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Mechanisms and kinetics of particle separation by size and density in an activated gravity flow of granular material

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Abstract: A study was carried out to examine the effects of interaction and efficiency of the particle separation process by size and density in fast gravity flows activated by longitudinal momentum by acting the particles of the open surface of the flow with a rough conveyor belt. It has been established that the momentum effect leads to a zone formation in the central part of the flow. The zone has extremely high values of the shear rate and the gradient of the voids fraction, ensuring an intensive occurrence of the quasi-diffusion separation effect. In the region adjacent to the base of the flow, momentum action leads to a decrease in the proportion of voids and an increase in the shear rate, which contributes to the intensification of particle size segregation. In the flow region adjacent to the surface of the momentum action, high values of the voids fraction and the temperature of the granular medium provide favorable conditions for the occurrence of quasi-diffusion effects of mixing and separation. As a result of momentum action, the separation efficiency increases if the direction of the quasi-diffusion separation and segregation flows coincides (separation by density), and the efficiency decreases when the directions of these flows are different (separation by size). With increasing intensity of the momentum action, the zone of intense shear deepens, and the efficiency of density separation increases, reaching its maximum value when the zone deepens by 0.5–0.55 of the layer thickness. With a further increase in the intensity of the momentum action, the separation efficiency decreases due to the expansion of the flow region in which the effect of quasi-diffusion mixing dominates. The conclusions based on the experimental results are confirmed by the method of mathematical modeling of the dynamics of particle separation by density in an activated gravity flow.

Keywords: granular material; activated gravity flow; longitudinal momentum; separation by size and density; segregation; quasi-diffusion separation; shear rate; void volume fraction.

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Механизмы и кинетика сепарации частиц по размеру и плотности в активированном гравитационном потоке зернистого материала

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Аннотация: Проведено исследование эффектов взаимодействия и эффективности процесса сепарации частиц по размеру и плотности в быстрых гравитационных потоках, активированных продольными импульсами путем воздействия на частицы открытой поверхности потока шероховатой лентой конвейера. Установлено, что импульсное воздействие приводит к формированию в центральной части потока зоны с экстремально большими значениями скорости сдвига и градиента доли пустот, обеспечивающими интенсивное протекание эффекта квазидиффузионной сепарации. В области, прилегающей к основанию потока, импульсное воздействие приводит к снижению доли пустот и повышению скорости сдвига, что способствует интенсификации сегрегации частиц по размеру. В области потока, прилегающей к поверхности импульсного воздействия, высокие значения доли пустот и температуры зернистой среды обеспечивают благоприятные условия для протекания квазидиффузионных эффектов перемешивания и сепарации. В результате импульсного воздействия эффективность сепарации повышается, если направление потоков квазидиффузионной сепарации и сегрегации совпадает (сепарация по плотности), и эффективность снижается при различном направлении названных потоков (сепарация по размеру). С повышением интенсивности импульсного воздействия зона интенсивного сдвига углубляется, и эффективность сепарации по плотности увеличивается, достигая максимального значения при углублении зоны на 0,5…0,55 толщины слоя. При дальнейшем повышении интенсивности импульсного воздействия эффективность сепарации снижается вследствие расширения области потока, в которой доминирует эффект квазидиффузионного перемешивания. Выводы по результатам эксперимента подтверждены методом математического моделирования динамики сепарации частиц по плотности в активированном гравитационном потоке.

Ключевые слова: зернистый материал; активированный гравитационный поток; продольные импульсы; сепарация по размеру и плотности; сегрегация; квазидиффузионная сепарация; скорость сдвига; объемная доля пустот.

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1. Introduction

In most practically important cases, flows of granular materials in technological processes and natural phenomena occur in the mode of rapid shear deformations [1–3]. Under conditions of rapid shear deformations, a specific mechanism for the formation of stresses appears, the magnitude of which is determined by the intensity of the exchange of impact momentum between particles of the material through the shear surface [2, 4, 5]. If shear deformations are caused by the action of gravity, then the corresponding flows are called fast gravity flows. Examples of this kind of flows can be the movement of materials on gravity slopes during transportation and the formation of embankments in tanks and bunkers, rotating pipes and drums, as well as natural processes of transforming the relief of the earth's surface (mudflows, rockfalls, underwater movements of rocks, expansion of dunes, etc.).

Fast gravity flows of granular materials, in general, are characterized by high heterogeneity of structural and kinematic parameters [6–8]. Thin-layer gravity flows on a rough slope are characterized by particularly high heterogeneity due to the relatively large contribution of boundary effects at the base and open surface of the flow [8, 9].

The intense interaction of particles under conditions of fast gravity flows is accompanied by the effects of their mixing and separation [6, 8, 9], the physical mechanisms and kinetics of which depend not only on the structural and kinematic characteristics of the flow, but also on the degree of heterogeneity of the latter. This is quite fully confirmed by the results of studies of the dynamics of the particle separation process according to a set of properties in a fast gravity flow presented in [6, 8, 10, 11]. As a result of the research, an equation for separation dynamics that describes the distribution of particles of the control component under the influence of convection, mixing and separation fluxes was obtained. The separation flux is expressed as a result of the conjugation of segregation fluxes caused by the relaxation of stresses concentrated on nonuniform particles in local flow conditions and quasi-diffusion separation initiated by the effects of spatial heterogeneity of the gravitational flow [8, 10, 11]. For the case of a two-dimensional flow, the dynamics of the concentration field $c(x, y, \tau)$ of control particles in a gravitational flow is determined as

$$\frac{\partial c(x, y, \tau)\rho_b}{\partial \tau} = -\frac{\partial uc\rho_b}{\partial x} + \\ + \frac{\partial}{\partial y}\left(\rho_b\left(D_{\mathrm{dif}}\frac{\partial c}{\partial y} - D_{\mathrm{m}}c\frac{\partial \ln s}{\partial y} - K\Delta Mc\right)\right), \qquad (1)$$

where $\rho_b = \rho(1-\varepsilon(y))$ is a local value of the bulk density of the granular medium, kg·m^{-3}; D_{dif} and D_{m} are rate coefficients of quasi-diffusion mixing and quasi-diffusion separation, respectively, which are calculated analytically, m^2·s^{-1}; K is a segregation coefficient, (H·c)$^{-1}$; $u(y)$ is an averaged local velocity value in the shear direction, m·s^{-1}; ΔM is a driving force of segregation, calculated analytically excess moment of gravity, friction and impact impulses acting on the test particle, N·m [8, 12]; $\varepsilon(y)$ is a volume fraction of voids, $m^3 \cdot m^{-3}$; x, y are coordinates in the direction of the slope and in the direction of the normal to the base of the flow, respectively; τ is time, s; $s(\varepsilon)$ is a local value of the average distance between particles, m.

It is important to note that in equation (1) the only kinetic parameter that requires experimental determination is the segregation coefficient K [8, 11, 12]. Research carried out in [12] made it possible to propose a method for experimentally determining the coefficient K as the relative velocity of the transverse movement of a control particle in a gravity flow of granular material per unit of excess moment of the forces acting on it. Moreover, it has been established that the segregation coefficient determined in this way exhibits the properties of a kinetic constant for a fairly wide range of changes in the properties of particles and flow parameters.

The complex of kinetic parameters (ΔM, D_{dif}, D_{m}), which determine the intensity of mixing and separation fluxes in equation (1), is expressed analytically as functions of particle properties, structural and kinematic parameters of the gravity flow based on the concept of a granular medium under conditions of the fast shear flow as "gas of solid particles" [1, 4, 13]. The quasi-diffusion separation coefficient for a binary mixture of cohesionless spherical particles having diameters d_i, masses m_i, restitution coefficients for collisions of uniform particles k_i is calculated by determining the ratio of the velocities of quasi-diffusion movements of nonuniform particles

$$D_{\text{m}} = \frac{\bar{m}(c)(\bar{V}')^2}{2F\bar{k}} \left(\frac{d_1^2 k_1}{m_1 \bar{d}^2} - \frac{d_2^2 k_2}{m_2 \bar{d}^2} \right), \tag{2}$$

where F is an average value of the frequency of particle collisions in local conditions of their interaction calculated from the dissipation energy value [5, 8, 11] subject to the law of conservation of energy generated by gravity shear, s^{-1}; $\bar{V}' = F\,s$ is an averaged value of the local value of the velocity of particle fluctuations, $m \cdot s^{-1}$; $\bar{d}(c)$ is an average particle diameter, m; $\bar{m}(c)$ is an average particle mass, kg; $\bar{k}(c)$ is an average value of the restitution coefficient upon collision of mixture particles [8, 11].

The coefficient of quasi-diffusion mixing is calculated as a function of the averaged values of the local values of the distance between particles and the rate of their fluctuations [8, 11] by analogy with the molecular kinetic theory [14]

$$D_{\text{dif}} = \frac{1}{3} s \bar{V}'. \tag{3}$$

The analysis of dependencies (1) – (3) indicates a significant dependence of the kinetic parameters of the processes of mixing and separation of nonuniform particles in a fast gravity flow on its structural and kinematic parameters. In this case, determining role in the formation of distributions of nonuniform particles is played by the shear rate and the volume fraction of voids in the flow, which, according to the equation of state of the granular medium under conditions of fast gravity flow [8], are in mutual correlation. Dilatancy (volume fraction of voids) and shear kinetic stress in the flow increase in proportion to the square of the shear rate. As a consequence, the shear rate directly affects the magnitude of the driving force for segregation [12]. At the same time, it is obvious that the rate of particle fluctuations also increases in proportion to the shear rate, which, together with an increase in the volume fraction of voids, leads to a significant increase in the diffusion permeability of the medium intensifying the quasi-diffusion effects of mixing and separation.

The extremely high significance of the shear rate is also manifested in its influence on the kinetics of quasi-diffusion separation. The quasi-diffusion fluxes of particles of the mixture components intensify not only due to increasing shear rate, but also due to the special role of the shear rate in the formation of the conditions (driving force) of quasi-diffusion separation [8, 11]. A necessary condition for initiating a quasi-diffusion separation flux is the presence of a gradient in the volume fraction of voids. In a gravity flow, such a condition can be formed under the influence of two factors, one of which is heterogeneity of the shear rate, and the other is heterogeneity of lithostatic pressure.

The above analysis indicates the primary and ambiguous role of shear velocity in the formation of separation and mixing flows in a fast gravity flow of granular material. In this case, not only the magnitude of the shear rate is of great importance, but also the nature of its change in the flow. This is largely confirmed by the results of the analytical study presented in [15], on the basis of which we developed a comprehensive research method aimed at finding ways to intensify the effects of particle separation by size and density with active variation of the shear rate in the gravity flow.

2. Research Methods and Model Materials

To purposefully vary the shear rate in the fast gravity flow for increasing the efficiency of particle separation by size and density, it is necessary to determine not only the direction of exploratory research, but also the method of identifying structural and kinematic parameters under unusual flow conditions. In this regard, it is advisable to use the results and methodology of the study [15], carried out in the form of a virtual experiment. The experiments in experiment [15] were carried out with virtual variation of speed profiles. In this case, the void fraction profile, which is missing for a complete flow characteristic, was determined taking into account the correlation of flow parameters in accordance with the equation of state of the granular medium under the fast gravity flow. The equation of state describes the relationship between dilatancy $\bar{\varepsilon}$, lithostatic pressure p and temperature of the granular medium (kinetic energy of particles in their mutual movements) taking into account the physical and mechanical properties of the material [8, 15].

Since dilatancy is correlated with the volume fraction of voids, and the temperature of the granular medium is proportional to the square of the shear rate du_x/dy, it seems possible to write the equation of state in the following form [15]

$$\bar{\varepsilon} = \frac{\psi}{p}\left(\frac{du_x}{dy}\right)^2. \qquad (4)$$

Equation (4) includes the coefficient ψ, which, under certain assumptions [15], can be considered a constant, depending only on the physical and mechanical properties of particles.

As part of a virtual experiment, the influence of shear rate on the efficiency of separation in the gravity flow, which was assessed by the value of the variation coefficient of the test component distribution, was studied [15]. The experiment was carried out using the method of mathematical modeling of separation dynamics based on equation (1) under boundary conditions reflecting the absence of transfer of the test component at the base and open surface of the flow and its uniform initial distribution. To add certainty to the results, linear velocity profiles characterized by uniform values of shear velocity in the flow volume, varied in the experiment, were specified. A mixture of glass bead fractions was used as a model material: +3.25–3.5 mm (88 %) – base component; +3.6–3.75mm (12 %) – test component.

The results of the virtual experiment in the form of the dependence of the variation coefficient on the shear rate in the gravity flow are presented in Fig. 1.

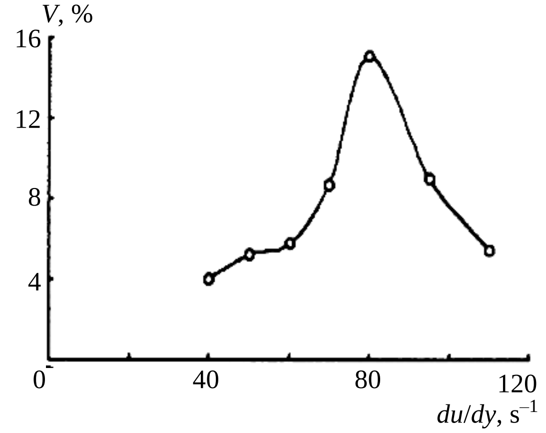


Fig. 1. Coefficient of variation in the composition of a bidisperse mixture of bead particles as a function of shear rate in the fast gravity flow [15]

Alternative options for mathematical modeling of the dynamics of the separation process with and without taking into account the effects of segregation and quasi-diffusion separation allowed us to draw a conclusion regarding the physical nature of the extreme value of heterogeneity in the shear rate range (80 ± 5) s^{-1}.

Modeling options indicate an increasing dominant role of the segregation effect in the ascending section of the dependence of the variation coefficient on the shear rate in the range of its moderate values, when there are sufficiently high values of the volume fraction of the solid phase in the flow. A further increase in the shear rate is accompanied by an increase in the volume fraction of voids in the flow, which creates conditions for the intense occurrence of quasi-diffusion effects and contributes to the transition of the dominant role from segregation to quasi-diffusion separation. Since in most of the gravity flow the effects of quasi-diffusion separation and particle size segregation have the opposite direction, a decrease in the variation coefficient is observed in this region. It is obvious that with a further increase in the shear rate, the nature of the dependence will not change, since the separation effects are increasingly suppressed by quasi-diffusion mixing, the effect of which increases with the increase in the rate of particle fluctuations and the volume fraction of voids in the flow (3).

The analysis of the kinetic laws of separation and the results of a virtual experiment clearly indicate the decisive and ambiguous role of shear rate in the manifestation of the interaction effects between nonuniform particles in the fast gravity flow [15]. The magnitude of the shear rate and the nature of its change along the height of the bed determine the distribution of the void volume fraction in the flow, the intensity and conditions for coupling the effects of separation and mixing. This conclusion initiated an experimental and analytical study within the framework of this work, the main objective of which is to determine the conditions of the activated gravitational flow that ensures an increase in separation efficiency.

Variation in shear rate is proposed due to the transfer of additional momentum to particles through the open surface of the gravity flow. Additional momentum is provided by placing a conveyor belt with adjustable belt speed above the open surface of the flow. The belt has roughness equal in size to half the diameter of the particles and the lower branch of the tape is in direct contact with the particles on the bed surface. If the speed of the belt is higher than the velocity of the particles on the bed surface, then the particles receive an additional momentum in the direction of the slope, which, under the influence of the effect of pseudo-viscous friction [1], penetrates in the volume of the gravity flow. It is obvious that the magnitude of the momentum and the depth of its penetration into the bed will be determined by the relative speed of the belt and the longitudinal velocity of the particles on the open surface of the flow.

For a comprehensive analysis of the separation effects that are generated as a result of such a momentum action, it is necessary to have information about the corresponding hydrodynamic effects that determine the conditions for the interaction of nonuniform particles in the shear flow. If there is a complex of information in the form of velocity profiles and the volume fraction of voids, it becomes possible to mathematically model the dynamics of particle separation in alternative options that take into account or neglect certain effects of particle interaction in accordance with equation (1).

Mathematical modeling of the dynamics of the process is carried out by integrating equation (4) using a numerical method with the Crank-Nicholson difference scheme [16]. The initial and boundary conditions for the separation dynamics equation (4) are formulated for the case of the gravity flow on a rough chute in the following traditional form [8, 10–12]

$$D_{\text{dif}}\frac{\partial c}{\partial y} = cD_{\text{m}}\frac{\partial \ln s}{\partial y} = Kc\Delta M\big|_{y=0,h} = 0; \qquad (5)$$

$$c(0,x,y) = c_0, \quad c(t,0,y) = c_0, \qquad (6)$$

where c_0 is an average concentration of the test component in the flow.

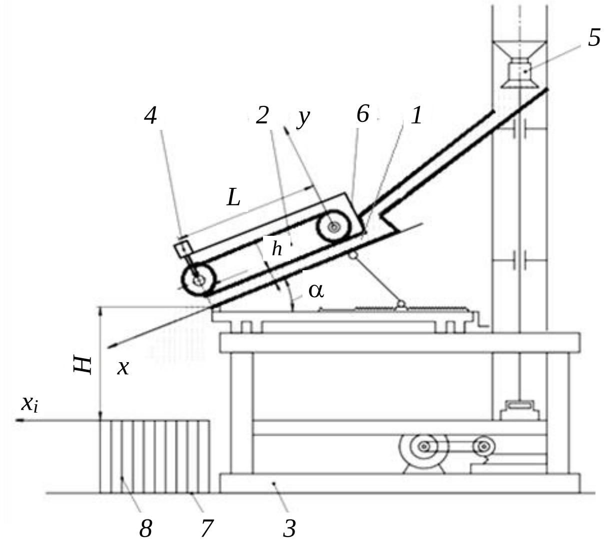


Fig. 2. Scheme of the experimental unit:
1 – rectangular channel with a rough bottom; *2* – conveyor with a rough belt; *3* – frame; *4* – conveyor drive; *5* – feeder dispenser; *6* – reflector; *7* – cuvette; *8* – cuvette cells

In this study, the profiles of the velocity and volume fraction of voids in the activated gravity flow were obtained using an experimental-analytical method [8, 11, 12], based on the analysis of the characteristics of the particle flow at the stage of their fall from a rough chute. These profiles were obtained using the experimental unit shown in Fig. 2.

To analyze the structural and kinematic characteristics of the flow, initial data is required, which includes an experimentally determined distribution function of the material along the horizontal coordinate x_1 with a known particle fall height H, layer thickness h, distribution formation time, slope angle α, true and bulk density of particles. The correlation of the particle coordinate at the chute threshold y with its horizontal coordinate of the material distribution x_1 is carried out using a system of equations [8, 11, 12]: material balance, the law of a freely falling body having an initial velocity, and the equation of state of a granular medium under rapid shear, which in a slightly transformed form is represented by dependence (4).

Unlike the traditional method, according to which a single boundary condition is specified corresponding to the condition of particle adhesion on the rough surface of the slope ($u = 0$ at $y = 0$) when the height of the roughness is equal to half the diameter of the particles, in the presented version of the method it is necessary to write the boundary condition on the upper boundary flow. This condition consists in the equality of the particle velocity at the upper boundary of the flow to the speed u_b of the rough conveyor belt ($u = u_b$ at $y = h$, where h is the height of the bed).

In addition, in contrast to the traditional condition of zero lithostatic pressure on the open surface of the gravity flow, in the flow activated by additional longitudinal momentum, the particles of the bed, including its open surface, are under the influence of the so-called "dispersion" pressure [17]. Dispersion pressure is due to the presence of a normal component of external momentum action on the shear movement of particles near the open surface of the flow. In this case, the ratio of the longitudinal and normal components of the momentum action is determined by the dynamic coefficient of internal friction. In connection with the problems of identifying the named coefficient at the upper boundary of the flow, the work proposes to determine the dispersion pressure using the equation of state of the granular medium under conditions of the fast shear flow. In accordance with the adapted form of the equation of state (4), we have the following expression for pressure

$$p = \frac{\psi}{\varepsilon}\left(\frac{du_x}{dy}\right)^2. \qquad (7)$$

If we assume that the value of the coefficient ψ of the state equation remains invariant under conditions of the activated gravity flow, then the dispersion pressure p_d can be represented by a component of the total pressure p. At the upper boundary of the flow, the total pressure will correspond to the dispersion pressure, and with deepening into the bed it will increase by the amount of lithostatic pressure. Thus, the dispersion pressure can be determined using (7) for known values of the shear rate initiated by the rough tape and the volume fraction of voids in the upper boundary zone of the flow. It seems possible to express the shear rate, to a first approximation, as the ratio of the difference in the velocities of the belt and particles on the surface of the flow in the absence of momentum action to the thickness of the bed.

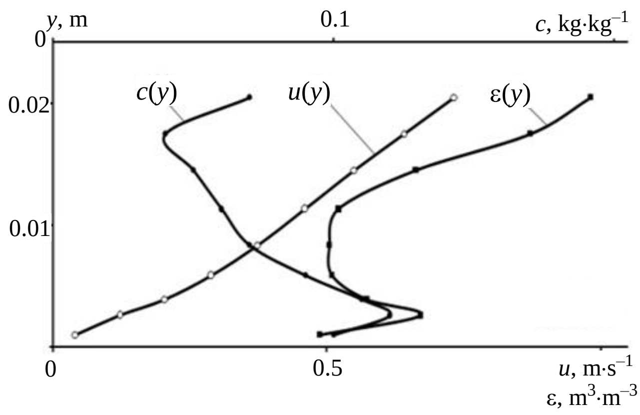
Fractions (+3.25–3.5 and +4.0–4.25 mm) of glass beads (to prepare a binary mixture of particles differing in size) and fractions (+3.6–3.75 mm) of beads and silica gel uniform in size (for preparing a binary mixture of particles that differ in density) were used as model materials.

3. Results and Discussion

3.1. Experimental study of particle size separation in the activated gravity flow

At the first stage of the study, an experiment was carried out, the purpose of which was to assess the influence of momentum action through the open surface of the gravity flow on its structural and kinematic characteristics and the possibility of achieving separation effects with a change in shear rate, similar to those demonstrated as a result of the virtual experiment (Fig. 1).

In the study at this stage, a bidisperse mixture of bead particles was used with a test fine fraction (+3.25–3.5 mm) content of 5–8 % in its mixture with the fraction (+4.0–4.25 mm). The concentration of the mixture was varied in an arbitrary manner in the experiments, due to the high tendency of the mixture to segregate. Figures 3*a* and 3*b* show the profiles of velocity, volume fraction of voids and concentration distributions of test particles in gravity flows on



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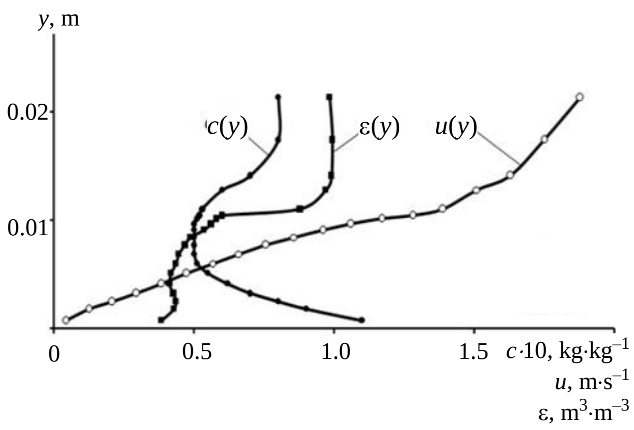

(*b*)

Fig. 3. Profiles of velocity $u(y)$, volume fraction of voids $\varepsilon(y)$ and concentration distribution of test small particles $c(y)$ in a binary mixture of bead fractions (+3.25–3.5 and +4.0–4.25 mm) in gravity flows: *a* – in the absence of additional momentum action; *b* – in the activated gravity flow (belt speed 1.9 m·s^{-1})

a rough chute in experiments in the absence of momentum action and when an additional momentum is imparted to the flow. Additional momentum is provided by a rough conveyor belt moving at a speed of 1.9 $m \cdot s^{-1}$.

In order to reduce the random error of the experimental results and its statistical assessment, parallel experiments were performed to check the variation of the samples of measured values using the Student’s t-test at a confidence probability level of 95 %. The relative value of the mean square error did not exceed 9 %.

A comparative analysis of the profiles in Fig. 3*a* and 3*b* allows us to detect the following effects of impulse action. Under the influence of the momentum pulse, not only does the shear rate systematically increase along the height of the bed, but also the formation of regions in the flow, the boundary between which runs through the surface with an extreme value of the shear rate. The flow region adjacent to the belt surface and receiving additional momentum from it finds itself in conditions of more intense chaotic movements of particles. As a result, in accordance with the equation of state of the granular medium under fast shear flow (4), the formed regions can be characterized as regions with low (at the base of the flow) and high (at the surface of the belt) temperatures of the granular medium. A similar conclusion should also be made when analyzing the characteristics of the structure of the named flow regions, based on the results of the study [18].

Since the temperature values of the granular medium in the named regions of the flow differ multiple times, then, in accordance with the same equation of state (4), it becomes the reason for an extremely sharp, almost hopping, increase in dilatancy during the transition from the lower region of the flow with a low temperature into the upper flow region having a high temperature. It is quite natural that a sharp increase in dilatancy is accompanied by the formation of a high gradient of the volume fraction of voids in the direction from the base to the surface of the gravity flow, as demonstrated by the corresponding profile $\varepsilon(y)$ in Fig. 3*b*. Due to the fact that the gradient of the volume fraction of voids directly determines the magnitude of the gradient of the average distance between particles, which, in accordance with equation (1), is the driving force of quasi-diffusion separation, the formation of a high gradient leads to a high local intensification of the named effect.

Based on the foregoing, it seems possible to assert that at the boundary of the contact of flow regions with low and high temperatures of the granular medium, a zone that functions similarly to a quasi-diffusion separator with a high degree of activity and localization in the volume of the bed is formed. Under the influence of quasi-diffusion separation in this zone, selective transfer of particles with a high fluctuation rate occurs in the direction of the temperature gradient of the granular medium, i.e. into the flow region with a high volume fraction of voids, where conditions for chaotic movements with a large free path are created for such particles. Under the influence of this effect, particles with a low fluctuation rate move in the opposite direction, i.e. into the flow region with a high concentration of solids.

This conclusion is confirmed by a comparative analysis of the distribution profiles of the concentration of test particles in flows without additional momentum action (Fig. 3*a*) and with momentum imparted by a rough belt (Fig. 3*b*). In the absence of the pulse impact (Fig. 3*a*), the concentration of test small particles systematically, with the exception of boundary values, increases towards the base in almost the entire volume of the flow. This indicates the dominance of the segregation effect in the flow in accordance with the mechanism of shear flow separation [12], formally similar to the mechanisms of percolation and kinetic sieving, on which most developed models of the kinetics of the separation process are based [19–23]. As a result of the momentum effect, the distribution of the concentration of test particles undergoes a significant and fundamental change, indicating a significant contribution of the quasi-diffusion separation effect to the process of distribution of nonuniform particles [8, 10, 11]. First of all, this conclusion is confirmed by an increase in the concentration of test fine particles in the direction from the base of the slope in the flow region covering the zone of intense shear deformation and the region located above it with a high temperature of the granular medium (profile $c(y)$ in Fig. 3*b*). An increase in the concentration of small particles in the direction opposite to the lithostatic pressure gradient, reaching values exceeding their average concentration in the flow, contradicts the segregation effect. At the same time, the coincidence in the indicated region of the directions of the concentration gradients of small particles and the volume fraction of voids makes it possible to explain the fundamental change in the concentration profile under momentum action by the intensification of the effect of quasi-diffusion separation (1).

Thus, based on the analysis of the structural and kinematic parameters of the gravity flow of the granular medium under conditions of additional momentum action, the following conclusion can be drawn. In contrast to the conditions of the virtual experiment [15], which made it possible to study with sufficiently high certainty the effect of shear rate on separation efficiency (Fig. 1), in this case there are nonlinear velocity profiles, which are characterized by non-uniform values of the shear rate in the flow volume.

The presence of regions with fundamentally different flow conditions in flows between two parallel inclined rough planes that have a relative speed of movement fundamentally distinguishes them from Couette flows [24–26]. It is obvious that the formation of regions in the flow with fundamentally different flow conditions is a consequence of the coupling of momentums generated by gravity and additional activating influence. In this we can note a certain analogy with the evolution of the stress-strain state in the processes of formation of structures of composite mixtures [27–30].

Taking into account the discovered features of the structure of the activated gravity flow, which occur as a result of the formation in its central part of a brightly localized zone with intense shear deformation and a pronounced effect of quasi-diffusion separation, within the framework of the work, it is proposed to use the separation coefficient to reflect the dependence of the separation efficiency on the shear rate. The value of the separation coefficient is determined either by the degree of depletion (depletion coefficient) of particles with a high fluctuation rate from the flow region with a low temperature of the granular medium, or by the degree of enrichment (enrichment coefficient K_e) of these particles in the flow region with a high temperature of the granular medium. It is obvious that the values of the separation coefficient K_s in alternative versions of its definition are the same, i.e. the coefficients of depletion K_d and enrichment K_e are equal to each other: $K_s = K_d = K_e$. These coefficients are determined as follows:

$$K_s = K_d = \frac{c_0 - c_1}{c_0}\left(\frac{G_1}{G_0}\right); \qquad (8a)$$

$$K_s = K_e = \frac{c_2 - c_0}{c_0}\left(\frac{G_2}{G_0}\right), \qquad (8b)$$

where c_0, c_1, c_2 is an average concentration of test particles in the flow as a whole, in the flow region with a low temperature of the granular medium located below the zone of intense shear, and in the region of the flow with a high temperature of the granular medium located above the zone of intense shear, respectively, kg·kg^{-1}; G_0, G_1, G_2 are mass flow rates of material in the gravity flow and in its regions with low and high temperatures of the granular medium, respectively, kg·s^{-1}; $G_0 = G_1 + G_2$.

The findings of the impact of the additional momentum intensity in the gravity flow on the efficiency of particle size separation in it are presented in Fig. 4 in the form of a dependence of the separation coefficient on the shear rate. Taking into account the no-slip condition at the upper and lower boundaries of the flow, the average value of the shear rate in the flow was conventionally determined as the ratio of the belt speed to the bed height. The experimental results shown in Fig. 4 were obtained at belt speeds from 1 to 2 m·s^{-1}.

In order to analyze the effect of additional momentum, the findings on the separation coefficient in the activated gravity flow are presented in comparison with the separation coefficient in the gravity flow without external influence. The average shear rate in the flow without additional momentum action, which is defined as the ratio of the particle velocity on the open flow surface to the bed height, was 36 s^{-1}. In this regard, the experimental value of the coefficient K_s, corresponding to the minimum value of the shear rate in Fig. 3, was obtained for the gravity flow in the absence of additional momentum action. In this case, the boundary between the regions of separated fractions in the flow is established by the average concentration of test particles. In one of the regions, local concentration values are systematically higher (enrichment region), and in the other – lower (depletion region) compared to the average concentration.

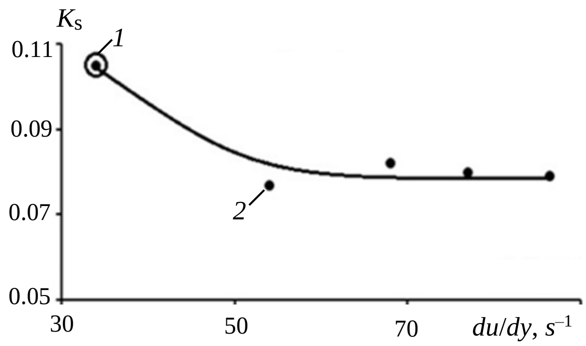

Fig. 4. Dynamics of changes in the separation coefficient of a binary mixture of bead fractions (+3.25–3.5 and +4.0–4.25 mm) depending on the shear rate in gravity flows:
1 – in the absence of additional momentum action;
2 – in activated gravity flows

The resulting dependence $K_S(du/dy)$ demonstrates a decrease in the separation coefficient with increasing intensity of the additional momentum action. This effect is quite fully explained by the results of a previous analysis of the velocity profiles, volume fraction of voids and concentration distributions of test particles shown in Figs. 3*a* and 3*b*. It should be noted that with an increase in the additional momentum effect on the gravity flow, the quasi-diffusion effects of mixing and separation intensify due to the formation of a region with a high temperature of the granular medium in the upper part of the flow. Moreover, for particles differing in size (Fig. 3*b*), the quasi-diffusion separation flux in this region has a direction opposite to the segregation flux, which dominates the separation process in the fast gravity flow without activation (Fig. 3*a*). The counter direction of segregation and quasi-diffusion separation fluxes with increasing intensity of the latter is accompanied by the loss of the dominant role of segregation and a decrease in separation efficiency. In addition, a decrease in separation efficiency is facilitated by an increase in the intensity of the quasi-diffusion mixing flow (3) with an increase in the distance between particles and the rate of particle fluctuations due to an increase in the shear rate and the volume fraction of voids under the influence of additional momentum.

3.2. Separation of particles by density in the activated gravity flow

The analysis of physical mechanisms of transformation of structural-kinematic parameters set out in Section 3.1 and, accordingly, the conditions for generating separation effects in the fast gravity flow under the influence of additional longitudinal momentum allows us to formulate a hypothesis regarding the intensification of the process. According to research, an increase in separation efficiency with this method of flow activation can be achieved in cases where either the direction of the segregation flows and quasi-diffusion separation coincide, or the segregation flow is negligibly small. It is obvious that this type of coupling options for separation flows is provided when the particles differ in density, roughness and elasticity [8, 11]. Under the influence of the quasi-diffusion separation effect, less dense, smooth and more elastic particles will migrate in the direction of the void volume fraction gradient (into the flow region with a high temperature of the granular medium), which coincides with the direction of segregation of such particles.

In order to test this hypothesis, an experimental and analytical study of the dynamics of particle separation by density in the activated gravity flow was carried out as part of the work. A mixture of uniform in size fractions (+3.6–3.75 mm) of beads and silica gel (test component) with a density of 2500 and 700 kg·m^{-3}, respectively, was used as a model material. The experimental research technique was fully consistent with that used previously when studying the dynamics of particle size separation. The study of the influence of an additional momentum on the efficiency of separation in the gravity flow of particles of different densities was carried out in the range of shear rates, which was controlled by varying the speed of the rough conveyor belt adjacent to the open surface of the layer from 1 to 2 m·s^{-1}.

The results of the study in the form of profiles of velocity, volume fraction of voids and concentration of control particles (silica gel) in gravity flows without additional momentum action and in the case of its activation (belt speed 1.5 m·s^{-1}) are shown in Figs. 5*a* and 5*b*, respectively.

A comparative analysis of the profiles indicates fundamental changes in the structural and kinematic characteristics of the gravity flow under conditions of additional momentum action. Such changes were noted earlier in the gravity flow of particles differing in size (Fig. 3). First of all, such changes include the formation in the central part of the flow in a zone with an extremely high intensity of shear deformation, which is part of the above flow region with high temperature values of the granular medium and the volume fraction of voids. Below this zone, a flow region with a low temperature of the granular medium is formed.

Particularly noteworthy is the extremely low value of the volume fraction of voids in the region of the activated gravity flow located below the zone with extremely high shear rates. At first glance, this result seems paradoxical, since the decrease in the volume fraction of voids under the influence of additional momentum to values close to the volume fraction of voids in the stationary bulk bed occurs under conditions of increasing shear rate in the specified flow region. The physical mechanism of the observed effect of contraction of the lower part of the flow under the influence of an additional momentum can be explained by the presence of fundamentally different boundary conditions at the base and upper boundary of the flow.

Under the influence of the pseudo-viscous effect in the fast shear flow of the granular medium, the energy of an additional longitudinal momentum distributed along the upper boundary of the flow penetrates deep into the bed intensifying the mutual

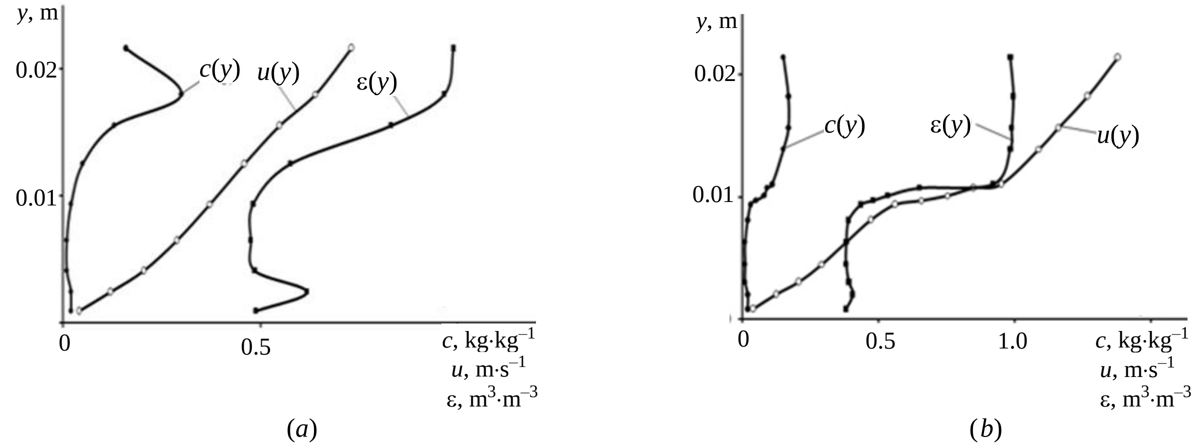

Fig. 5. Profiles of velocity $u(y)$, volume fraction of voids $\varepsilon(y)$ and distribution of test particles concentration $c(y)$ in a mixture of monodisperse particles (+3.6–3.75 mm) of beads and silica gel in gravity flows:
a – in the absence of additional momentum influence; *b* – when activated by longitudinal momentum (belt speed 1.5 $m \cdot s^{-1}$)

movements of particles mainly in the flow region located above the zone of intense shear deformation. In this case, relatively high values of the temperature of the granular medium (the kinetic energy of particles in their mutual movements) are achieved in the named flow region. Due to the condition of sticking at the base of the bed and the dissipation of energy of the additional momentum as it penetrates deep into the layer (according to the effects of pseudo-viscous friction, elasticity and friction), in the flow region located below the zone of intense shear, the temperature of the granular medium turns out to be significantly lower than in the upper part of the flow. The higher temperature of the granular medium in the upper part of the flow leads to effect of "dispersion pressure" [17] causing contraction of the lower part of the flow.

In this regard, it seems possible to assert that the formation of a zone with intense shear deformation is accompanied by the formation of flow regions with an extremely large difference in the volume fraction of voids. Thus, the zone of intense shear deformation is located at the boundary between flow regions characterized by a large difference in the average distance between particles. As a consequence, favorable conditions for the manifestation of the effect of quasi-diffusion separation under the influence of a large gradient of the average distance between particles, i.e. the driving force of this effect, are formed in this zone [8, 11].

The findings in the form of velocity profiles, volume fraction of voids and concentration of test particles (silica gel) in the gravity flow are shown in Figures 6–8 for different magnitudes of momentum action (conveyor belt speed). For the purpose of comparative assessment of the effect of the gravity flow activation, the figures show profiles obtained in the absence of additional impulse action, when the belt speed is zero.

The analysis of the evolution of velocity profiles shows that with increasing momentum action, shear rates increase throughout the entire volume of the flow, but the most significant changes in the rate of deformation of the granular medium are observed in the flow region with a high temperature of the granular medium and, especially, in the vicinity of the zone with extreme values of the shear rate. With increasing momentum action, this zone deepens and expands with an increase in the flow region with a high temperature of the granular medium.

In all cases, even at small values of the additional momentum, fundamental changes in the velocity profiles and the volume fraction of voids are observed. One of the main features of the ongoing profile changes is that together with the formation of an intense shear zone in the flow, there is a sharp decrease in the volume fraction of voids (an increase in the fraction of the solid phase) in the flow area adjacent to its base. This is evidenced by a comparison of profiles 1 and 2 in Figs. 6 and 7. When increasing intensity of the momentum action, the volume fraction of voids increasingly approaches its extreme minimum value, corresponding to the porosity of the repose state of bulk material.

It is important to note that an increase in the volume fraction of the solid phase in the flow region adjacent to its base occurs under the action of a longitudinal momentum with increasing shear rate. A change in the structural and kinematic characteristics in the indicated direction contributes to an increase in stress concentrations on non-uniform particles, which, according to the kinetic laws of segregation [2, 8, 12], directly increases the driving force of the mentioned separation effect. This conclusion is especially clearly confirmed by the results of a study

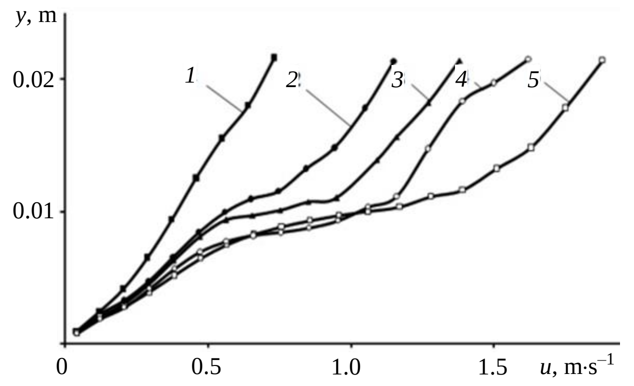

Fig. 6. Velocity profiles in gravity flows of a mixture of monodisperse particles (+3.6–3.75 mm) of beads and silica gel for different values of longitudinal momentum generated at conveyor belt speeds, m·s^{-1}: *1* – 0.0; *2* – 1.2; *3* – 1.5; *4* – 1.7; *5* – 1.9

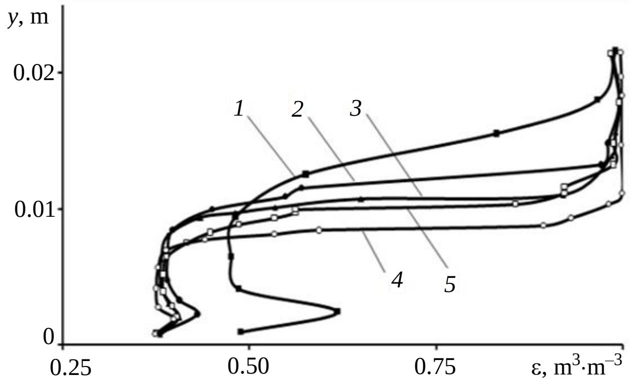

Fig. 7. Profiles of the volume fraction of voids in gravity flows of a mixture of monodisperse particles (+3.6–3.75 mm) of beads and silica gel for different values of longitudinal momentum generated at conveyor belt speeds, m·s^{-1}: *1* – 0.0; *2* – 1.2; *3* – 1.5; *4* – 1.7; *5* – 1.9

of particle size separation in gravity flows in the absence and presence of additional momentum action, presented in Figs. 3*a* and 3*b*, respectively. A comparison of the concentration profiles in Figs. 3*a* and 3*b* shows that in the flow region adjacent to its base, the impact of an additional momentum is accompanied by a significant increase in the concentration gradient of test particles, which indicates intensified segregation.

At the same time, conditions are formed for the intensive occurrence of quasi-diffusion separation in the remaining flow region, in which the momentum action leads to an increase in quasi-diffusion permeability and the gradient of the volume fraction of voids. Thus, it seems possible to highlight two aspects of increasing the efficiency of separation in the gravity flow under the influence of additional longitudinal momentum action. Firstly, the momentum effect helps to intensify the effect of quasi-diffusion separation in the flow region adjacent to the surface of the direct momentum effect. Secondly, the momentum effect contributes to the intensification of segregation in the flow area adjacent to its base.

If, in accordance with the physical and mechanical properties of the mixture particles, the direction of the effects of segregation and quasi-diffusion separation for the test particles coincides, then as a result of the additional momentum action, the separation efficiency increases. For example, for a mixture of monodisperse particles of different densities, segregation of test particles with low density occurs in the direction opposite to the lithostatic pressure gradient, i.e. towards the upper boundary of the flow. This direction coincides with the gradient of the volume fraction of voids in the upper part of the flow, in the direction and under the influence of which quasi-diffusion separation of less dense particles occurs.

Thus, the findings made it possible to obtain the information necessary to theoretically substantiate the feasibility of applying longitudinal momentum to the fast gravity flow of the granular material to increase separation efficiency. The feasibility of using longitudinal momentum should be justified taking into account the conjugation of the direction of segregation and quasi-diffusion separation fluxes and the possibility of intensifying the corresponding effects in the flow volume.

Since the concentrations of test particles were obtained by the direct weight method, the geometric similarity of the distribution profiles of the concentration (Fig. 8) and the volume fraction of voids (Fig. 7) is an indirect, but quite convincing confirmation of the reliability of changes in the structural and kinematic characteristics of the gravity flow that occur under the influence of additional longitudinal momentum. This is explained by the fact that for particles differing in density, the dominant separation effect is quasi-diffusion separation [8, 10, 11], the driving force of which is the gradient of the average distance between particles, proportional to the gradient of the volume fraction of voids. In this regard, particles with a high fluctuation rate are distributed in the flow revealing gradients in the concentration distribution similar to gradients of the volume fraction of voids.

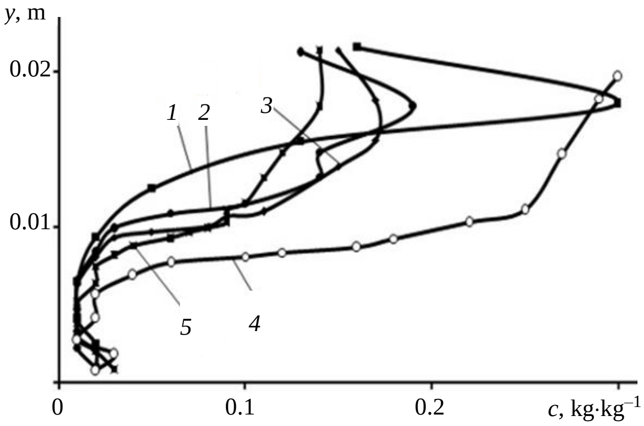

Fig. 8. Concentration profiles of test particles in gravity flows of a mixture of monodisperse particles (+3.6–3.75 mm) of beads and silica gel for different values of longitudinal momentum generated at conveyor belt speeds, $m \cdot s^{-1}$:
1 – 0.0; *2* – 1.2; *3* – 1.5; *4* – 1.7; *5* – 1.9

In the profiles (Fig. 8) for flows with momentum action, a zone of high concentration gradient of test particles is clearly visible, the dislocation of which in the flow volume completely coincides with the location of zones of intense shear deformation (Fig. 6) and a high gradient of the volume fraction of voids (Fig. 7). With an increase in the intensity of the momentum effect, the zone of extreme values of velocity gradients, volume fraction of voids and concentration of the test component deepens without reducing the magnitude of the gradients (curves *2–4* in Figs. 6–8). However, as the zone approaches the base of the bed at a distance equal to approximately two particle diameters and a further increase in the intensity of the momentum action, signs of impending destruction of the zone are observed (curves *5* in Figs. 6-8). First of all, the destruction of the zone is evidenced by a sharp decrease in the velocity gradients, the volume fraction of voids and the concentration of the test component and a shift in the extreme values of the gradients towards the upper boundary of the flow.

The findings on the impact of the intensity of momentum action on the efficiency of particle separation by density in the gravity flow are presented in Fig. 9 in the form of a dependence of the separation coefficient (8) on the shear rate. The technique for determining the average shear rate and separation coefficient was similar to the technique used in the case of differences in particle size. For a comparative assessment of the effect of momentum action in the separation process, the dependence $K_s(du/dy)$ was constructed taking into account the separation coefficient and the average value of the shear rate in the gravity flow without momentum action. These indicators are determined in full accordance with the procedure used to find them in the gravity flow of particles of various sizes (Fig. 4).

The obtained dependence $K_s(du/dy)$ for particles differing in density (Fig. 9) is fundamentally different from the similar dependence for particles of different sizes (Fig. 4). In the studied range of pulse impact intensity and corresponding shear rates, the values of the separation coefficient exceed the value of the coefficient in the gravity flow without momentum impact. The $K_s(du/dy)$ dependence reveals an extreme value of the separation coefficient corresponding to an average shear rate of 65–70 s^{-1}. If the physical mechanism of the increase in the separation coefficient with an increase in the intensity of the momentum effect was previously explained by the increase in the driving force of the effects of quasi-diffusion separation and segregation, then the mechanism for the decrease in the coefficient after reaching a certain extreme value requires additional analysis and explanation.

A hypothetical explanation for the decrease in the separation coefficient with increasing momentum action may be an increase in the volume of flow with a high temperature of the granular medium associated with a deepening of the zone of intense shear deformation. As the zone deepens, the above-located rarefied flow region expands, in which, due to small values of the gradient of the volume fraction of voids, the role of quasi-diffusion mixing increases. Intense quasi-diffusion mixing of particles in a rarefied region of the flow is facilitated by the high diffusion permeability of the medium, proportional to the average distance between particles and the velocity of their chaotic movements [31].

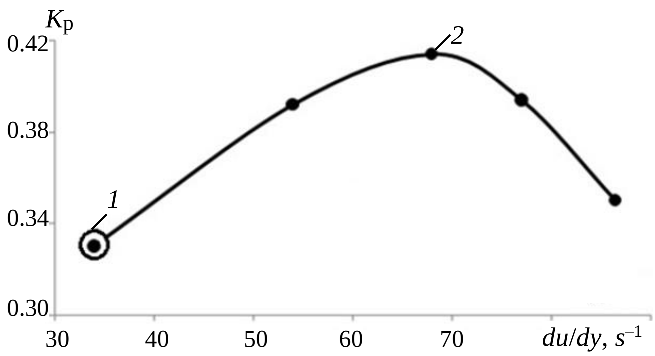

Fig. 9. Dynamics of changes in the separation coefficient of a mixture of monodisperse particles (+3.6–3.75 mm) of beads and silica gel depending on the shear rate in gravity flows: *1* – in the absence of additional momentum action; *2* – in activated gravity flows

It is important to note the similar nature of the dependences of the separation efficiency indicators on the shear rate presented in Figs. 1 and 9. A fairly high degree of analogy of the dependences occurs despite the fundamental difference in the objects to which they relate. In a virtual flow (Fig. 1), the shear rate is uniform throughout the volume of the layer and the particles differ in size, but in a real activated flow (Fig. 9) there is a high heterogeneity of the shear rate and separation of particles by density. In the first case, particle separation occurs predominantly under the influence of the segregation effect, and in the second case, quasi-diffusion separation plays a decisive role. At the same time, the dependences of the efficiency indicators reveal an extreme value of efficiency at close average values of the shear rate (80 and 70 s^{-1}). Taking into account that the physical nature of the decrease in separation efficiency in the cases under consideration is obviously general and is associated with the suppression of separation effects by quasi-diffusion mixing, we can assume the existence of a threshold average value of the shear rate in the gravity flow (70–80 s^{-1}). As the specified shear rate and the corresponding dilatancy of the medium are exceeded, the rate of particle fluctuations and their free path increase so much that quasi-diffusion mixing of particles begins to suppress separation effects.

Thus, the findings made it possible to obtain the information necessary to theoretically substantiate the feasibility of applying longitudinal momentum to the fast gravity flow of the granular material to increase separation efficiency. The feasibility of using longitudinal momentum should be justified taking into account the option of combining the direction of segregation and quasi-diffusion separation fluxes with a certain combination of properties of nonuniform particles.

In order to analytically confirm the reliability of the conclusions based on the results of the experimental study, mathematical modeling of the dynamics of the separation process in the activated gravity flow (belt speed 1.5 $m \cdot s^{-1}$) of beads and silica gel granules of uniform size was carried out. The modeling method and the conditions for its implementation are described in Section 2. Figure 10 shows the concentration distribution profiles of low-density particles in the activated gravity flow. In order to provide conditions for a comparative analysis of separation effects and identify areas of their dominance, modeling was carried out for options with and without taking into account the effect of quasi-diffusion separation. To assess the adequacy of the modeling results, the latter are presented in comparison with the experimentally obtained concentration distribution profile. The velocity and volume fraction of voids profiles presented in Figs. 5, 6 were used as initial data for modeling. The adequacy of the modeling results was established by comparing the variances of adequacy and reproducibility using Fisher's F test at a significance level of 5 %.

The analysis of concentration profiles shows that the presence of a zone with a high concentration gradient in the central part of the flow, characteristic of the experimental profile, is detected only when modeling taking into account the flow of quasi-diffusion separation ($D_m \neq 0$).

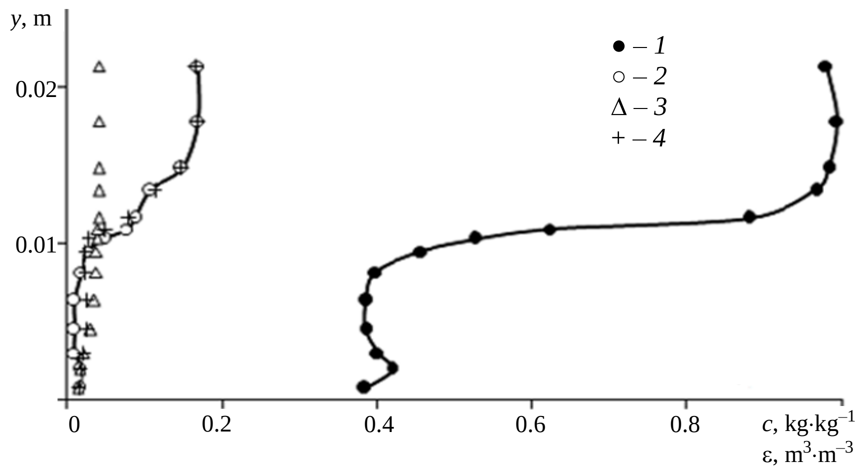

Fig. 10. Profiles of the volume fraction of voids (1) and the concentration of silica gel granules (2) – (4) in the activated (belt speed 1.5 $m \cdot s^{-1}$) fast gravity flow in a mixture with beads:
1, 2 – experimental results; *3* – simulation results based on (1) at $D_m = 0$; $K \neq 0$;
4 – simulation results based on (1) at $D_m \neq 0$, $K \neq 0$

The coordinates of the location of the zone in the bed completely coincide with the coordinates of extremely large gradients of the volume fraction of voids in the flow, which are a condition for ensuring high values of the driving force of quasi-diffusion separation. The combination of the listed features of the profiles presented in Figure 10 indicates not only the dominant role of the effect of quasi-diffusion separation, but also indicates the determining importance of the factor in the formation of a zone of intense shear deformation in the distribution of nonuniform particles.

4. Conclusion

An experimental and analytical study was carried out on the effectiveness of particle separation by size and density when activating shear deformations in fast gravity flows of granular materials. Activation is carried out by imparting to the particles on the open surface a flow of additional momentum in the direction of the slope using a rough conveyor belt. It was established that under the influence of momentum in the central part of the flow a zone with extremely high values of shear rate, gradient of the void fraction and, as a consequence, the intensity of the quasi-diffusion separation effect is formed. Below the zone there is a flow region in which the momentum action is accompanied by a decrease in the proportion of voids and an increase in the shear rate with a corresponding intensification of particle size segregation. In the flow region located above the zone of intense shear, the momentum effect leads to an increase in the proportion of voids and the temperature of the granular medium and, as a consequence, to the intensification of the quasi-diffusion effects of mixing and separation.

Momentum action increases separation efficiency if the direction of quasi-diffusion separation and segregation fluxes for the test component coincides (for example, density separation). Momentum action reduces the efficiency of particle separation by size, for which the direction of the said fluxes is opposite. With increasing shear rate, the zone of intense shear deepens, and the efficiency of density separation increases reaching its maximum value when the zone deepens by 0.5–0.55 of the layer thickness. With a further increase in the intensity of the momentum action, the separation efficiency decreases due to the expansion of the flow region in which the effect of quasi-diffusion mixing dominates. The results of mathematical modeling confirm the decisive role of the quasi-diffusion separation effect in the zone of intense shear of the activated gravity flow in the process of particle separation by density.

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6. Conflict of interests

The authors declare no conflict of interest.

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