z \in \Delta_k$ for $k > r$. If $z \in (D_k, D_{k+1}]$ then $g_k(u, z) \ge 0$ according to lemma 4. Using lemmas 9 and 10 we obtain $p \ge p(z, k) \ge p(D_k, k)$. Therein $p(D_k, k) + q(D_k, k) = \lim_{y \to D_k + 0} h(y) \ge c / (1 - \lambda_n)$, because $D_k > z^*$. Then $p(D_k, k) \ge p(D_k, k - 1)$, according to assertion (b) of lemma 10. According to assertion (*a*) of this lemma $p(y, k - 1)$ does not decrease in y in Δ_{k-1}. Hence, $p(D_{k-1}, k - 1) \le p(D_k, k - 1) \le p(D_k, k)$. Repeating the reasoning let us prove that $p \ge p(D_k, k) \ge p(D_{k-1}, k - 1) \ge \ldots \ge p(D_{r+1}, r + 1)$. Note that $F(u(D_{r+1}, r + 1), D_{r+1}) = 0$ according to lemma 6 and $g_{r+1}(u(D_{r+1}, r + 1), D_{r+1}) = 0$ in (9), therefore applying lemma 9 when $k = r$, $u = u(D_{r+1}, r + 1)$ and $z = D_{r+1}$, we obtain $p(D_{r+1}, r + 1) \ge p(D_{r+1}, r)$. Finally, according to assertion (a) of lemma 10, $p(z^*, r) \le p(D_{r+1}, r) \le p$.

Thus, if $z^* \in \Delta_r$, then $p(z^*, r) \le p$ for any concedent's strategy $u = (p, q) \in U$. If $z^* \in (D_r, D_{r+1})$ or $r = n - 1$ and $z^* = D_n$, then $p^* = p(z^*, r) \le p$. Assume $z^* = D_r$ and $r < n$. Then it follows from assertion (b) of lemma 7 that $p(D_r, r) + q(D_r, r) = \lim_{y \to D_r + 0} h(y) \ge c / (1 - \lambda_n)$, and according to assertion (b) of lemma 10, $p(z^*, r - 1) \le p(z^*, r) \le p$. Now from definition p^* follows $p^* \le p$. So, for any concedent's strategy $u = (p, q) \in U$ we obtain $p^* \le p$. We proved above that $u^* \in U$, therefore $p^* = \min\{p \mid (p, q) \in U\}$.

Theorem 3 demonstrates that if the concedent declares a concession with parameters p^* and q^*, then z^* is included in the concessionaire's response, and it will have no grounds to refuse to create this particular heat generating capacity. Therein the concedent's goal is achieved in the equilibrium at the minimal tariff.

Consequence 3. In the equilibrium the concessionaire's NPV is zero.

Proof. By convention u^* is one of strategies $u(z^*, r)$ and $u(z^*, r - 1)$ or their linear combination. Then $F(u^*, z^*) = 0$ follows from lemma 6 in the first two cases and from linearity $F(u, z)$ in u – in the third case.

One-Parametric Model

The concession mechanism offered above with $q > 0$ implies penalisation of the concessionaire for each unit of unsatisfied demand. Let us examine the results which may be obtained from a one-parametric model which has no (equals zero) parameter q.

As before, we assume that the concessionaire develops the capacity of z, and then on each day i of the heating season it chooses the rate of its usage $x_i(z)$, maximising the current profit. The concedent makes the first move defining the tariff $p \ge 0$, and the concessionaire makes the second move choosing the capacity $z \in \{0\} \cup [z_0, D_n]$. An analogue of condition (1) which in the two-parametric model guarantees that the concessionaire does not stop heat supply even at the minimal temperature, in the one-parametric model is the inequality $p(1 - \lambda_n) \ge c$. (15)

If it holds then at the capacity of z average daily temperature t_j on the day i of the heating season the concessionaire will choose $x_i(z) = \min\{z, D_j\}$.

Concedent's gain function is: $H_1^1(p, z) = -p$, if $z \in [z_1, D_n]$ and $p(1 - \lambda_n) \ge c$, otherwise $H_1^1(p, z) = 0$.

The net present expected profit of the concessionaire is written as

$F_1(p, z) = -A(z) + \alpha[pE\mu(z) - cEx(z)]$. (16)

It follows from (3), (5) and (16) that

$F_1(p, z) = F(u, z) + \alpha qE\nu(z)$, where $u = (p, q)$, (17)

where $F(u, z)$ – the concessionaire's expected NPV within the planned period in the two-parametric model. If condition (15) is met, then analogues of assertions of lemma 3 hold: function $F_1(p, z)$ is continuous in z within the interval of $[D_1, D_n]$ and concave in z within this interval; it is differentiable in z in $[z_0, D_n]$ everywhere, except for points D_k, and in these points it has one-sided derivatives in z.

When $z \in \Delta_k$ from (16) and lemma 1 follows that

$F_1(p, z) = -A(z) + \alpha[z(pM_1(k) - cM_2(k)) + pM_3(k) - cM_4(k)] =$

$$-A(z)+ +\alpha\left[z\sum_{j=1}^{k}D_j b_j\left(p\left(1-\lambda_j\right)-c\right)+\sum_{j=k+1}^{n}b_j\left(p\left(1-\lambda_j\right)-c\right)\right]. \quad (18)$$

Designation. For $z \in \Delta_k$ assume $g_k^1(p,z) = -A'(z) + \alpha[pM_1(k) - cM_2(k)]$.

The value $g_k^1(p,z)$ – derivative of function $F_1(p, z)$ in z when $z \in (D_k, D_{k+1})$, left-side derivative when $z = D_{k+1}$ and right-side derivative when $z = D_k$. If $u = (p, q)$, then $g_k(u, z) = g_k^1(p,z)$.

The concessionaire's gain function: $H_2^1(p,z) = F_1(p, z)$, if $z \in [z_0, D_n]$, and $H_2^1(p,0) = 0$.

The concessionaire's strategy is function $z(p)$ which indicates the concessionaire's response to any concedent's strategy p.

Definition and Designations

$Z_1(p) = \mathrm{Argmax}\{F_1(p, z) \mid z \in [z_0, D_n]\}$, $R_1(p) = \mathrm{Argmax}\{H_2^1(p,z) \mid z \in \{0\} \cup [z_0, D_n]\}$.

The pair of strategies $\overline{z}(p)$ and $\overline{p}$ is a equilibrium if $\overline{z}(p) \in R_1(p)$ for all $p \geq 0$ and

$H_1^1(\overline{p},\overline{z}(\overline{p})) = \max\{H_1^1(p,\overline{z}(p)) \mid p \geq 0\}$.

Let us assume that $P = \{p \mid p \geq c / (1 - \lambda_n), R_1(p) \cap [z_1, D_n] \neq \emptyset\}$.

For $z \in \Delta_k$ assume $p_1(z, k) = cM_2(k) / M_1(k) + A'(z) / \alpha M_1(k)$.

$R_1(p)$ – representation of the concessionaire's response. If $z \in Z_1(p)$ then $z \in R_1(p)$ is equivalent to $F_1(p, z) \geq 0$. The concedent looks for the minimal value of p out of P. At any $p > 0$ the concessionaire chooses $z(p) \in R_1(p)$. All $z \in R_1(p)$ are equal for the concessionaire, therefore let us assume that $z(p) \in Z_1(p)$ if $p \in P$. Both $p_1(z, k - 1)$ and $p_1(z, k)$ are defined for $z = D_k \in (D_1, D_n)$. An analogue of lemma 4 with replacement of $Z(u)$ with $Z_1(p)$ and $g_k(u, z)$ in $g_k^1(p,z)$ holds.

Lemma 11. Assume $z \in \Delta_k$.

(a) $g_k^1(p_1(z,k),z) = 0$ and $p_1(z, k) = p(z, k) + q(z, k)$. If $z \in (D_k, D_{k+1}]$ then $p_1(z, k) = h(z)$.

(b) $z \in Z_1(p)$ if and only if one of the following conditions is fulfilled: $z = D_1$ and $p \leq p_1(z, k)$; $z \in (D_k, D_{k+1})$ and $p = p_1(z, k)$; $z = D_{k+1} < D_n$ and $p_1(z, k) \leq p \leq p_1(z, k + 1)$; $z = D_n$ and $p \geq p_1(z, n - 1)$.

Proof. Assertion (a) follows from definitions $g_k^1(p_1(z,k),z)$, $p_1(z, k)$, $u(z, k)$ and $h(z)$. Function $g_k^1(p,z)$ does not decrease in p, therefore any relation between $g_k^1(p,z)$ and zero (less than, equal or greater than) is equivalent to the identical relation between p and $p_1(z, k)$. Now assertion (b) follows from the analogue of lemma 4 for one-parametric model.

6. Comparison of Equilibrium in the One-Parametric and Two-Parametric Models

Let us assume that $u^* = (p^*, q^*)$ and $z(u)$ are equilibrium strategies of participants in the two-parametric model, $z^* = z(u^*) \in (D_r, D_{r+1}]$. Let us also assume that p_1 and $z_1(p_1)$ are equilibrium strategies of participants in the one-parametric model where $z_1(p_1) = z_1^* \in (D_s, D_{s+1}]$.

Theorem 4. $z_1^* \geq z^*$ and $p_1 \geq \max\{p^*, p^* + q^*\}$.

Proof. If it is granted that $z_1^* < z^*$. Then $p_1 \in P$ and $z_1^* \in Z_1(p)$. From assertion (a) of lemma 7 follows $h(y) < c / (1 - \lambda_n)$ for $y \in [z_1^*, z^*)$. If $z_1^* \neq D_{s+1}$ then $p_1 = h(z_1^*)$ according to 11. If $z_1^* = D_{s+1}$ then from $z_1^* < z^*$ follows $D_{s+1} < D_n$ and using lemma 11 we obtain

$p_1 \leq p(z_1^*,s+1) + q(z_1^*,s+1) = \lim_{y \to z_1^*} h(y) < c / (1 - \lambda_n)$.

In all cases $p_1 < c / (1 - \lambda_n)$, which contradicts $p \in P$.

Consequently, $z_1^* \geq z^*$, hence $k(z_1^*) = s \geq r = k(z^*)$. If $q(z_1^*,s) \geq 0$ then from lemma 11 follows $p_1 \geq p_1(z_1^*,s) = h(z_1^*) \geq p(z_1^*,s)$. Let us assume that $p_1(z_1^*,s) = h(z_1^*) \geq p(z_1^*,s) < 0$. Then with $p_1 < p_1(z_1^*,s) = h(z_1^*) \geq p(z_1^*,s)$ we have:

$$F_1(p_1,z_1^*) = p_1 E\mu(z_1^*) - cEx(z_1^*) < p(z_1^*,s)E\mu(z_1^*) - cEx(z_1^*) - q(z_1^*,s)E\nu(z_1^*) = = F(u(z_1^*,s),z_1^*) = 0,$$

And this contradicts the condition of $z_1^* \in R_1(p_1)$. It means that $p_1 \geq p_1(z_1^*,s)$ in all cases.

Function $p(z, k(z))$ does not decrease in z (see lemmas 9 and 10), therefore from $z^* \neq D_{r+1}$ it follows that $p^* = p(z^*, r) \leq p_1(z_1^*,s)$. If $z^* = D_{r+1}$ then $s > r$; from definition of p^* and lemma 10 we obtain $p^* \leq p(z^*, r + 1) \leq p(z_1^*, s))$. In all cases $p_1 \geq p^*$ because $p_1 \geq p_1(z_1^*,s)$.

It is obvious from definition of u^* that either $p^* + q^* = c / (1 - \lambda_n)$, or $g_r(u^*, z^*) \leq 0$. Consequently, if $p < p^* + q^*$ two variants are possible : $p < c / (1 - \lambda_n)$ or $g_r^1(p,z^*) < g_r^1(p^*+q^*,z^*) = g_r(u^*,z^*) \leq 0$.

Function $g_k^1(p,z)$ does not increase in Δ_k, and for concave function $F_1(p, z)$ the left-hand derivative in any point of z is not less than the right-hand derivative, therefore $g_r^1(p,z^*) < 0$ implies $g_s^1(p,z^*) < 0$. In both cases we have a contradiction with $z_1^* \in R_1(p_1)$. In means that $p_1 \geq p^* + q^*$.

Thus, in the two-parametric model the equilibrium capacity and equilibrium price of heat energy do not exceed the corresponding indicators of the one-parametric model.

The following result shows that $z_1^* = z^*$ and $p_1 > p^*$ under "normal" conditions (when $q^* \geq 0$).

Consequence 2. If $q^* \geq 0$ then in the one-parametric model there is equilibrium with the following parameters $z_1^* = z^*$ and $p_1 = p^* + q^*$.

Proof. Let us assume that $p = p^* + q^*$. Inasmuch as $p_1 \geq p^* + q^*$ (theorem 4) it is sufficient to prove that $p \in P$ and $z^* \in R_1(p)$. If $p' = p + q$ and $u = (p, q)$ then $g_k^1(p',z) = g_k(u, z)$ for all k and $z \in \Delta_k$. Then from $z^* \in Z(u^*)$ of lemma 4 and its analogue for one-parametric model it follows that $z^* \in Z_1(p)$. From $z^* \in R(u^*)$, (17) and $q^* \geq 0$ follows $F_1(p, z^*) \geq F_1(p^*, z^*) \geq F(u^*, z^*) \geq 0$, therefore $p_1 \in P$.

In the two-parametric model the expected NPV of the concessionaire in equilibrium equals zero. In the one-parametric model this indicator may be positive (which means that the aggregate expected discounted costs of the community party (the consumer) exceed the expected discounted costs of the concessionaire). For example, $F_1(p_1, z^*) > F_1(p^*, z^*) > F(u^*, z^*)$ in the situation described by consequence 2 with $q^* > 0$.

Results and Conclusions

In the two-parametric model the concedent defines the required consistency of heat supply σ and chooses parameters p and q (tariff and penalty for failure to fulfill normative delivery of heat). With these parameters the concessionaire, maximising NPV, chooses z (the enterprise capacity), and defines the amount of generation of heat energy each day. We have proven the existence of, and found the method of creating Stackelberg equilibrium. The properties of this equilibrium are as follows: a capacity of z^* ensures sufficient consistency of heat supply; the concessionaire utilises the full extent this capacity to satisfy the normative demand; the price p^* of heat energy covers its price cost c; the net present expected profit of the concessionaire is zero; and finally, p^* is the minimum price at which the abovementioned conditions may be fulfilled.

If the concessionaire's expenses for the modernisation of the heat supply system K_1 are too high the "penalty" q^* becomes negative. Inasmuch as $p^* \geq c$ the concessionaire's lost profits correspond to the unsatisfied demand. If $q^* < 0$ this short-received profit is reimbursed by the concedent which pays $|q^*|$ for each unit of unsatisfied demand. Reimbursements to the concessionaire are included in many real concession agreements as a "guaranteed minimum income" (GMI). Our model indicates in which cases such payments are necessary, and helps to define their amount. It is true that if $z^* \in \Delta_r$ then $u^* = u(z^*, r)$, or $u^* = u(z^*, r-1)$, or $u^* = (1-\gamma)u(z^*, r-1) + \gamma u(z^*, r)$. For the first two cases, assertion (a) of consequence 1 indicates the concessionaire's maximum expenses for the transmission system project with which $q^* \geq 0$. It is easy to calculate such threshold value for the third version of definition u^* as well. In particular, $q^* \geq 0$ if the concessionaire does not invest in modernising heat supply systems. By reducing the concessionaire's contribution towards modernisation of the heat supply systems one can always ensure non-negativeness of the penalty in the equilibrium.

The concessionaire's expected NPV in equilibrium equals zero, nevertheless it is possible to ensure normal entrepreneurial profit for it by choosing the discount rate δ, i.e. the price of investments. However, it follows from (11) that when δ increases p^* grows as well, whereby the consumers' payments may be socially unacceptable.

Conclusion

As shown in section 6 the two-parametric model is more beneficial for the community party / consumers in terms of value indicators. An important advantage of this model, in our opinion, is that it is better protected from the concessionaire's opportunistic behaviour, and makes violation of the agreement unprofitable for it. If the concessionaire develops the capacity of $z < z^*$ then with equal equilibrium values of parameters (p^* and q^*) its net present expected profit will not be positive (as a rule, it will be negative).

The one-parametric model does not possess such a property. With the equilibrium value of heat energy (p_1) and capacity $z < z_1^*$, the concessionaire's profit may be positive (though not maximal). If that is granted, for example, the initial capacity z_0 is positive and $z_0 \in \Delta_k$. Then with $z = z_0$ and price p_1, in accordance with (18) the concessionaire will have the following profit

$$F_1(p_1,z_0) = -A(z_0) + \alpha[z_0 \sum_{j=1}^{k} D_j b_j \left(p_1\left(1-\lambda_j\right)-c\right) + \sum_{j=k+1}^{n} b_j \left(p_1\left(1-\lambda_j\right)-c\right)]. \quad (19)$$

From the definition of equilibrium follows the proposition $p_1(1-\lambda_n) \geq c$, and so the expression in square brackets is non-negative. If it equals zero then $F_1(p_1, z) < 0$ for all $z > 0$, and this contradicts the condition $p_1 \in P$. Hence, the addend on the right side of formula (19) is positive. In such a circumstance, an unscrupulous concessionaire may get a positive profit without making any investments or having necessary operating expenses, but selling heat energy at price p_1. In the two-parametric model such concessionaire would have gone bankrupt paying penalties for short-delivery of heat energy, and in the one-parametric model the concedent may simply resort to administrative and legal remedies. This all goes to say that an economic mechanism which inhibits the concessionaire's opportunistic behaviour is "embedded" in the two-parametric model. There is no such mechanism in the present generally accepted one-parametric model. Therefore, practical implementation of the two-parametric model is possible only after modification of the concession legislation.

A simplified version of the model was tested successfully in the master's thesis by S.A. Klimentieva (Novosibirsk State University, Faculty of Economics, 2018, advisor – A.B.Khutoretsky), using construction of a boiler-house in the microdistrict of Razdolny, in the town of Berdsk, as an example (at the time of writing, the project has not been implemented). The calculated price of heat energy turned out to be 9.6% less than the current actual price, and 7.5% less than the design price, and the design capacity and capital costs 12% less than corresponding project parameters.

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