

DOI: <https://doi.org/10.17323/j.jcfr.2073-0438.15.4.2021.66-87>

JEL classification: B40, G21, F65



Approaches to Building Default Probability Models for Financial Instruments of Project Financing at Long Time Horizons

Alfia Vasilieva

Associate Director of the Project and Structured Finance Ratings Group,
Analytical Credit Rating Agency (ACRA), Moscow, Russia,
alfiavaf@mail.ru, [ORCID](#)

Abstract

Project financing is one of the priority tools for stimulating the country's economic growth around the world, which allows the implementation of large-scale and capital-intensive projects, providing favorable credit conditions with insufficient creditworthiness of the project beneficiaries.

As a rule, project financing instruments are long-term (10–30 years, depending on the type of transaction), so this asset class is interesting for the implementation of the task of building long-term models for assessing credit risk associated with the introduction in 2018 of the new international financial reporting standard IFRS 9 “Financial Instruments”.

The new standard requires financial institutions to calculate their expected credit loss (ECL) at the time of granting loans and other banking products exposed to credit risk, taking into account different time horizons, which significantly changes the traditional approaches to assessing credit risk by commercial banks.

As part of this work, a model was built to assess the long-term probability of default for the portfolio of assets of a Russian commercial bank belonging to the project finance segment in accordance with the requirements of the International Financial Reporting standard IFRS 9 “Financial Instruments”. At present, the topic of this work is extremely relevant and may be of interest both for commercial banks that are faced with the problem of improving credit risk assessment models

Keywords: IFRS 9, expected credit losses, credit risk assessment stages, project finance, pdlife-time

For citation: Vasilieva, A. Approaches to Building Default Probability Models for Financial Instruments of Project Financing at Long Time Horizons. *Korporativnye finansy = Journal of Corporate Finance Research*.2021;15(4): 66-87. doi: <https://doi.org/10.17323/j.jcfr.2073-0438.15.4.2021.66-87>

The journal is an open access journal which means that everybody can read, download, copy, distribute, print, search, or link to the full texts of these articles in accordance with CC Licence type: Attribution 4.0 International (CC BY 4.0 <http://creativecommons.org/licenses/by/4.0/>).

Introduction

Project financing (PF) is a method of long-term borrowing for large projects by means of “financial engineering” based on loan on cash flow created only by the project without recourse to the borrower. The fundamental feature of project financing is that for implementation of a certain project a special project enterprise is established (SPV, SPE) which attracts resources (not only funds) for project implementation, implements the project and squares accounts with creditors and project investors using the funds (cash flows) generated by the project itself [1]. For decades project financing was a preferable way of financing of large-scale infrastructure projects all over the world. A series of studies emphasized its importance, especially for the countries with emerging market economies and accentuated the correlation between investment in infrastructure and economic growth.

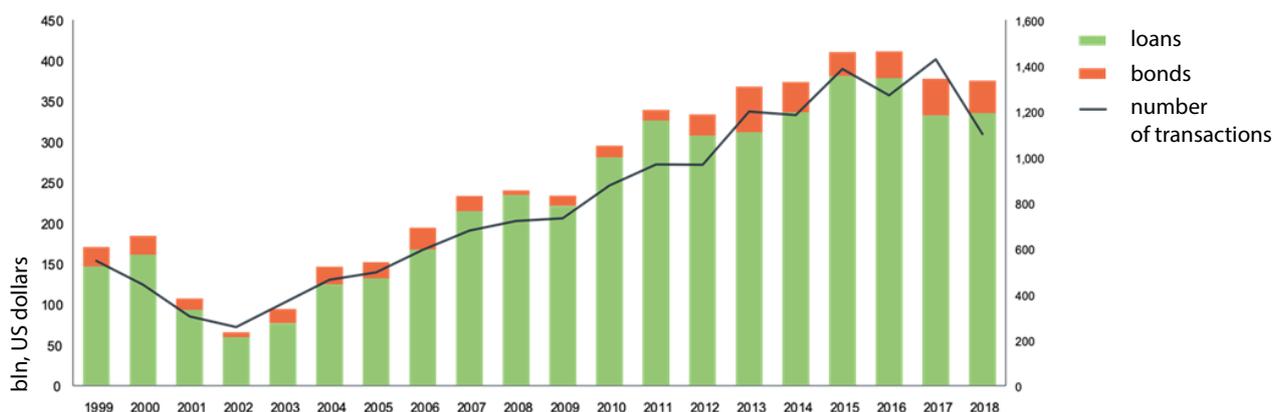
Long-termness is a distinctive feature of project financing transactions – implementation of some projects takes 30 years. Large-scale capital-intensive projects usually require significant initial investments and generate the revenue sufficient only to cover expenses in the long-term. Thus, some authors point out in their papers that on average

loans aimed at financing projects have a longer maturity than other syndicated credits [5–7]. This distinctive feature makes this assets class an interesting research object as a part of the task of constructing long-term models of assessing credit risk according to requirements of IFRS 9.

In the recent 15–20 years investors’ interest to transactions of project financing has been growing across the globe. First of all, it is due to financing of infrastructure facilities’ construction under public-private partnership (PPP): from 1999 through 2019 the volume and number of performed transactions increased more than twice (Figure 1). Projects are increasingly financed by issue of bonds backed by cash-flows from implementation of infrastructure projects based on PPP. According to international practices default of this tool occurs less frequently even in economic turbulence periods [8].

As on September 2020 over 3,000 PPP projects were implemented in Russia. Their total value exceeds 4.5 trillion roubles and the share of private investments amounts to 3.1 trillion roubles (69%). The total value of PPP projects amounts to 44% of expenses for infrastructure which have been planned for implementation of national projects in 2019 [8].

Figure 1. Global Volume of the Project Financing Market



Source: Moody’s, ACRA.

Review of Methods and Models of Assessment of Credit Risk of Assets at a Long Time Horizon

On the basis of PD estimates MPD values are calculated for each life period of an agreement. Then they are used to calculate ECL for the whole lifetime of the agreement.

The model is calibrated on the basis of the PIT (point in time) principle:

$$PD_t^{PIT} = \beta_t \cdot PD_t^{TTC}, \quad (1)$$

where PD_t^{PIT} – default probability on the basis of the PIT calibration;

PD_t^{TTC} – default probability on the basis of the TTC calibration;

β_t – scaling coefficient defined on the basis of data about the current default rate of the portfolio

The following methods are applied to evaluate PD:

- Use of external data about defaults;
- Methods based on the migration matrix;
- Methods based on approximation of historic default levels;
- Approach based on exponential curve extrapolation (a simplified approach).

Use of external data on defaults. This method implies a PD assessment based on migration of ratings information on which is provided by external rating agencies (S&P, Moody’s, Fitch Ratings, ACRA).

In case the Bank has no statistics to build a migration matrix using internal data the migration matrix built on external data is used. Depending on the purpose of modelling statistics of one or several rating agencies may be applied.

In case of inversion in the data of external matrices the matrix is adjusted (by experts or applying mathematical methods of reducing the function to a monotone function) PD assessment on the basis of migration matrices. The migration matrix is a square matrix which elements contain probabilities of change (transition probabilities) of the rating category of a corresponding Borrower.

$$M = \begin{bmatrix} p_{11} & \dots & p_{1,n} \\ \dots & \dots & \dots \\ p_{n-1,1} & \dots & p_{n-1,n} \\ 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

where p_{ij} – probability of transition to the j rating category in a certain period of time provided it belongs to the i rating category.

The Bank uses the rating scale of internal credit ratings to build the migration matrix (Appendix 1).

The Bank does not set upper and lower bounds of the values of the probability of default. According to IFRS 9 assessment of the probability of default is unbiased. Subsequently, the conservatism concept enshrined in the model of probability of default assessment as per IRB Basel II [10] cannot be used to calculate PD in accordance with IFRS 9 and as part of upgrade of IRB of PD models in order to meet requirements of IFRS 9 such material adjustments are excluded (adjustment “PG not less than 0.03%” established in accordance with 483-P is also excluded) [4].

This is with the exception of the RF rating adjustment (the borrower’s rating is not higher than the RF rating): this adjustment is preserved.

Depending on availability of data when constructing a migration matrix enlarged or initial rating categories may be used (for example, combining of ratings 7-, 7, 7+ into one category).

Assessment of probabilities of transition is determined by cohort analysis:

$$\hat{p}_{ij} = \frac{N_{ij}(t)}{N_i(t-1)}, \quad (2)$$

where $N_{ij}(t)$ – number of migrations from state I into state j in the t period;

$N_i(t-1)$ – number of transactions in state I in the $t-1$ period.

The probability of default at a 1-year horizon.

A one-year migration matrix M_0 is constructed on the basis of statistics of observations for 12 calendar months. Shorter periods may be used in order to take into consideration the most relevant information.

An average one-year migration matrix is calculated by computing the arithmetic mean of one-year migration matrices obtained every quarter (month).

A one-year probability of default (PD_t) for each rating category is defined as the likelihood of transition into the

state of “10-default”. In the migration matrix (PD_t) is indicated in the last column of the one-year transition matrix.

If statistical frequency of defaults does not correspond to the probability of default in each rating grade in the Bank’s master scale scaling is performed.

The adjustments performed are recorded in the Report on the Model Development.

When assessing the PD indicator on the basis of migration matrices the following main assumptions are contemplated:

- further transitions to rating grades depend only on the current rating but not on previous ratings (property of Markov process);
- probabilities of migration do not depend on a certain time point, i.e. the transition rates are unchanged in time (homogeneity) [9].

A formula to calculate the probability of default for the lifetime of a financial instrument:

$$M_T = M_1^T, \quad (3)$$

where T – lifetime of a financial instruments.

The column in the multiyear matrix which shows a probability of transition into default is the cumulative probability of default of a certain period (cPD). Use of the migration matrix allows to take into consideration complete information on migration of ratings when calculating the probability of default for the lifetime.

Profiles of cumulative PDs are made by evaluating parameters of cumulative DR distribution.

On the basis of the Weibull distribution:

Parameters of the Weibull distribution k and λ are assessed using a linear regression of double logarithm of the survival function. The survivorship function is defined by the following formula:

$$S(t) = 1 - F(t; k, \lambda), \quad (4)$$

where $F(t; k, \lambda)$ is a two-parameter Weibull distribution function.

$$F(t; k, \lambda) = cDR(t; k, \lambda) = \begin{cases} 1 - e^{-\left(\frac{t}{\lambda}\right)^k}, & t > 0 \\ 0, & t \leq 0. \end{cases}, \quad (5)$$

where $k > 0$ defines the shape of the distribution function. $k < 1$ is indicative of a decrease of default rate in time, $k = 1$ points at stability of default rate in time, $k > 1$ is indicative of increase of default rate in time;

$\lambda > 0$ is a scale parameter which regulates survivorship time [11].

On the basis of the modified Weibull distribution:

- Modeling of cumulative PD is made by selecting such distribution parameters which describe most accurately the behaviour of cumulative default rates.

A two-parameter modified Weibull distribution function is as follows:

$$F(t, \alpha, \beta) = cDR(t, \alpha, \beta) = \begin{cases} \frac{1 - e^{-\left(e^{-\alpha t^\beta}\right)}}{(1 - e^{-1})}, & t > 0, \\ 0, & t \leq 0 \end{cases} \quad (6)$$

where α and $\beta < 0$ are parameters of the modified Weibull distribution;

$cDR(t, \alpha, \beta)$ is the cumulative default rate per year [11].

Construction of the model for the project financing segment

The target segment of this model is customers pertaining to the area of applying the Project Financing models in accordance with the bank's methodology.

When developing of the present model the default definition stated in the section Terms and Definitions was used. The fact of assigning to a borrower of the "10-default" rating was taken into consideration as default characteristic.

At the date of developing the model Lt PD of the Project Financing segment 1,171 observations were available (key "TIN + reporting date", 325 default observations) for 116 borrowers since April 2009. In the segment of Project Financing (Developers) 1,334 observations were available (key "TIN + reporting date", 565 default observations) for 141 borrowers since April 2009. Moreover, the sample contains observations concerning 15 borrowers pertaining to both segments.

In view of the above the approach to modeling of Lt PD on the basis of the migration matrices constructed of consolidated internal data on changing ratings for both segments was considered for the segments of Project Financing and Project Financing (Developers).

When combining rating groups the number of observations in a rating group amounted from 292 to 890 observations (the largest number of observations was in the "default" rating group and the smallest number of observations was in "good" rating groups). As long as it is important for the segments with the borrowers characterized by credit risk above average to have a number of observations in "bad" ratings sufficient for rating migration analysis we made the conclusion on applicability of the approach to modeling of Lt PD by means of constructing the rating migration matrix on the basis of internal data.

Table 1. Values before adjustment

	1	2	3	4	5	Default
...
2	15.75%	78.91%	4.94%	0.40%	0.00%	0.00%
...

The approach to getting multiyear PD using rating migration matrices. Computation of an average one-year migration matrix.

The migration matrix is indicative of the likelihood that a borrower with a certain rating as at the beginning of the year will have the following:

- the same rating (shown on the principal diagonal);
- rating with a higher probability of default (in the migration matrix such values are indicated above the principal diagonal);
- rating with a lower probability of default (in the migration matrix such values are indicated under the principal diagonal);
- rating of default state (in the migration matrix such values are indicated in column D, default).

When computing a one-year rating migration matrix using the data for a period exceeding 1 year probabilities of migration of the final one-year migration matrix are obtained by averaging probabilities of migration of several matrices. Averaging is performed by calculating the arithmetic mean.

A one-year migration matrix with averaged probabilities of migration is applied as the basic matrix to compute multi-year matrices.

When empiric default rates deviate from model ones (PD of the master scale) PD of the basic migration matrix (the last column – an average one-year default rate, DR) is adjusted to harmonize with PD of the Bank's master scale.

PD adjustment of the basic migration matrix is also necessary in case of inversions (PD of "bad" ratings is lower than PD of "good" ratings).

Adjustment may be performed both by means of calculating the coefficient by which the actually obtained DR is to be multiplied / divided and by means of permutation of PD of a corresponding master scale into the last column of the basic migration matrix. If the basic migration matrix is constructed on the basis of rating groups the weighted average of the number of observations in the rating group of PD of the master scale is calculated for the last column.

After adjustment of the values of the last column the transition probabilities of the basic matrix are adjusted in such a way that the sum of probabilities of transition of each line was 100% (by proportional change of probabilities of rating transitions of each line).

Adjustment example

Table 2. Values after adjustment

	1	2	3	4	5	Default
...
2	15.55%	77.89%	4.88%	0.39%	0.00%	1.29%
...

Sum of probabilities of transition (except for default) before adjustment = 100%, sum of probabilities of transition after adjustment (except for default) = 98.71%.

Adjusted probability of transition from rating 2 to rating 1 is calculated by the following formula:

$$15.55\% = 15.75\% \cdot (98.71\% / 100\%).$$

Other probabilities in the line are adjusted in the same way.

Calculation of cumulative PD estimates

Estimates of cumulative PD are obtained using migration matrices by means of raising a one-year migration matrix to the corresponding power. For example, in order to get a cumulative PD for N years it is necessary to raise the matrix to the N power in accordance with the Formula (1):

$$M = M_0^N, \quad (7)$$

where M is a migration matrix for N years;

M_0 – a one-year migration matrix.

The column in the multiyear matrix indicating a probability of transition to the default state is the cumulative probability of default of a corresponding period (cPD).

Advantages of use of a complete migration matrix as opposed to use of just the column indicating the probability of default when calculating the cumulative probability of default consist in recording of complete information on rating migration when the probability of default is calculated for several years [12–15].

Adjustment of Probabilities of Transition of Ratings of the Basic Migration Matrix

A probability of rating transition should decrease monotonically when moving from the principal diagonal to extreme columns of the migration matrix. It means that the probability of transition of ratings to neighboring rating groups is higher in comparison to the probability of rating transition “skipping” 2 or 3 ratings.

Probabilities of transition are adjusted using mathematical methods (for example, approximation of nonmonotonic data series by a monotone function).

In practice parameters of the function (exponential, logarithmic) used for adjustment may be selected applying Excel (“search for solution” package, trend adding).

After obtaining the cumulative probability of default for consolidated rating groups it is converted into the conditional probability of default to calculate conditional PD

for each rating inside rating groups applying logarithmic interpolation.

On the basis of the conditional PD obtained at the previous stage final marginal PD is calculated (exclusive of forecasting information).

The above transitions of the probability of default profiles are performed by the following formulae.

The cumulative PD is determined as:

$$cPD_t = \begin{cases} cPD_{t-1} + (1 - cPD_{t-1}) \cdot PD_t, & t > 0 \\ 0, & t = 0. \end{cases} \quad (8)$$

The marginal PD is determined as:

$$MPD_t = PD_t \cdot (1 - cPD_{t-1}) = cPD_t - cPD_{t-1}. \quad (9)$$

Due to a non-linear character of change of PD ratings when moving along a rating scale it is not recommended to apply the linear approach to interpolation. See below the approach to interpolation which takes into consideration the non-linear character of PD.

Interpolation consists of several main stages.

1st stage.

Each rating is assigned a numerical value (Table 3).

Table 3. Numerical values of ratings

Rating	Numerical value
1+	1
1	2
1–	3
2+	4
2	5
2–	6
3+	7
3	8
3–	9
4+	10
4	11
4–	12
5+	13
5	14

Rating	Numerical value
5-	15
6+	16
6	17
6-	18
7+	19
7	20
7-	21
8+	22

Rating	Numerical value
8	23
8-	24
9	25
10	26

2nd stage.

Average-weighted ratings and PD corresponding to them expressed in terms of numerical values are calculated. Weights are the number of observations in each corresponding rating (Table 4).

Table 4. Example of calculated actual ratings and cPD corresponding to them

Actual average numerical value of the rating group	PD-1	PD-2	PD-3	PD-4	PD-5	PD-6
19.7	11.74%	11.10%	10.88%	10.53%	10.11%	10.11%
23.9	31.97%	29.02%	25.34%	21.64%	18.22%	18.22%

Table 5. cPD of the “8+” rating obtained by interpolation

Numerical value of the rating	PD-1	PD-2	PD-3	PD-4	PD-5	PD-6
22	19.37%	18.69%	17.21%	15.56%	13.91%	13.91%

3rd stage. PD is calculated for each rating on the basis of corresponding PD of rating groups.

For the *j* rating $PD_{j,t}$ for the period of *t* ($t \in [1, N]$, $t \in \mathbb{N}$) is calculated as per the following formula:

$$PD_{j,t} = PD_{i,t} \cdot \frac{PD_{i+1,t} \left(\frac{b_j - a_i}{a_{i+1} - a_i} \right)}{PD_{i,t}}, \quad (10)$$

where b_j is a numerical value of the *j* rating which is between the numerical values of the rating groups *i* and *i+1*;

a_i, a_{i+1} are numerical values of rating groups *i* and *i+1* respectively;

$PD_{i,t}, PD_{i+1,t}$ are conditional probabilities of default calculated applying migration matrices for the *t* period for rating groups *i* and *i+1* respectively (Table 5).

At the next stage adjustment is performed: PD of the first year is set equal to PD of the master scale.

Forecasting macroeconomic information is taken into consideration by adjusting conditional PD of the 1st and 2nd year with consideration to the forecasted one-year default rate for 2 years (2 values of DR) from the date of reports.

TTC PD is the probability of default average for the whole economic cycle which assessment is based on all information available about the borrower. TTC PD is stable in time

and is not correlated to the economic cycle. The calculated transition probability and default probability obtained after multiplication of the one-year migration matrix according to art. 7.1.3 are mean values computed on the basis of rating results for approximately 8 years (01.04.2009–01.07.2017) which covers various stages of the economic cycle, i.e. they are TTC PD estimates.

In order to take into consideration forecasting macroeconomic information it is necessary to adjust estimates of TTC PD ratings obtained for the model with consideration to the forecasted default rate.

PIT calibration is performed on the basis of Bayes’ formula where the rating PD is scaled according to the forecasted default rate and CDT.

In order to convert one-year PD values Bayes’ formula is applied.

$$PD_i^{New} = \frac{(1 - CDT) * DR_{New} * PD_i}{CDT * (1 - DR_{New}) * (1 - PD_i) + (1 - CDT) * DR_{New} * PD_i}, \quad (11)$$

where PD_i^{New} – PIT PD of the *i* rating which corresponds to a new forecasting default rate DR_{New} ;

PD_i – conditional PD of rating grade *i*;

DR_{New} – forecasting default rate;

CDT – average default rate calculated by the economic cycle.

The data source for developing the model is the corporate data warehouse (area of corporate data warehouse which stores the information of the data warehouse of CRM) with a set of presentations which contain data from various points of view (loan portfolio, agreement information etc.).

Data from the segments of Project Financing and Project Financing (Developers) was used for analysis.

Analysis comprises 114 customers from the model of Project Financing (994 observations, 315 of them are default ones) and 128 customers from the model of Project Financing (Developers) (1,082 observations, 506 of them are default ones).

Data from the segments of Project Financing and Project Financing (Developers) was used for analysis (hereafter – the PF portfolio). Due to a significant reduction in the number of borrowers in the PF portfolio in 2015–2017 in comparison to 2009–2013 we decided to analyze the PF portfolio as one segment without division into PF and PF (Developers).

See detailed information on the model in the Table 6. The first 4 lines are the models developed after October 2011, the last 2 lines are the models developed before October 2011 (in 2009).

Table 6. Number of unique records with a breakdown into models (PF portfolio)

Grouping of observations

No.	Model	Number of records
	Project Financing (Developers) v.2.0 (archive)	625
	Project Financing v.2.0 (archive)	527
	Project Financing (Developers) v.2.1	69
	Project Financing v.2.1	55
	Project Financing (Developers) v.1 (archive)	640
	Project Financing v.1 (archive)	563

Due to insufficient number of observations for individual ratings we made rating groups of 1 to 9 ratings in order to construct migration matrices.

When making rating groups we took into consideration the following:

- rating groups comprise ratings close in terms of risk level;
- the number of observations in a rating group should be sufficient to model probabilities of ratings transition (Table 7).

Table 7. Number of observations related to grades/groups

Rating group	Rating grade	Number of observations in a rating grade	Number of observations in a rating group
	1+	–	
	1	–	
	1–	–	
	2+	–	
	2	–	
345	2–	–	330
	3+	–	
	3	1	
	3–	–	
	4+	9	
	4	12	

Rating group	Rating grade	Number of observations in a rating grade	Number of observations in a rating group
345	4–	38	330
	5+	91	
	5	97	
	5–	82	
6	6+	139	401
	6	165	
	6–	97	
7	7+	115	292
	7	110	
	7–	67	
89	8+	57	593
	8	266	
	8–	73	
	9	197	

Analysis of data in terms of assigning to a stage of the project

As a part of model development we studied existence of dependence between the default rate and a project stage. When assessing borrowers' projects assigned to the segments of Project Financing and Project Financing (Developers) in the Bank the basic rating defining module comprises the factor of the Project Stage. Since the wording of factors changed in re-development of models we decided

to combine the observations data in three groups: 1) A (initial financing); 2) B (work performance); 3) C (completion) (Table 8).

Analysis showed that over 65% observations pertain to the C stage (completion) and 18% – to the B stage (work performance). Apart from that, 84.3% of projects were at the completion (C) stage out of 312 observations as at 2015–2017 (Table 9).

Table 8. Description of values of the factor of the Project Stage model depending on the model version

Project version	Values of the factor of the Project Stage model	Designation of the factor of the Project Stage model
Project Financing v.1	A: Initial financing – <idea> level	A
	B: Preparation and beginning of work – <planning> level	
	C: Work performance	B
	D: Ready business – <completion> level	C
Project Financing v.2	A: Initial financing – <idea> level	A
	B: Preparation and beginning of work – <planning> level	
	C: Work performance	B
	D: Ready business – <completion> level	C

Project version	Values of the factor of the Project Stage model	Designation of the factor of the Project Stage model
Project Financing (Developers) v.1	A: Construction "from greenfield"	A
	B: Ditch / foundation	B
	C: House case constructed	
	D: Finishing	C
	E: Finishing constructing / restructuring of an existing building	
	F: Repairs of an existing building for further use	
Project Financing (Developers) v.2	Development project's stage (in percent)	
	< 20%	A
	> 20% < 70%	B
	> 70%	C

Table 9. Number of observations by the type of the project stage

Project stage	Number of observations	Share of observations in the total amount, %
A	134	5.6
B	459	18.3
C	1,632	65.2
(Empty)	280	11.2
Total	2,505	100

On the basis of the results of studies we took the decision not to divide the initial sample into project stages and not to determine individual models for various stages for the following reasons:

- 1) The major part of observations pertain to the stage of project completion (65%).
- 2) The rating calculated on the basis of the one-year default probability model takes into consideration the fact of project affiliation to a certain stage.
- 3) A single sample will allow to develop a stable PD Lifetime model.

Results of modeling of TTC LT PD before taking into consideration forecasting macroeconomic information

Basic Prerequisites

When constructing one-year migration matrix we adopted the following prerequisites:

- default is an absorbing state, i.e. getting out of the default state is not considered;
- in case of several ratings calculated on the basis of the same reports we used for calculation the rating with the last date of calculation;
- within the period (one year) we eliminated migrations into the state of "no rating (no re-rating)", i.e. if as at the beginning of the considered period a customer was assigned a rating and at the end of the year there was no information on the calculated rating such rating was considered in the calculation as remaining in the same rating. The prerequisite was introduced to meet the modeling purposes – the event of "no re-rating" was not simulated, change of the rating while the borrower is in the Bank's portfolio is simulated.
- assigning of "10" rating to the borrower was considered as an event of default (provided it did not equal 10 as at the previous date).

Average (Basic) One-Year Migration Matrix

Calculation of the basic migration matrix

One-year matrices were calculated as follows:

- a one-year probability that a borrower with a certain rating as at the beginning of the year will in one year have the same or a different rating was calculated per quarters for a one-year interval (one-year matrices were calculated per quarters);
- data was analyzed from 01.04.2009 to 01.07.2017 (30 matrices in total);
- the matrix obtained by averaging of 30 matrices was taken as the basic one-year matrix;

- for the PF portfolio the data was combined in rating groups 345 (3, 4+, 4, 4-, 5+, 5, 5-), 6 (6+, 6, 6-), 7 (7+, 7, 7-), 89 (8+, 8; 8- and 9). It was necessary to consolidate ratings into groups due to an insufficient number of observations in individual rating grades (Table 10).

Adjustment of the Last Column of the Basic Migration Matrix

Due to an insufficient number of default observations we adjusted probabilities of default on the basis of weighted PD of corresponding ratings in the master scale of the Bank. PD are weighted by the number of observations in each rating (Table 11 and 12).

Table 10. Average one-year migration matrix

Risk category	345	6	7	89	10
345	77.6%	13.5%	4.4%	1.3%	3.2%
6	18.3%	41.5%	23.9%	9.8%	6.5%
7	2.8%	13.7%	45.6%	30.5%	7.3%
89	3.7%	2.0%	6.6%	72.5%	15.2%
10	0.0%	0.0%	0.0%	0.0%	100.0%

Table 11. Weighted PD for rating groups

Rating group	Rating grade	Number of observations in the rating grade	Rating PD	PD* number of questionnaires	PD of the rating group
345	1+	–	0.01%	–	
	1	–	0.02%	–	
	1–	–	0.04%	–	
	2+	–	0.08%	–	
	2	–	0.16%	–	
	2–	–	0.32%	–	
	3+	–	0.45%	–	
	3	1	0.58%	0.006	2.42%
	3–	–	0.75%	–	
	4+	9	0.96%	0.086	
	4	12	1.23%	0.148	
	4–	38	1.58%	0.600	
	5+	91	2.03%	1.847	
	5	97	2.61%	2.532	
	5–	82	3.36%	2.755	

Rating group	Rating grade	Number of observations in the rating grade	Rating PD	PD* number of questionnaires	PD of the rating group
6	6+	139	4.31%	5.991	5.50%
	6	165	5.54%	9.141	
	6-	97	7.12%	6.906	
7	7+	67	9.14%	10.511	11.48%
	7	110	11.74%	12.914	
	7-	115	15.08%	10.104	
89	8+	73	19.37%	11.041	30.60%
	8	266	24.89%	66.207	
	8-	57	31.97%	23.338	
	9	197	41.06%	80.888	

Table 12. Basic one-year migration matrix (after reducing the master scale to PD)

Risk category	345	6	7	89	10
345	78.3%	13.6%	4.4%	1.3%	2.4%
6	18.5%	42.0%	24.2%	9.9%	5.5%
7	2.7%	13.1%	43.6%	29.2%	11.5%
89	3.1%	1.6%	5.4%	59.3%	30.6%
10	0.0%	0.0%	0.0%	0.0%	100.0%

Reducing the Basic Migration Matrix to the Monotone Type

The basic matrix is reduced to the monotone type against the principal diagonal by using smoothing functions. Values of transition probability are adjusted line-by-line except for the values in the last column and the principal diagonal.

In order to eliminate zero probabilities of transition and non-monotonic values above the principal diagonal we used the decreasing function $y = a \cdot \exp(-b \cdot t)$. Its parameters were selected applying the Search for Solution package in Excel (Table 13).

Table 13. The basic one-year migration matrix (after reducing to PD of the master scale and reducing to the monotone type)

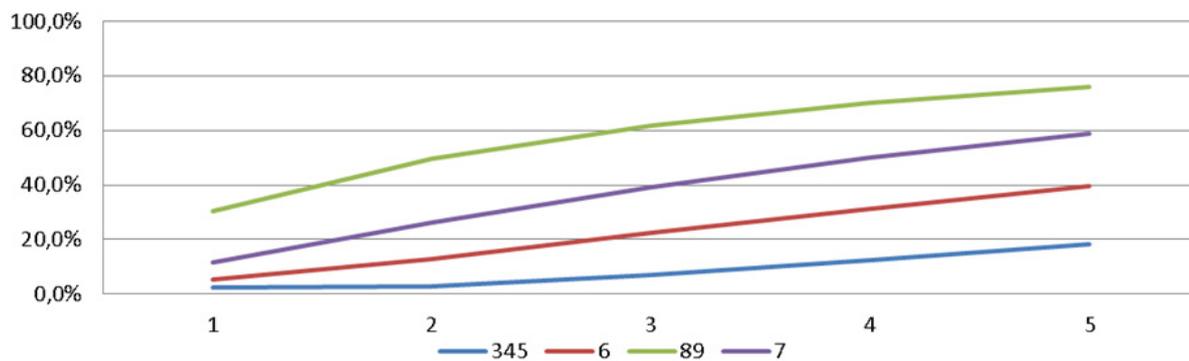
Risk category	345	6	7	89	10
345	78.3%	13.6%	4.3%	1.4%	2.4%
6	18.5%	42.0%	24.2%	9.9%	5.5%
7	0.0%	15.7%	43.6%	29.2%	11.5%
89	2.0%	3.1%	5.0%	59.3%	30.6%
10	0.0%	0.0%	0.0%	0.0%	100.0%

Results of calculation of cumulative PD

The cumulative PDs were calculated by raising to power of the adjusted basic one-year migration matrix (Table 14 and Figure 2). Multiyear matrices were calculated for the period not exceeding 5 years.

Table 14. The cumulative probability of default

Risk category	1 st year	2 nd year	3 rd year	4 th year	5 th year
345	2.4%	6.0%	10.8%	16.7%	23.2%
6	5.5%	14.0%	23.7%	33.1%	41.6%
7	11.5%	26.3%	39.6%	50.5%	59.2%
89	30.6%	49.6%	61.9%	70.2%	76.2%
10	100.0%	100.0%	100.0%	100.0%	100.0%

Figure 2. Cumulative probabilities of default calculated on the basis of the basic matrix after adjustments

Reducing to the Master Scale, Results' Interpolation

Cumulative probabilities of default were transformed into conditional probabilities of default in accordance with dependence [12] for further reducing of TTC PD for the first year to the Bank's master scale and calculation of the conditional PD for each rating inside rating groups by means of logarithmic interpolation.

$$PD(t) = \frac{cPD(t) - cPD(t-1)}{1 - cPD(t-1)}. \quad (12)$$

The conditional PDs for rating categories 345, 6, 7, 89 were calculated by means of logarithmic interpolation.

The conditional PDs for ratings 1+, 1, 1-, 2+, 2, 2-, 3+, 3, 3- were fixed at the Bank's master scale level.

Table 15. The conditional probabilities of default obtained after reducing to the master scale and interpolation

Risk category	1 st year	2 nd year	3 rd year	4 th year	5 th year
1+	0.01%	0.01%	0.01%	0.01%	0.01%
1	0.02%	0.02%	0.02%	0.02%	0.02%
1-	0.04%	0.04%	0.04%	0.04%	0.04%
2+	0.08%	0.08%	0.08%	0.08%	0.08%
2	0.16%	0.16%	0.16%	0.16%	0.16%
2-	0.32%	0.32%	0.32%	0.32%	0.32%
3+	0.45%	0.45%	0.45%	0.45%	0.45%
3	0.58%	0.58%	0.58%	0.58%	0.58%
3-	0.75%	0.75%	0.75%	0.75%	0.75%
4+	0.96%	1.06%	1.15%	1.21%	1.26%

Risk category	1 st year	2 nd year	3 rd year	4 th year	5 th year
4	1.23%	1.51%	1.76%	1.96%	2.11%
4-	1.58%	2.14%	2.70%	3.17%	3.53%
5+	2.03%	3.65%	5.17%	6.58%	7.76%
5	2.61%	4.16%	5.77%	7.20%	8.32%
5-	3.36%	5.44%	7.28%	8.67%	9.63%
6+	4.31%	7.11%	9.17%	10.44%	11.15%
6	5.54%	9.05%	11.28%	12.33%	12.72%
6-	7.12%	11.40%	13.47%	14.24%	14.35%
7+	9.14%	14.04%	15.81%	16.23%	16.02%
7	11.74%	16.72%	18.07%	18.09%	17.56%
7-	15.08%	19.39%	19.79%	19.18%	18.23%
8+	19.37%	22.02%	21.40%	20.17%	18.83%
8	24.89%	27.31%	24.42%	21.96%	19.89%
8-	31.97%	28.40%	25.02%	22.30%	20.09%
9	41.06%	32.26%	27.05%	23.45%	20.76%

Marginal PDs were calculated on the basis of the conditional PDs (Table 15).

For ratings 1+, 1, 1-, 2+, 2, 2-, 3+, 3, 3- mPD were fixed at the Bank's master scale level.

For ratings 7, 7-, 8+, 8, 8- and 9 values of marginal PDs were adjusted in order to eliminate intersections (mPD should not decrease when the rating moves from 1+ to 9).

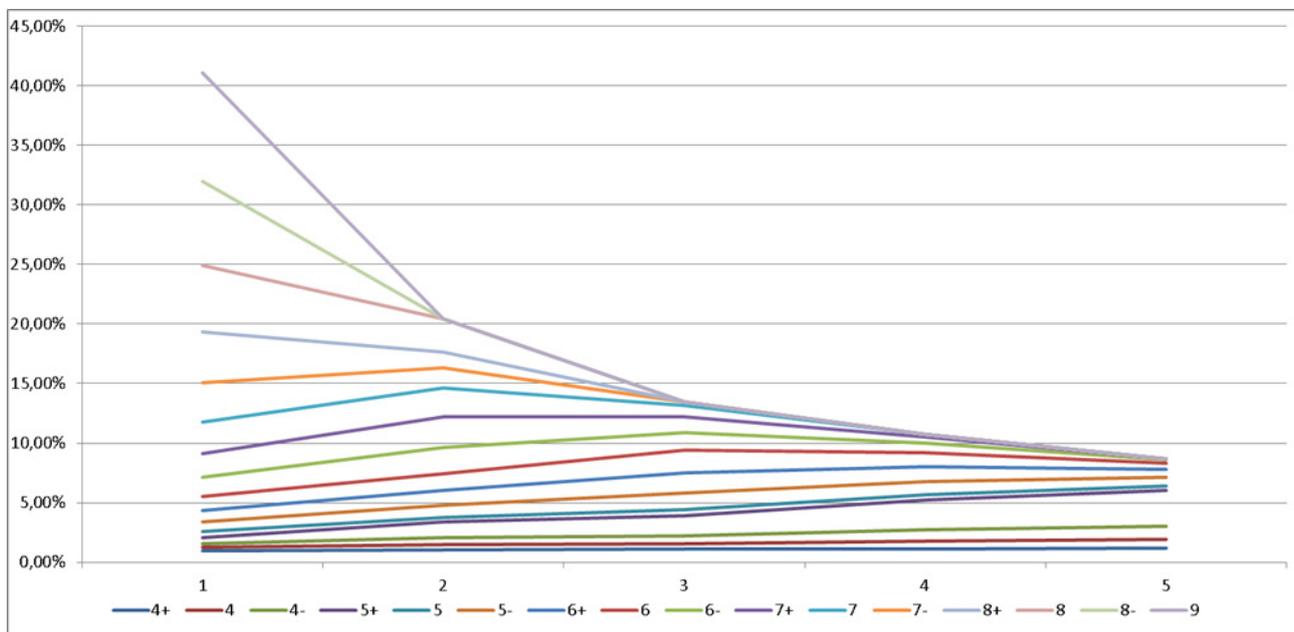
The final values of mPD (without regard to forecasting information) are presented in Table 16 and Figure 3.

Table 16. Marginal TTC profiles of multiyear default probabilities for the PF portfolio (adjusted)

Risk category	1 st year	2 nd year	3 rd year	4 th year	5 th year
1+	0.01%	0.01%	0.01%	0.01%	0.01%
1	0.02%	0.02%	0.02%	0.02%	0.02%
1-	0.04%	0.04%	0.04%	0.04%	0.04%
2+	0.08%	0.08%	0.08%	0.08%	0.08%
2	0.16%	0.16%	0.16%	0.16%	0.16%
2-	0.32%	0.32%	0.32%	0.32%	0.32%
3+	0.45%	0.45%	0.45%	0.44%	0.44%
3	0.58%	0.58%	0.57%	0.57%	0.57%
3-	0.75%	0.74%	0.74%	0.73%	0.73%
4+	0.96%	1.05%	1.13%	1.17%	1.20%
4	1.23%	1.49%	1.71%	1.87%	1.97%

Risk category	1 st year	2 nd year	3 rd year	4 th year	5 th year
4-	1.58%	2.11%	2.60%	2.97%	3.21%
5+	2.03%	3.58%	4.88%	5.89%	6.49%
5	2.61%	4.05%	5.39%	6.33%	6.79%
5-	3.36%	5.25%	6.65%	7.34%	7.46%
6+	4.31%	6.81%	8.15%	8.43%	8.06%
6	5.54%	8.55%	9.69%	9.40%	8.50%
6-	7.12%	10.58%	11.08%	10.14%	8.77%
7+	9.14%	12.76%	12.35%	10.67%	8.82%
7	11.74%	14.75%	13.28%	10.90%	8.82%
7-	15.08%	16.46%	13.55%	10.90%	8.82%
8+	19.37%	17.75%	13.55%	10.90%	8.82%
8	24.89%	20.51%	13.55%	10.90%	8.82%
8-	31.97%	20.51%	13.55%	10.90%	8.82%
9	41.06%	20.51%	13.55%	10.90%	8.82%

Figure 3. The marginal default probabilities (adjusted) for the PF portfolio



Comparative analysis of the results of development of the basic model for the Construction and Rental Business segments with the results of development of the basic model for project financing

Comparison of obtained mPD estimates for project financing (exclusive of forecasting information) to estimates for the Construction and Rental Business segments revealed

that for “good” rating grades (3+, 3, 3-, 4+, 4, 4-, 4+, 4, 4-, 5+, 5, 5-) the obtained estimates are better (the probability of default is lower) than for the Construction and Rental Business segments. This may be due to the fact that the PF portfolio contains a third less observations in “good” rating grades (Table 17).

For “average” and “bad” rating grades the obtained estimates are a little worse (the probability of default is higher) than for the Construction and Rental Business segments. It should be noted that the PF portfolio contains 2.5 times

as much observations in “bad” rating grades (8+, 8, 8–, 9) as in the Construction and Rental Business segments. It is important to take this feature into consideration when assessing the final mPD value in order to calculate ECL.

Table 17. Number of observations for rating grades

Rating grades	Number of observations for the PF portfolio	Number of observations for the Construction and Rental Business segments
2–, 3+, 3, 3–, 4+, 4, 4–, 4+, 4, 4–, 5+, 5, 5–	274	979
6+, 6, 6–	278	457
7+, 7, 7–	228	258
8+, 8, 8–, 9	475	178
10	821	414
Total	2,076	2,286

The effect of adding the obtained mPD estimates of the PF portfolio (instead of the estimates for the Construction and Rental Business segments) to calculation of ECL for the corporate portfolio as of 01.01.2018 amounted to 0.9 million roubles (or + 0.1 %).

Thus, the computed estimates do not lead to overvaluation of the ECL amount and show the specific character of the PF portfolio in the best way. Therefore, they should be used in ECL calculation in the Bank.

Adjustment of one-year PD values taking into consideration a Macro forecast

Table 18 represents the final one-year conditional PDs which indicate the probability of default taking into consideration influence of macroeconomic information.

Table 18. The final conditional PDs which indicate the probability of default taking into consideration influence of macroeconomic information (PF portfolio)

Scale	PD TTC	Forward PD (PD PIT for the 1 st and 2 nd year, PD TTC for the 3 rd to 5 th year)				
		1 st year	2 nd year	3 rd year	4 th year	5 th year
1+	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
1	0.02%	0.01%	0.02%	0.02%	0.02%	0.02%
1–	0.04%	0.03%	0.04%	0.04%	0.04%	0.04%
2+	0.08%	0.06%	0.08%	0.08%	0.08%	0.08%
2	0.16%	0.12%	0.16%	0.16%	0.16%	0.16%
2–	0.32%	0.24%	0.31%	0.32%	0.32%	0.32%
3+	0.45%	0.33%	0.44%	0.45%	0.45%	0.45%
3	0.58%	0.43%	0.57%	0.58%	0.58%	0.58%
3–	0.75%	0.55%	0.74%	0.75%	0.75%	0.75%
4+	0.96%	0.71%	1.05%	1.15%	1.21%	1.26%
4	1.23%	0.91%	1.49%	1.76%	1.96%	2.11%
4–	1.58%	1.17%	2.11%	2.70%	3.17%	3.53%

Scale	PD TTC	Forward PD (PD PIT for the 1 st and 2 nd year, PD TTC for the 3 rd to 5 th year)				
		1 st year	2 nd year	3 rd year	4 th year	5 th year
5+	2.03%	1.50%	3.60%	5.17%	6.58%	7.76%
5	2.61%	1.93%	4.09%	5.77%	7.20%	8.32%
5-	3.36%	2.49%	5.35%	7.28%	8.67%	9.63%
6+	4.31%	3.21%	7.00%	9.17%	10.44%	11.15%
6	5.54%	4.14%	8.91%	11.28%	12.33%	12.72%
6-	7.12%	5.34%	11.23%	13.47%	14.24%	14.35%
7+	9.14%	6.89%	13.84%	15.81%	16.23%	16.02%
7	11.74%	8.92%	16.48%	18.07%	18.09%	17.89%
7-	15.08%	11.56%	19.13%	19.79%	19.84%	20.05%
8+	19.37%	15.02%	21.73%	21.55%	22.09%	22.96%
8	24.89%	19.61%	26.98%	24.82%	26.54%	29.26%
8-	31.97%	25.70%	29.80%	28.51%	32.07%	38.24%
9	41.06%	33.89%	34.42%	35.26%	43.79%	63.11%

Table 19 represents the final one-year marginal PDs which indicate the probability of default taking into consideration influence of macroeconomic information and participate in ECL estimate.

Table 19. The final marginal PDs which indicate the probability of default taking into consideration influence of macroeconomic information (PF portfolio)

Scale	PD TTC	MPD				
		1 st year	2 nd year	3 rd year	4 th year	5 th year
1+	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
1	0.02%	0.01%	0.02%	0.02%	0.02%	0.02%
1-	0.04%	0.03%	0.04%	0.04%	0.04%	0.04%
2+	0.08%	0.06%	0.08%	0.08%	0.08%	0.08%
2	0.16%	0.12%	0.16%	0.16%	0.16%	0.16%
2-	0.32%	0.24%	0.31%	0.32%	0.32%	0.32%
3+	0.45%	0.33%	0.44%	0.45%	0.44%	0.44%
3	0.58%	0.43%	0.57%	0.57%	0.57%	0.57%
3-	0.75%	0.55%	0.73%	0.74%	0.73%	0.73%
4+	0.96%	0.71%	1.04%	1.13%	1.18%	1.21%
4	1.23%	0.91%	1.47%	1.72%	1.88%	1.98%
4-	1.58%	1.17%	2.08%	2.61%	2.98%	3.22%
5+	2.03%	1.50%	3.54%	4.91%	5.93%	6.53%
5	2.61%	1.93%	4.01%	5.43%	6.38%	6.84%

Scale	PD TTC	MPD				
		1 st year	2 nd year	3 rd year	4 th year	5 th year
5-	3.36%	2.49%	5.22%	6.71%	7.42%	7.53%
6+	4.31%	3.21%	6.78%	8.25%	8.54%	8.17%
6	5.54%	4.14%	8.54%	9.85%	9.55%	8.64%
6-	7.12%	5.34%	10.63%	11.32%	10.36%	8.95%
7+	9.14%	6.89%	12.89%	12.68%	10.96%	9.06%
7	11.74%	8.92%	15.01%	13.75%	11.28%	9.13%
7-	15.08%	11.56%	16.92%	14.16%	11.38%	9.22%
8+	19.37%	15.02%	18.47%	14.33%	11.52%	9.33%
8	24.89%	19.61%	21.69%	14.57%	11.71%	9.49%
8-	31.97%	25.70%	22.14%	14.87%	11.96%	9.69%
9	41.06%	33.89%	22.75%	15.29%	12.29%	9.96%

References

1. Prokofiev S.E., Murar V.I., Rashkeeva I.V., Elesina M.V. Project finance: The nature and significance. *Internet-zhurnal Naukovedenie*. 2014;(6):2. (In Russ.). <http://dx.doi.org/10.15862/02EVN614>
2. Cohen B.H., Edwards G.A. Jr. The new era of expected credit loss provisioning. *BIS Quarterly Review*. 2017;(March). URL: https://www.bis.org/publ/qtrpdf/r_qt1703f.pdf
3. Vasilyeva A.F., Zhevaga A.A., Morgunov A.V. Methods of credit risk management for corporate clients under variability of the requirements of financial reporting standards. *Upravlenie finansovymi riskami = Financial Risk Management Journal*. 2017;(4):258-268. (In Russ.).
4. Bank of Russia Regulation of 06.08.2015 No. 483-P. On the procedure for calculating the amount of credit risk based on internal ratings. URL: http://www.consultant.ru/document/cons_doc_LAW_186639/ (In Russ.).
5. Sorge M. The nature of credit risk in project finance. *BIS Quarterly Review*. 2011;(December). URL: https://www.bis.org/publ/qtrpdf/r_qt0412h.pdf
6. Hainz C., Kleimeier S. Project finance as a risk-management tool in international syndicated lending. *SSRN Electronic Journal*. 2006. <http://dx.doi.org/10.2139/ssrn.567112>
7. Wilson R. Risk measurement of public projects. In: Lind R.C. Discounting for time and risk in energy policy. Washington, DC: Resources for the Future, Inc.; 1982:205-249.
8. Vasilyeva A.F, Jorge Trujillo Cesar Augusto et al. Infrastructure investment and PPP: How will credit ratings help attract new players? Research by the Analytical Credit Rating Agency. Moscow: ACRA; 2020. 33 p. URL: <https://www.acra-ratings.ru/upload/iblock/a30/osnriwpqczjm43txih1646uhgiddg38g.pdf> (In Russ.).
9. Vasilyeva A., Frolova E. Methods of calculation of expected credit losses under requirements of IFRS 9. *Korporativnye finansy = Journal of Corporate Finance Research*. 2019;13(4):74-86. <https://doi.org/10.17323/j.jcfr.2073-0438.13.4.2019.74-86>
10. Guidance on credit risk and accounting for expected credit losses. Basel: Bank for International Settlements; 2015. 36 p. URL: <https://www.bis.org/bcbs/publ/d350.pdf>
11. Vasilieva A., Frolova E. Development of the 'inner assessment model' of long-term default probability for corporate borrowers in the trade segment of the economy in accordance with IFRS 9. *Korporativnye finansy = Journal of Corporate Finance Research*. 2020;14(1):91-114. <https://doi.org/10.17323/j.jcfr.2073-0438.14.1.2020.91-114>
12. International Financial Reporting Standard (IFRS) 9: Financial instruments. 2018. URL: http://www.consultant.ru/document/cons_doc_LAW_201982/ (In Russ.).
13. Guidelines on the application of the definition of default under Article 178 of Regulation (EU) No 575/2013. EBA/GL/2016/07. URL: <https://www.eba.europa.eu/sites/default/documents/files/documents/10180/1721448/052c260f-da9a->

4c86-8f0a-09a1d8ae56e7/Guidelines%20on%20default%20definition%20%28EBA-GL-2016-07%29_EN.pdf?retry=1

14. Marshall J. An introduction to reliability and life distributions. Coventry: The University of Warwick; 2018. URL: <https://studylib.net/doc/13322936/an-introduction-to-reliability-and-life-distributions-obj...>
15. Kuznetsova Yu.I., Zhuravlev I.B. Application of the Bayesian estimate of the probability of a rare event to determining the probability of default of a counterparty. *Upravlenie finansovymi riskami = Financial Risk Management Journal*. 2013;(2):94-102. (In Russ.).

Appendix 1.

Master scale of the bank

Scale of PJSC Bank XXX	Probability of default (PD)	Lower bound of the probability of default	Upper bound of the probability of default
1+	0.01%	0.00%	0.02%
1	0.02%	0.02%	0.03%
1-	0.04%	0.03%	0.07%
2+	0.08%	0.07%	0.14%
2	0.16%	0.14%	0.27%
2-	0.32%	0.27%	0.38%
3+	0.45%	0.38%	0.49%
3	0.58%	0.49%	0.63%
3-	0.75%	0.63%	0.81%
4+	0.96%	0.81%	1.04%
4	1.23%	1.04%	1.33%
4-	1.58%	1.33%	1.71%
5+	2.03%	1.71%	2.20%
5	2.61%	2.20%	2.82%
5-	3.36%	2.82%	3.63%
6+	4.31%	3.63%	4.66%
6	5.54%	4.66%	5.99%
6-	7.12%	5.99%	7.69%
7+	9.14%	7.69%	9.88%
7	11.74%	9.88%	12.69%
7-	15.08%	12.69%	16.30%
8+	19.37%	16.30%	20.93%
8	24.89%	20.93%	26.89%
8-	31.97%	26.89%	34.54%
9	41.06%	34.54%	100%
10	100%	100%	100%

Appendix 2.

Substantiation of the Approaches Chosen for PD Modeling

Method	Advantages	Drawbacks	Application
Weibull distribution	The model approximates the simulative DR to the observed ones significantly better than migration matrices Simplicity of use	Requires a large amount of data on defaults	Applied in case of a large amount of data on defaults (to build DR for several years)
Migration matrices	Use existing information of the segment more heavily and therefore may be built with a smaller number of defaults A convenient mathematical apparatus Opportunity to make adjustments (for example, in the master scale)	At long periods of time (over 9 years) the results are exaggerated significantly but this may be disregarded due to discounting	Applied in case of insufficient data on defaults (for the Weibull distribution)
Generator matrix	Use existing information of the segment more heavily and therefore may be built with a smaller number of defaults A convenient mathematical apparatus Convenient mathematical methods of adjustment Opportunity to obtain cPD estimates for nondiscrete time periods Opportunity to obtain nonzero PD for high ratings even in the absence of observed defaults	Highly complex including use No intuitive explanation when introducing adjustments in the master scale At long periods of time (over 9 years) the results are exaggerated significantly but this may be disregarded due to discounting	May be used instead of migration matrices if: Adjustments in the master scale are not necessary and Use of the generator matrix decreases the number of adjustments introduced by experts / manually

Source: [9].

Appendix 3.

Terms and definitions

Probability of default	– The probability (in percent) of default in relation to customer's obligations within one year defined by means of the model of default probability assessment
Internal credit rating	– an indicator which provides a complex characteristic of the customer's/project's solvency and is calculated on the basis of risk factor indicators
Default	– failure to fulfill obligations of repayment of borrowings by the borrower (default is recorded in accordance with bank's rules)
Cumulative probability of default (cPD)	– the probability of default at any point within the T period (accumulated probability of default, cumulative probability of default)
Marginal probability of default, mPD (t)	– an unconditional probability that default will occur within the future t period which is a part of the T period (marginal probability of default)
Observation	– a block of data about the customer/project as of a certain date
Rating	– In accordance with Reports on Development of the Internal Model of Default Probability Assessment "Project Financing v.2.1", "Project Financing (developers) v.2.1".
Rating group	– an aggregate of several rating grades placed in the neighboring positions in the rating scale which are consolidated in order to provide a sufficient number of observations for statistical analysis
Risk segment	– a group of rating objects determined in accordance with the Rules for Classification of Bank's Credit Requirements to which the same model of default probability assessment is applied
Rating scale	– gradation of rating estimates in accordance with Appendix 1
Conditional probability of default, PD (t)	– a conditional probability that default will occur within the future t period which is a part of the T period provided default does not take place before the t period (conditional probability of default)
Lifetime probability of default, Lt PD	– a probability of default within the contractual term of a financial instrument (lifetime probability of default)

Appendix 4.

Abbreviations

cDR	– cumulative default rate
cPD	– cumulative probability of default
Dpd	– days past due
DR	– default rate
mPD (t)	– marginal probability of default, unconditional probability of default in the t period
PD	– probability of default
PD for 12 months	– probability of default in the following 12 months
PIT	– point-in-time calibration
TTC	– through-the-cycle calibration
IFRS 9	– International Financial Reporting Standard (IFRS) 9 Financial Instruments
ECL	– expected credit losses
SP AACR	– software package Accounting and Analysis of Credit Risk

Contribution of the authors: the authors contributed equally to this article.

The authors declare no conflicts of interests.

The article was submitted 06.11.2021; approved after reviewing 08.11.2021; accepted for publication 14.11.2021.