DOKLADY EARTH SCIENCES, 2025, Vol. 520, No. 1, pp. 135–139

OCEANOLOGY

INTERNAL GRAVITY WAVES IN THE OCEAN WITH SHEAR FLOWS EXCITED BY NON-STATIONARY SOURCES

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Presented by Academician of the Russian Academy of Sciences M. V. Flint on August 12, 2024

Received August 12, 2024
Revised September 11, 2024
Accepted September 16, 2024

Abstract. The problem of internal gravity wave generation by a localized oscillating disturbance source in the ocean of finite depth with background shear currents is considered. Model representations of the buoyancy frequency and the shear current distribution by depth are used to construct analytical solutions in the linear approximation. Under the Miles-Howard assumption, an integral representation of the solution is constructed as a sum of wave modes. Using the stationary phase method, an asymptotic representation of the solution for an individual mode is obtained. The spatial transformation of the phase structures of wave fields is studied depending on the oscillation frequency of the disturbance source and the main characteristics of the shear currents. Experimentally measured shear flows in abyssal channels are shown and compared with the results of laboratory modeling.

Keywords: *internal gravity waves, stratified ocean, shear flows, far fields, asymptotics, non-stationary source*

DOI: 10.31857/S26867397250116e3

Due to the progress in the study of large-scale oceanic wave processes, the study of the dynamics and propagation of internal gravitational waves (IGW) in the ocean, taking into account the presence of currents, is an urgent task [2, 5, 10, 16, 20]. In the real ocean, internal gravitational waves propagate against the background of background shear ocean currents, so the vertical and horizontal dynamics of shear currents are largely related to these waves. In the ocean, such currents can manifest themselves, for example, in the area of the seasonal thermocline and have a noticeable effect on the dynamics of IGW [16-18]. Intense natural currents are Antarctic bottom water flows that flow around underwater ridges at abyssal depths. Their velocities near the bottom often reach 40–50 cm/s [16-18]. Bottom water flows around underwater ridges in the straits generate intense internal waves, for example in the Kara Gate Strait or the Strait of Gibraltar [16-18]. The depths in the straits are less than in the abyssal faults and vary from tens to hundreds of meters. Unsteady or oscillating sources of disturbances are one of the mechanisms for generating intense internal gravitational waves in natural (ocean, Earth's atmosphere) and artificial stratified environments. Such sources of IGW excitation can be both natural (collapse of the turbulent mixing region, rapid movement of the ocean floor, spread of intense atmospheric disturbances) and anthropogenic (underwater and above-ground explosions). [2, 5, 6, 7, 19, 20]. To simulate IGW generation, a steep slope of a transverse ridge in straits can be considered as a point source in the real ocean, and, for example, wave generation by periodic currents on the slopes of transverse ridges in straits can be considered as a possible mechanism for IGW excitation [5, 16-18]. To a first approximation, it can be assumed that background currents with a vertical velocity shift are weakly dependent on time and horizontal coordinates, so if the scale of the horizontal flow change is much larger than the lengths of the IGW, and the scale of the temporal variability is much larger than the periods of the IGW, then such currents can be considered stationary and horizontally homogeneous [2, 5, 10]. In the general formulation, describing the dynamics of IGW in the ocean with background fields of shear currents is a very difficult

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task as early as in a linear approximation. [2, 4, 5, 10, 15, 19, 20].

In the Boussinesq approximation, the vertical component of small perturbations of the IGW velocity W satisfies the equation [2, 3, 5, 9]

$$\frac{D^2}{Dt^2}\left(\Delta+\frac{\partial^2}{\partial z^2}\right)W-\frac{D}{Dt}\left(\frac{d^2U}{dz^2}\frac{\partial W}{\partial x}+\frac{d^2V}{dz^2}\frac{\partial W}{\partial y}\right)+$$

$$+N^2(z)\Delta W=\frac{D}{Dt}\left(\frac{\partial}{\partial z}\left(\frac{Dq}{Dt}\right)\right) \quad (1)$$

$W=0$, at $z=0, -H$

where $\frac{D}{Dt}=\frac{\partial}{\partial t}+U(z)\frac{\partial}{\partial x}+V(z)\frac{\partial}{\partial y}$, $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, $N^2(z)=-\frac{g}{\rho^*}\frac{d\rho_0(z)}{dz}$ are the square of the Brent-Weiss frequency (buoyancy frequency), $(U(z),V(z),0)$ are the components of the velocity of the background shear flow on the horizon z, $\rho_0(z)$ is the undisturbed density, ρ^* is the characteristic value of the density [2, 5], $q=q(x,y,z,t)$ is the distribution density of the sources. Problem (1) is considered in a vertically finite $-H<z<0$ and horizontally unlimited $-\infty<x, y<+\infty$ layer. At the bottom of $z=-H$, the vertical velocity component W is zero; at the surface of $z=0$, the "hard cap" approximation is used: $W=0$, which filters out the surface mode and has little effect on the main characteristics of the IGW [2, 5]. Next, the Miles-Howard stability condition for the Richardson number is assumed to be fulfilled: $Ri(z)=N^2(z)\Big/\left(\left(\frac{dV}{dz}\right)^2+\left(\frac{dU}{dz}\right)^2\right)>1/4$, this means that the corresponding spectral problem does not have complex eigenvalues [4, 11, 14]. The characteristic values of Richardson numbers in the waters of the World Ocean in the absence of dynamic instability of background shear currents can range from 2 to 20 [16-18]. The frequency of buoyancy is assumed to be constant: $N(z)=N=const$. Background shear flow is one—dimensional and linear:

$$V(z)\equiv 0,\ U(z)=U_0+\frac{U_0-U_H}{H}z,$$
$$U_0=U(0),\ U_H=U(-H).$$

The Miles-Howard stability condition is satisfied for the Richardson number:

$$Ri=N^2/\left(\frac{dU}{dz}\right)^2=\frac{N^2H^2}{(U_0-U_H)^2}>\frac{1}{4}.$$

A point harmonic mass source located at a depth is considered: $q(x,y,z,t)=Q\exp(i\omega t)\delta(x)\delta(y)\delta(z-z_0)$, $Q=const$, ω is the oscillation frequency of the source when at large distances from an oscillating source of disturbances at $r=\sqrt{x^2+y^2}\to\infty$ asymptotics of solutions along a certain direction, forming the angle α with the positive direction of the Ox axis is constructed using the stationary phase method [1, 2, 12]

$$W(x,y,z,z_0,t)=\sum_{n=1}^{\infty}W_n(x,y,z,z_0,t)$$

$$W_n(x,y,z,z_0,t)\sim$$

$$\sim\sum_{j=1}^{J(\alpha)}\frac{Q\exp\left(i\left(\omega t-\Theta_n\left(\nu_j^n,\omega\right)+\delta_j\right)\right)F_n\left(\nu_j^n,\omega,z,z_0\right)}{\sqrt{2\pi r\chi_n\left(\nu_j^n,\omega\right)}\dfrac{\partial B\left(\mu_n\left(\nu_j^n,\omega\right),\nu_j^n,\omega\right)}{\partial S_\alpha}}$$

$$\Theta_n(\nu,\omega)=\mu_n(\nu,\omega)x+\nu y,$$

$$F_n(\nu,\omega,z,z_0)=\frac{\varphi_n(\omega,\nu,z)}{d_n(\omega,\nu)}\left(\frac{df(z_0)}{dz_0}\frac{\varphi_n(\omega,\nu,z_0)}{\omega-f(z_0)}+\right.$$
$$\left.+\frac{\partial\varphi_n(\omega,\nu,z_0)}{\partial z_0}\right)$$

$$d_n(\omega,\nu)=\frac{\partial\varphi_0(\omega,\mu_n(\nu,\omega),\nu,-H)}{\partial\mu}$$

$$\frac{\partial\varphi_H(\omega,\mu_n(\nu,\omega),\nu,-H)}{\partial z}$$

$$B(\mu_n(\nu,\omega),\nu,\omega)=\varphi_0(\omega,\mu_n(\nu,\omega),\nu,-H)$$
$$\frac{\partial\varphi_H(\omega,\mu_n(\nu,\omega),\nu,-H)}{\partial z}$$

$$\chi_n(\nu,\omega)=\left|\frac{\partial^2\mu_n(\nu,\omega)}{\partial\nu^2}\right|\left(1+\left(\frac{\partial\mu_n(\nu,\omega)}{\partial\nu}\right)^2\right)^{-3/2}$$

where $\varphi_n(\omega,\nu,z)=\varphi_0(\omega,\mu_n(\nu,\omega),\nu,z)=\varphi_H(\omega,\mu_n(\nu,\omega),\nu,z)$, $\mu_n(\nu,\omega)$ are eigenfunctions and eigenvalues of the vertical spectral problem, which are expressed in terms of modified Bessel functions of the imaginary index [3, 9], $f(z)=\mu_n(\nu,\omega)U(z)$, $\nu_j^n=\nu_j^n(\alpha)$, $j=1,2,..,J(\alpha)$: all such real roots of the equation are $\frac{\partial\mu_n(\nu,\omega)}{\partial\nu}=-\text{tg}\pm$ for which the corresponding stationary points are $\left(\nu_j^n,\mu_n\left(\nu_j^n,\omega\right)\right)$ of the phase function $\Theta_n\left(\nu_j^n,\omega\right)$ lie on the curve $l_n^+(\alpha)$, $\chi_n(\nu,\omega)$ is the curvature of this curve. Phase shift δ_j is equal to $-\frac{\pi}{4}$ or $-\frac{3\pi}{4}$ depending on whether the curve is reversed $l_n^+(\alpha)$ at point $v_j^n(\alpha)$ with a convexity and concavity to the chosen direction S_α The asymptotics of the stationary phase becomes inapplicable near the corresponding wave fronts (caustics), since each caustic is generated by some inflection point of the corresponding dispersion curve, that is, a point at which the curvature of this curve vanishes [1, 2, 12].

Two models of linear shear currents characteristic of the conditions of the World Ocean were used for numerical calculations: unidirectional (the shear current does not change the direction of its propagation throughout the ocean depth) and multidirectional (bottom and near-surface currents are multidirectional). The Richardson number for the flow models used is $Ri=25$, the calculations are given for the first wave mode. Fig.1 shows the results of calculations of lines of equal phase (solid lines) and wavefronts (dashed lines) for unidirectional shear flow, Fig. 2 for multidirectional flow.

Numerical calculations show that the variability, ambiguity, and qualitative diversity of the obtained dispersion relations determine the nature of the generation of various types of waves. In particular, at relatively low oscillation frequencies of the source, only annular (transverse) waves are excited, and in some cases more than two wave packets of such waves can be excited simultaneously. The number of simultaneously excited wave packets is determined by the total number of individual branches of the dispersion curves. At high frequency values, only longitudinal (wedge-shaped) waves of two types are generated, and as the oscillation frequency increases, the half-wave angle of the wavefronts decreases. It can also be noted that there are frequency values at which the wavefront half-solution angle is close to 90°. Therefore, at these frequency values, due to the ambiguity of the dispersion relations, the wave pattern of the excited fields is a complex wave system with both longitudinal and transverse wave properties. For certain types of wave packets, an increase in phase leads to an approximation of the corresponding line of equal phase to the origin (the position of the source of disturbances), and for other types of waves, to a distance from it. For a multidirectional type of flow, a wave pattern in the form of a wave cross is obtained. In this case, all wave vibrations propagating from the source of disturbances can be localized inside the wave fronts (caustics).

Strong bottom currents in the ocean are observed in the Vema fault in the Mid-Atlantic Ridge at 11°N [16-18]. At the meridian of about 41° W, the near-bottom flow with velocities of about 15–20 cm/s flows around an underwater ridge located across the fault. Next, a stream of dense Antarctic bottom water rushes down the slope for about 8 km, the stream accelerates as it rolls down and then descends to a height of about 250–300 m. According to the measurement data, the flow accelerates to 39 cm/s at a depth of 4000–4500 m and then slows down, since its kinetic energy is insufficient to overcome stratification (Fig. 3). In [7, 8, 13, 15], the flow around underwater obstacles was modeled in laboratory experiments, numerical calculations were performed and theoretical estimates of the parameters were proposed IGW, which are generated when flowing around.

In [7, 8, 13, 15], examples of laboratory modeling for several flow parameters are given, as well as a numerical calculation of the internal vibrations that occur during such a flow. The flow structure in their formulation of the problem and

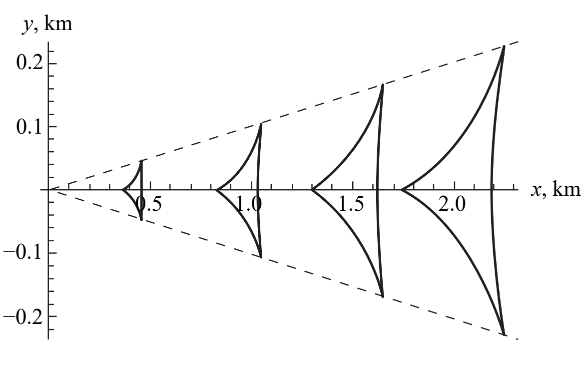

Fig. 1. Wave pattern of propagating waves from a source in the positive direction of the Ox axis, two wavefronts at x > 0.

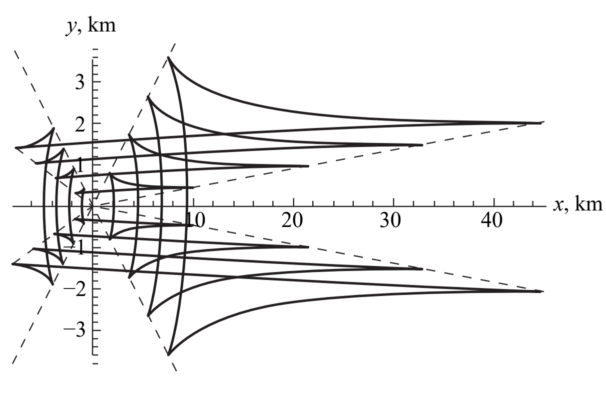

Fig. 2. Waves from the source in all directions; two wavefronts at $x > 0$, two wavefronts at $x < 0$.

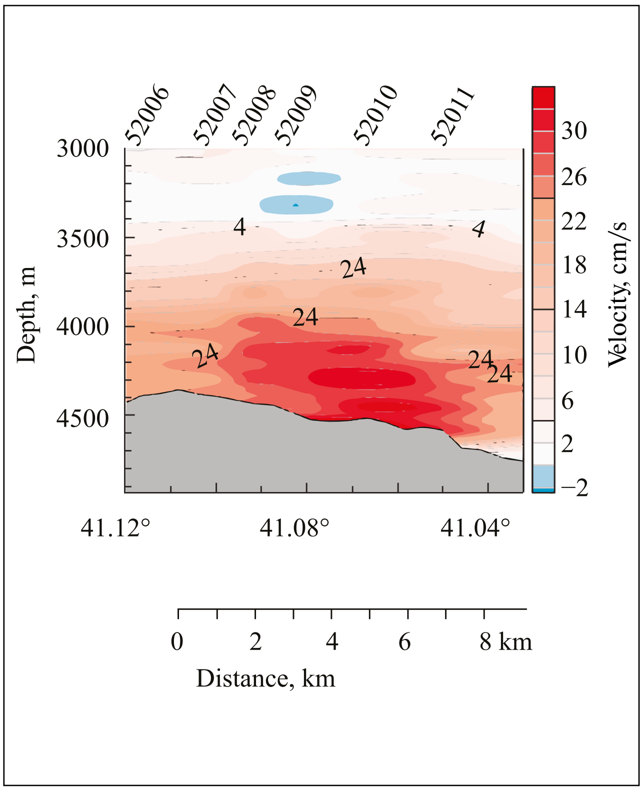

Fig. 3. The measured velocity field along the Vema abyssal fault in the tropical Atlantic during the flow of bottom water around a transverse underwater ridge. The numbers on the upper axis show the numbers of current profiling stations with a lowered Doppler current profiler. The maximum eastward flow velocities (from left to right) are observed after the current rolls down the slope.

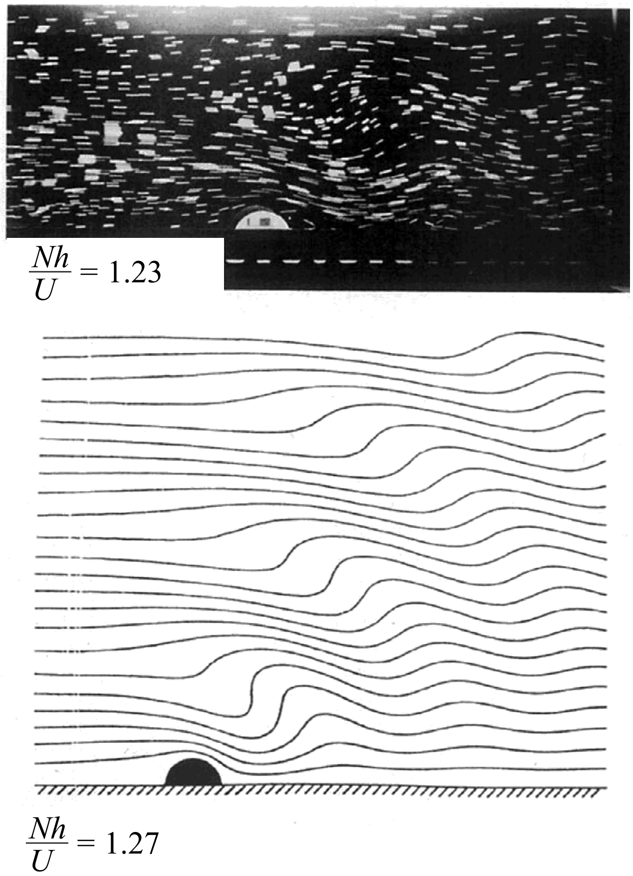

Fig. 4. Laboratory modeling (above) and numerical calculation (below) of the flow around an underwater obstacle for values of the *Nh/U* parameter close to those observed in the ocean.

experiment depends on a dimensionless number *Nh/U*, where *h* is the height of the obstacle and *U* is the maximum value of the shear flow velocity. According to the results of the estimates obtained, propagating columnar perturbations occur at $0.5 < Nh/U < 2$. Fig. 4 shows the flow lines in laboratory and numerical simulations for different values of the *Nh/U* parameter. The available measurements in the ocean correspond to the range of the dimensionless parameter proposed in [7, 8]: the Brunt-Vaisala wave frequency at a depth of 4000-4500 m is N=0.002 s^{-1}, the maximum shear flow velocity is 0.39 m/s, and a streamlined obstacle with a height of h=250–300 m [16-18]. Then the range of values of the dimensionless parameter *Nh/U* will be 1.28–1.53.

Thus, the obtained asymptotic results with different values of the physical parameters included in them make it possible to evaluate the characteristics of IGW observed in real oceanic conditions with currents, as well as calculate wave fields, including from non-local sources of disturbances of various physical nature. As a result of model multivariate calculations, the simulated wave system can be approximated to the wave patterns observed in field and laboratory conditions, which makes it possible to estimate the physical parameters of real sources of IGW generation in the marine environment and determine the main characteristics of initial disturbances by varying the model values of the initial parameters.

FUNDING

The work was carried out on the topic of state assignment No. FFGN-2024-0005 (V.V. Bulatov), No. FMWE-2024-0016 (I.Yu. Vladimirov, E.G. Morozov). The measurement data was obtained during voyages of IO RAS vessels with the support of RSF grant No. 21-77-20004

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