UDC 37.015.31
DOI: 10.23951/2782-2575-2022-1-75-83

ADDRESSING DIFFICULTIES IN TEACHING MATHEMATICS TO STUDENTS WITH MATHEMATICAL LEARNING DISABILITIES*

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Abstract. The essence and reason for the inability to master mathematics are described as a lack of working memory. This paper describes two main approaches to solving the problem of teaching mathematics to students with learning difficulties in mathematics (MLD): (1) training working memory and (2) reducing the load on working memory in the instructional process. It was found that the results of the first approach are ambiguous: Training working memory leads to its improvement, which is confirmed by the test results but may not lead to improvement of the mathematical learning process associated with the student's working memory. This justifies the primacy of the second approach. Both previously known methods for reducing the load on working memory in mathematics instruction are presented. A computer-based mathematics learning system developed by the author aims at automating basic computational skills (arithmetics, trigonometry, geometry). It is explained how to work with the developed computer-assisted learning system, which is based on the method of interval repetitions, and empirical data on the results of the system implementation are given.

Keywords: *mathematical skills, mathematics education, working memory, computer-assisted mathematics learning, interval repetitions, learning anxiety in mathematics.*

School children can be divided into Very Capable, Capable, Average, and Incapable according to the degree of development of their mathematical abilities [1, p. 191]. How does one deal with the inability to study mathematics? In the world of the digital economy, one cannot succeed in many fields without basic mathematical skills. After understanding the reason for the inability to learn mathematics, such teaching methods should be developed so that children with poor academic performance can get out of the vicious circle of failure and gain potential for growth and development.

The essence of mathematical learning disability

The differences in mathematical abilities of different students are reflected in the fact that "the same exercises, the same practical work, the same experience (as far as this Sameness can be judged) leads to different results in students with different abilities. For example, in some pupils (we call them Capable), these exercises lead to the development of the ability to see analytically and synthetically; in others, this is not the case" [1, pp. 252–253].

About students who are unable to learn mathematics, Krutetsky [1] states, "The study of mathematics is challenging for such students, despite their diligence; they cannot expect great success in mathematics, both in terms of speed of progress and level of achievement [1, p. 189].

* *Original Russian language version of the article:* Pigarev A.Yu. Metodicheskiye aspekty obycheniya matematike nesposobnykh shkol'nikov [Methodological Aspects of Teaching Mathematics to Incapable Students]. *Nauchno-pedagogicheskoye obozreniye – Pedagogical Review*, 2020, vol. 1 (29), pp. 37–45 (in Russian). DOI: 10.23951/2307-6127-2020-1-37-45

"Their mathematical skills are formed laboriously, with a large number of exercises, and are fragile-they fall apart easily when there are no exercises [1, p. 190]."

As for the age at which one can speak about the presence or absence of mathematical abilities, Krutetsky [1] says: "Mathematical abilities and their absence show up in the grades VI–VII in connection with the beginning of the systematic study of algebra and geometry. Furthermore, in grades IX–X, the abilities are completely reviled [1, pp. 101–102]."

Causes of the inability to study mathematics

Krutetsky in [1] did not disclose the reason for the inability to study mathematics. He only gives the following explanation: "...the inability to study mathematics has its root in a great difficulty in the selection of stimuli by the brain, such as generalized mathematical relations, functional dependencies, numerical abstractions and symbols, and the difficulty of working with them [1, p. 390]."

Within the framework of the computer metaphor, the concept of human working memory emerged, whose founder was Alan Baddeley, at the end of the twentieth century [2].

Working memory is a system that combines and transforms a limited amount of information it receives from the perceptual organs, short-term memory, and long-term memory. Working memory supports thinking processes; it is an interface between short-term and long-term memory and provides both the consolidation of long-term memory and the extraction of information from it [2].

According to the basic model proposed by Alan Baddeley [2], working memory consists of three main components, the "central executive, which acts as a supervisory system and controls the flow of information to and from subordinate systems: the phonological loop and the visuospatial sketchpad. The phonological loop stores verbal content, while the visuospatial sketchpad is responsible for visuospatial data. Both slave systems function only as short-term storage centers." (Fig. 1).

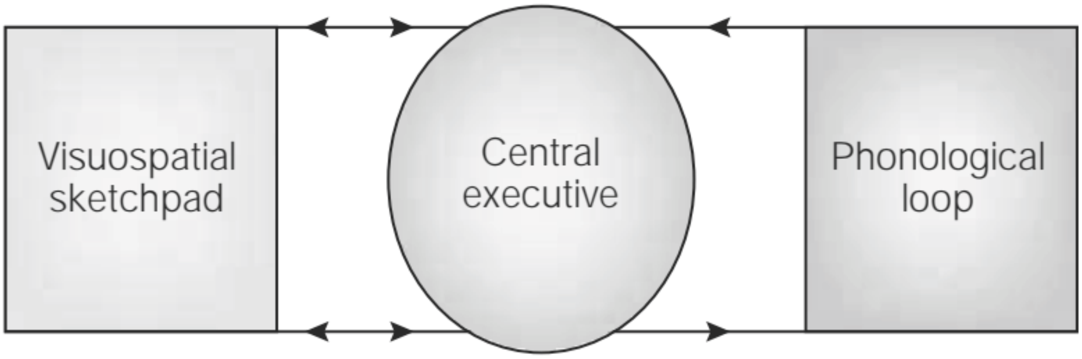

Fig. 1. A three-component model of working memory [2]

Later, a fourth subordinate system, an episodic buffer, was added to the model. It is responsible for the interaction of the visuospatial sketchpad and the phonological loop with long-term memory [2].

The extent of working memory was highly correlated with instructional outcomes, especially in mathematics subjects: "...memory skills clearly predicted mathematical skills and arithmetic skills [3, p.136]." This conclusion has been confirmed by many other studies, for example [4–7].

Thus, the lack of working memory is the cause of the inability to learn mathematics. All three components of the basic working memory model are essential for understanding mathematics. Mathematical ability is most strongly correlated with the visuospatial sketchpad, which is like a mental board on which numbers and variables are represented in calculation and comparison [4, 5]. The central executive and phonological loops determine the ability to solve verbal mathematical problems [6].

Based on the concept of working memory, two approaches to teaching mathematics to students with learning difficulties in mathematics have been developed.

Basic approaches to solving the problem of teaching mathematics to students with learning difficulties in mathematics

The first approach is to improve the student's working memory by exercising it or by seeking medical treatment if the deficit in working memory is some kind of injury, such as a concussion.

As for improving working memory with the help of a computer-assisted learning system, the initial results were auspicious, but later studies have called them into question.

The first work appeared in 2008 [8]. The authors argued that training a person's working memory with a computer-assisted learning system built based on an "n-back" task in a relatively short period (10–20 days) can not only increase the efficiency of a person's working memory but also develop the ability to learn and assimilate new knowledge. The authors "provided evidence for the transfer of working memory training results to the development of fluid intelligence." [8] According to their data, the more and the longer the training sessions were, the more significant was the improvement in fluid intelligence, which was determined by means of special tests.

This conclusion was refuted in [9, 2013]. Working memory training was based on the Cogmed platform (https://www.cogmed.com/). The training cycle consisted of 20–25 sessions. Each session lasted from 30 to 45 minutes and included eight exercises, and each exercise was repeated 15 times. The founder of this company, Professor Torkel Klingberg (M.D., Ph.D. Professor in Cognitive Neuroscience Karolinska Institutet), has published sixty-two articles and two popular science books on the benefits of the platform he created (http://www.klingberglab.se/torkel-klingberg/).

Working memory was assessed with the standardized test battery AWMA (Automated Working Memory Assessment) [9]. One of the authors created this test system in [9] Susan E. Gathercole. However, the Cogmed training exercises differ in content from the AWMA test system tasks, so the authors of [9] argue that working memory training improves working memory performance, as measured by "untrained" tests: "adaptive W.M. training significantly boosted performance on untrained W.M. tasks in children with low W.M. [9]".

Learning ability was measured with two test batteries: the Wechsler Abbreviated Scales of Intelligence (WASI) and the Kaufman Test of Educational Attainment (KTEA). Improving working memory performance based on the results of taking "untrained" tests did not improve learning performance based on the results of taking intelligence tests, leading the authors of [9] to conclude that working memory training is useless as a tool for improving academic performance. This study involved 810 schoolchildren.

On the other hand, the positive effect of training working memory on performance in mathematical disciplines is confirmed in [10, 2018]. In this study, 104 schoolchildren participated.

Schoolchildren in the same age group (8–13 years old) participated in both studies. However, the study conditions were different. In the experiment presented in [9], the training was carried out in a special laboratory; in [10], it took place in the school where the children were learning. In addition, the training programs were also different. In the study [10], there were also exercises to develop computational skills in addition to the classic working memory simulators.

Currently, specific working memory training benefits are still open and controversial. Many papers claim the benefits of such training, and even more papers question the effectiveness of this approach.

The second approach is to reduce the load on working memory during the learning process. When the load on working memory exceeds its limits, the student cannot process new material.

Methods for reducing the load on working memory in math class

1. Automate basic math skills (arithmetic, trigonometry, geometry) so that the computational process puts minimal load on working memory.

2. Use simple sentences when presenting new material, as compound and complex sentences overload weak working memory [11].

3. Keep statements and instructions short, containing no more than one "if" condition and relating directly to the current task. For example, one cannot give the following instruction until the previous instructions have been solved by all students in the class [11].

4. The reference materials for solving tasks should be at the level of the textbook on which the task is solved: on a tablet or smartphone, in printed form. Because if you look from a distant object (blackboard) to a close one (notebook), working memory can fail [11].

5. Mathematical facts (formulas, rules) frequently used to solve problems must be memorized because retrieving well-remembered information from long-term memory does not tax working memory, e.g., reduction formulas. There is a mnemonic rule that stresses working memory. Furthermore, if one memorizes frequently used formulas ($F(\pi \pm x)$, $F(\pi/2 \pm x)$), such a load can be eliminated [12].

6. Minimize the intensity of the new information flow by redistributing it over time. For example, teach a method for solving irrational equations in class, indicate that the solutions obtained must be checked by substitution, and postpone the justification for extra roots and the solution of irrational inequalities until the next lesson. Thus, extend the study of rules and exceptions as the lesson progresses [12].

7. Visualize the material in conceptual diagrams, charts, and mind maps. This reduces the load on working memory and provides a deeper awareness of the new information, leading to better retention [12].

8. Use mnemonics to anchor new information in long-term memory. Although mnemonics are not as commonly used in mathematics as in the humanities, they should not be neglected. For example, the Horse Rule in trigonometry combines the condition of replacing a function with a co-function using commonly accepted head movements to express agreement or disagreement [12]. Use a computer-assisted learning system.

A necessary condition for teaching the basics of elementary mathematics to students with MLD is individualization and a specially developed methodology: "A detailed study (during the year) of a group of MLD students has again convincingly shown that they have only an elementary level of generalization of the mathematical material available to them in the process of independent work. A higher degree of generalization (necessary for a satisfactory mastery of mathematics) came about only gradually due to very hard work and the direct help of the experimenter. In a number of cases, such generalization came about only as a result of work specially organized by the experimenter. In difficult cases, no generalization occurred at all." [1, p. 274–275]

As a result of his many years of research on mathematical ability, Krutetsky [1] concludes that "Students with mathematical learning problems need to be trained for a long time, with specially selected material covering all possible cases and combinations of insignificant features, so that a more or less elementary level of generalization is available to them" [1, pp. 258–259].

Prolonged training on specially selected exercises makes it possible to overcome the objective difficulties in learning mathematics for students with MLD since the automation of basic computational skills (arithmetic, trigonometry, geometry) reduces the load on working memory in solving relevant problems and frees its resources for understanding the conditions and searching for algorithms.

Computer programs can become a generator of such specially selected exercises and, at the same time, a tutor who suggests the order in which they should be performed and evaluates the result of the work. The author has developed a computer-assisted learning system available at http://www.workingmemory.ru/.

Figures 2–5 show screenshots, and Table 1 provides a brief description. In addition, a link to a video describing the methodology for working with the training exercises can be found on the main page of the following website: http://www.workingmemory.ru/.

Registration is free to get an objective picture of the work with the training exercises.

Fig. 2. Screenshot of the training exercise "Mental counting." Optimized for mobile phones

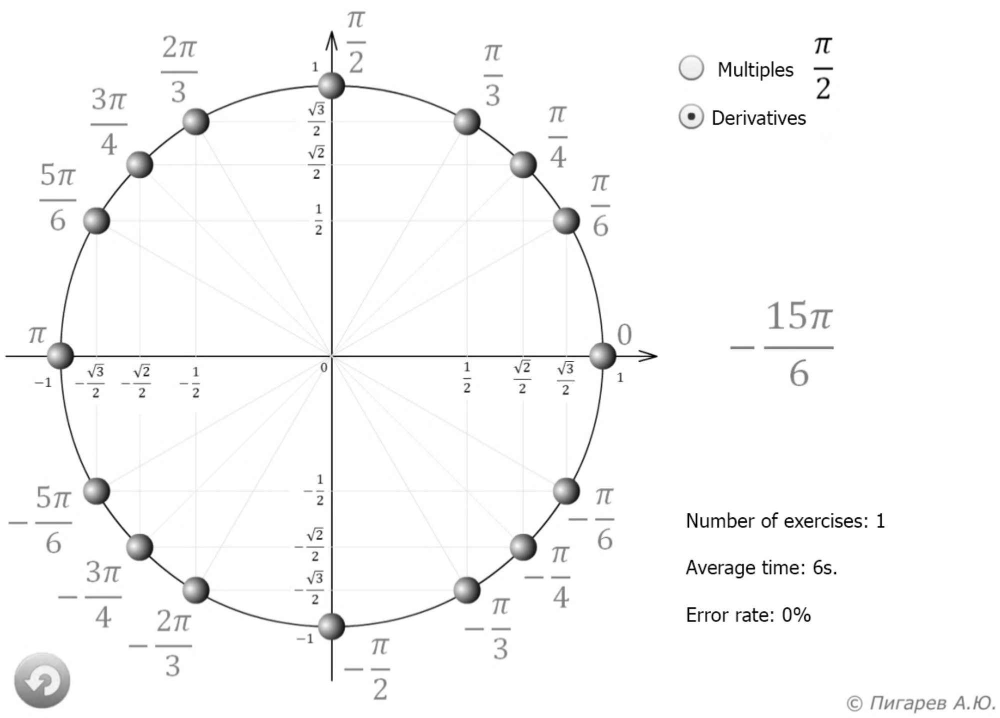

Fig. 3. Screenshot of the training exercise Unit Circle

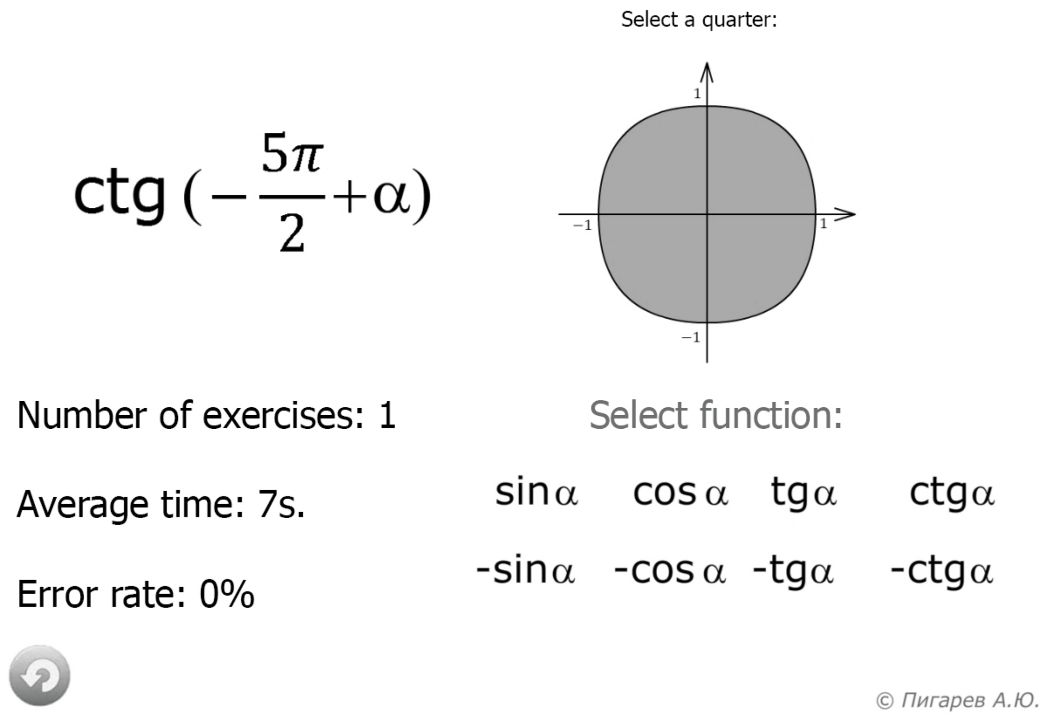

Fig. 4. Screenshot of the training exercise Reduction Formulas

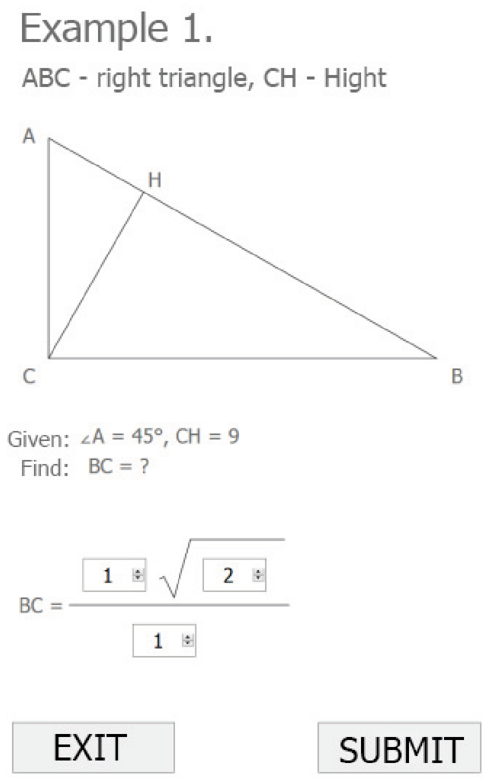

Fig. 5. Screenshot of the training exercise Right Triangle. Optimized for cell phones

Table 1

Brief description of computer-assisted training exercises published at: http://www.workingmemory.ru/

Training exercise name	Description	Numbers of tasks from the Unified State Exam in Mathematics (Russia, 2020)	Grades
Mental Counting	Develops mental counting skills, including the use of various methods and computational procedures	1–19	2–11
Unit Circle	Develops the ability to quickly find a point on a unit circle for a given numerical value in radians or degrees	5, 9, 10, 13	10–11
Reduction Formulas	Develops the ability to use reduction formulas without error when solving trigonometric equations and inequalities	5, 9, 13	10–11
Right Triangle	Develops the ability to automatically and accurately determine the relationship between the sides of a right triangle using the values of the trigonometric functions of the acute angles	6, 14, 16	8–11

Methodology for training exercises

1. Consistent study of the basic rules for performing arithmetic operations (arithmetic, trigonometric, or geometric).

2. Twenty minutes of daily practice for one month. One can see a positive dynamic by tracking the average practice time and error rate. The student subjectively feels that the arithmetic problems are solved easier, faster, and without errors.

3. After a one-month break, the training is repeated to consolidate computational skills in long-term memory.

The experiment participants were selected from the students of the preparatory course for the Unified State Examination in Mathematics (grades 10–11). Each participant was given a unique login and password. The results of each training exercise were stored as a separate dataset in the server database. The dataset contains a unique student login. At the time of writing, 31 students participated in the experiment.

As a result of the training, the average time to complete an exercise decreased significantly (Table 2), and the frequency of errors also decreased (Table 3). The decrease in the time for one exercise indicates a lower load on working memory, and the decrease in error frequency indicates a more efficient use of its resources.

The effect of the training is characterized by the feedback of the students themselves, both during the course: “Thanks to the training exercises, I got an excellent grade in school because I solved the problems in trigonometry optimally,” and after passing the exam in mathematics: “I did not make a single mistake in the second part of the exam...” These training exercises are not only a tool for teaching school children with MLD but also know-how for successful preparation for a state exam in mathematics.

Table 2

Average time in seconds to complete a training exercise considering the standard deviation

Training exercise	At the beginning of the training course	After the training course
Unit Circle	35 ± 20	13 ± 11
Reduction Formulas	33 ± 17	21 ±5
Mental Counting. Exercise in which one of the numbers is one-digit, and the other is two-digit	18 ± 7	7 ± 3
Mental Counting. Exercise on multiplication of two-digit numbers in the head.	57 ± 27	20 ± 11

Table 3

The percentage of errors in the number of correctly performed exercises considering the standard deviation (%)

Training exercise	At the beginning of the training course	After the training course
Unit Circle	54 ± 48	10 ± 8
Reduction Formulas	52± 32	29 ± 14

The thresholds of the indicators for working with the training exercises depend on the working memory and effort of the student. Therefore, the average deviations of the indicators are comparable to the average values in Tables 2 and 3.

The presence of time and computational error counters in the training exercises puts the student's working memory in the maximum effort mode, which indirectly contributes to its improvement through training. This effect was not quantified because the students' working memory was not measured. However, it should be considered as one of the arguments for the benefits of using such training exercises in the teaching process.

In any approach, it is important to consider the psycho-emotional state of the student. For example, poor academic performance and unsatisfactory grades can lead to Math Anxiety. Math anxiety is the fear of learning math and solving math problems. It is a feeling of tension associated with working with numbers and solving math problems in school or everyday life. Math anxiety develops in students at all grade levels because of negative past or present experiences. It is a lack of confidence in working with numbers and mathematical structures [13, pp. 166–167].

Anxiety of any kind leads to the release of the hormone cortisol in the blood. The primary function of cortisol is to direct the mind to the source of the anxiety and determine actions that will relieve the stress; pulse and blood pressure increase; the frontal lobes are no longer engaged in the study of mathematics or mathematical operations because they are preoccupied with a threat to personal safety. As a result, the student can no longer concentrate on learning and struggles with attention deficit disorder. The most crucial point is that anxiety affects the working memory's ability to store numbers in short-term memory and manipulate numerical expressions.

To reduce the level of anxiety, the teacher should provide individualized support to students: (1) develop in them a sense of confidence in dealing with numbers through a computerized training system and individually selected tasks appropriate to their level; (2) maintain a sense of accomplishment through grades that reflect the student's effort rather than their absolute level in mathematics; (3) encourage the development of a positive attitude toward mathematics as one of the greatest achievements of civilization.

Conclusion

The best results in teaching mathematics to students with mathematical learning disabilities can be achieved if we avoid overloading their working memory and encourage their positive attitude toward mathematics as a subject and as one of the greatest achievements of civilization. One method is to develop their computational skills using a computer-assisted training system developed by the author. This increases the efficiency of using working memory and frees up its resources for mastering problem-solving methods and absorbing new knowledge. Thus, the performance of students with mathematical learning disabilities in learning the basic course of school mathematics increases.

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